1. Números complexos na forma algébrica

Páginas 3 a 5

1.

1.1 $z = 2 + 3i$
$\text{Re}(z) = 2; \text{Im}(z) = 3$

1.2 $z = -3 + 5i$
$\text{Re}(z) = -3; \text{Im}(z) = 5$

1.3 $z = 1 - i$
$\text{Re}(z) = 1; \text{Im}(z) = -1$

1.4 $z = -2 - 4i$
$\text{Re}(z) = -2; \text{Im}(z) = -4$

1.5 $z = 5$
$\text{Re}(z) = 5; \text{Im}(z) = 0$

1.6 $z = 6i$
$\text{Re}(z) = 0; \text{Im}(z) = 6$

2.

2.1 $x^2 + 25 = 0 \Leftrightarrow x^2 = -25$
$\Leftrightarrow x = \pm\sqrt{-25}$
$\Leftrightarrow x = 5i \vee x = -5i$
C. S. $= \{5i, -5i\}$

2.2 $x^2 + 6x + 10 = 0 \Leftrightarrow x = \frac{-6 \pm \sqrt{6^2 - 4\times 10}}{2}$
$\Leftrightarrow x = \frac{-6 \pm \sqrt{-4}}{2}$
$\Leftrightarrow x = \frac{-6 \pm 2i}{2}$
$\Leftrightarrow x = -3 + i \vee x = -3 - i$
C. S. $= \{-3 + i, -3 - i\}$

2.3 $x^2 + 4x + 13 = 0 \Leftrightarrow x = \frac{-4 \pm \sqrt{4^2 - 4\times 13}}{2}$
$\Leftrightarrow x = \frac{-4 \pm \sqrt{-36}}{2}$
$\Leftrightarrow x = \frac{-4 \pm 6i}{2}$
$\Leftrightarrow x = -2 + 3i \vee x = -2 - 3i$
C. S. $= \{-2 + 3i, -2 - 3i\}$

2.4 $3x^2 - 6x + 15 = 0 \Leftrightarrow x^2 - 2x + 5 = 0$
$\Leftrightarrow x = \frac{2 \pm \sqrt{2^2 - 4\times 5}}{2}$
$\Leftrightarrow x = \frac{2 \pm \sqrt{-16}}{2}$
$\Leftrightarrow x = \frac{2 \pm 4i}{2}$
$\Leftrightarrow x = 1 + 2i \vee x = 1 - 2i$
C. S. $= \{1 + 2i, 1 - 2i\}$

2.5 $2x^2 + 3x + \frac{9}{4} = 0 \Leftrightarrow x = \frac{-3 \pm \sqrt{3^2 - 4\times 2\times \frac{9}{4}}}{2\times 2}$
$\Leftrightarrow x = \frac{-3 \pm \sqrt{-9}}{4}$
$\Leftrightarrow x = \frac{-3 \pm 3i}{4}$
$\Leftrightarrow x = -\frac{3}{4} + \frac{3}{4}i \vee x = -\frac{3}{4} - \frac{3}{4}i$
C. S. $= \left\{-\frac{3}{4} - \frac{3}{4}i, -\frac{3}{4} + \frac{3}{4}i\right\}$

2.6 $x^2 - 3x + 4 = 0 \Leftrightarrow x = \frac{3 \pm \sqrt{3^2 - 4\times 4}}{2}$
$\Leftrightarrow x = \frac{3 \pm \sqrt{-7}}{2}$
$\Leftrightarrow x = \frac{3}{2} + \frac{\sqrt{7}}{2}i \vee x = \frac{3}{2} - \frac{\sqrt{7}}{2}i$
C. S. $= \left\{\frac{3}{2} + \frac{\sqrt{7}}{2}i, \frac{3}{2} - \frac{\sqrt{7}}{2}i\right\}$

3.

3.1 $z^2 + 49 = 0 \Leftrightarrow z^2 = -49$
$\Leftrightarrow z = \pm\sqrt{-49}$
$\Leftrightarrow z = 7i \vee z = -7i$
C. S. $= \{7i, -7i\}$

3.2 $z^2 - 6z = -13 \Leftrightarrow z^2 - 6z + 13 = 0$
$\Leftrightarrow z = \frac{6 \pm \sqrt{6^2 - 4\times 13}}{2}$
$\Leftrightarrow z = \frac{6 \pm \sqrt{-1}}{2}$
$\Leftrightarrow z = \frac{6 \pm 4i}{2}$
$\Leftrightarrow z = 3 + 2i \vee z = 3 - 2i$
C. S. $= \{3 + 2i; 3 - 2i\}$

3.3 $z^3 + 2z^2 + 17z = 0$
$\Leftrightarrow z(z^2 + 2z + 17) = 0$
$\Leftrightarrow z = 0 \vee z^2 + 2z + 17 = 0$
$\Leftrightarrow z = 0 \vee z = \frac{-2 \pm \sqrt{2^2 - 4\times 17}}{2}$
$\Leftrightarrow z = 0 \vee z = \frac{-2 \pm \sqrt{-6}}{2}$
$\Leftrightarrow z = 0 \vee z = \frac{-2 \pm 8i}{2}$
$\Leftrightarrow z = 0 \vee z = -1 + 4i \vee z = -1 - 4i$
C. S. $= \{0, -1 + 4i, -1 - 4i\}$

3.4 $z^3 + 3z^2 + z - 5 = 0$
Sabe-se que 1 é uma das soluções da equação.
Recorrendo à regra de Ruffini, obtém-se:

	1	3	1	−5
1		1	4	5
	1	4	5	0

$z^3 + 3z^2 + z - 5 = 0$
$\Leftrightarrow z = 1 \vee z^2 + 4z + 5 = 0$
$\Leftrightarrow z = 1 \vee z = \frac{-4 \pm \sqrt{4^2 - 4\times 5}}{2}$
$\Leftrightarrow z = 1 \vee z = \frac{-4 \pm \sqrt{-4}}{2}$
$\Leftrightarrow z = 1 \vee z = \frac{-4 \pm 2i}{2}$
$\Leftrightarrow z = 1 \vee z = -2 + i \vee z = -2 - i$
C. S. $= \{1, -2 + i, -2 - i\}$

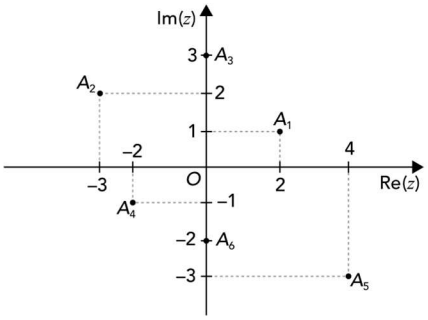
3.5 $z^4 - 1 = 0$
$\Leftrightarrow (z^2 - 1)(z^2 + 1) = 0$
$\Leftrightarrow z^2 - 1 = 0 \vee z^2 + 1 = 0$
$\Leftrightarrow z^2 = 1 \vee z^2 = -1$
$\Leftrightarrow z = 1 \vee z = -1 \vee z = i \vee z = -i$
C. S. $= \{1, -1, i, -i\}$

3.6 $z^4 + 17z^2 + 16 = 0$
Sabe-se que i e $-i$ são soluções da equação.
Recorrendo à regra de Ruffini, obtém-se:

	1	0	17	0	16
i		i	-1	$16i$	-16
	1	i	16	$16i$	0
$-i$		$-i$	0	$-16i$	
	1	0	16	0	

$z^4 + 17z^2 + 16 = 0$
$\Leftrightarrow z = i \vee z = -i \vee z^2 + 16 = 0$
$\Leftrightarrow z = i \vee z = -i \vee z^2 = -16$
$\Leftrightarrow z = i \vee z = -i \vee z = 4i \vee z = -4i$
C. S. $= \{i, -i, 4i, -4i\}$

4. Sejam A_1, A_2, A_3, A_4, A_5 e A_6 os afixos de z_1, z_2, z_3, z_4, z_5 e z_6, respetivamente.



4.1 $|z_1| = \sqrt{2^2 + 1^2} = \sqrt{5}$
4.2 $|z_2| = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$
4.3 $|z_3| = \sqrt{0^2 + 3^2} = 3$
4.4 $|z_4| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$
4.5 $|z_5| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$
4.6 $|z_6| = \sqrt{0^2 + (-2)^2} = 2$

5. $\overline{z_1} = 2 - i$
$-z_1 = -2 - i$
$\frac{1}{z_1} = \frac{1}{2+i} = \frac{1}{2+i} \times \frac{2-i}{2-i} = \frac{2-i}{2^2+1^2} = \frac{2}{5} - \frac{1}{5}i$
$\overline{z_2} = -3 - 2i$
$-z_2 = 3 - 2i$
$\frac{1}{z_2} = \frac{1}{-3+2i} = \frac{1}{-3+2i} \times \frac{-3-2i}{-3-2i} = \frac{-3-2i}{3^2+2^2} =$
$= -\frac{3}{13} - \frac{2}{13}i$
$\overline{z_3} = -3i$
$-z_3 = -3i$
$\frac{1}{z_3} = \frac{1}{3i} = \frac{1}{3i} \times \frac{-3i}{-3i} = -\frac{3i}{3^2} = -\frac{1}{3}i$
$\overline{z_4} = -2 + i$
$-z_4 = 2 + i$
$\frac{1}{z_4} = \frac{1}{-2-i} = \frac{1}{-2-i} \times \frac{-2+i}{-2+i} = \frac{-2+i}{2^2+1^2} = -\frac{2}{5} + \frac{1}{5}i$
$\overline{z_5} = 4 + 3i$
$-z_5 = -4 + 3i$
$\frac{1}{z_5} = \frac{1}{4-3i} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i} = \frac{4+3i}{4^2+3^2} = \frac{4}{25} + \frac{3}{25}i$
$\overline{z_6} = 2i$
$-z_6 = 2i$
$\frac{1}{z_6} = \frac{1}{-2i} = \frac{1}{-2i} \times \frac{2i}{2i} = \frac{2i}{2^2} = \frac{1}{2}i$

6. $z_1 = z_2 \Leftrightarrow 2 + (3 - a)i = 3 + b - 2i$
$\Leftrightarrow 2 = 3 + b \wedge 3 - a = -2$
$\Leftrightarrow b = -1 \wedge a = 5$
Assim, $a = 5$ e $b = -1$.

7.
7.1 $zw = (1 + 2i) \times (-2 + i) =$
$= -2 + i - 4i + 2i^2 =$
$= -4 - 3i$
7.2 $2z - 3w = 2(1 + 2i) - 3(-2 + i) =$
$= 2 + 4i + 6 - 3i =$
$= 8 + i$
7.3 $w^3 = (-2 + i)^3 =$
$= (-2 + i)^2 \times (-2 + i) =$
$= (4 - 4i + i^2) \times (-2 + i) =$
$= (3 - 4i) \times (-2 + i) =$
$= -6 + 3i + 8i - 4i^2 =$
$= -2 + 11i$
7.4 $(z + w)(z - w) =$
$= (1 + 2i - 2 + i) \times (1 + 2i + 2 - i) =$
$= (-1 + 3i) \times (3 + i) =$
$= -3 - i + 9i + 3i^2 =$
$= -6 + 8i$
7.5 $(z + w)^2 = (1 + 2i - 2 + i)^2 =$
$= (-1 + 3i)^2 =$
$= 1 - 6i + 9i^2 =$
$= -8 - 6i$
7.6 $(\bar{z} + w)i = (1 - 2i - 2 + i)i =$
$= (-1 - i)i =$
$= -i - i^2 =$
$= 1 - i$
7.7 $z \times \overline{w} + \bar{z} = (1 + 2i)(-2 - i) + 1 - 2i =$
$= -2 - i - 4i - 2i^2 + 1 - 2i =$
$= 1 - 7i$
7.8 $z^2 + \bar{z} \times \overline{w} =$
$= (1 + 2i)^2 + (1 - 2i) \times (-2 - i) =$
$= 1 + 4i + 4i^2 - 2 - i + 4i + 2i^2 =$
$= -7 + 7i$
7.9 $\frac{z}{w} = \frac{1 + 2i}{-2 + i} =$
$= \frac{1 + 2i}{-2 + i} \times \frac{-2 - i}{-2 - i} =$
$= \frac{-2 - i - 4i - 2i^2}{(-2)^2 - i^2} =$
$= \frac{-5i}{4 + 1} =$
$= -i$
7.10 $\frac{w}{z} = \frac{-2 + i}{1 + 2i} =$
$= \frac{-2 + i}{1 + 2i} \times \frac{1 - 2i}{1 - 2i} =$
$= \frac{-2 + 4i + i - 2i^2}{1^2 - (2i)^2} =$
$= \frac{5i}{1 + 4} =$
$= i$
7.11 $\frac{1 - i}{-z} = \frac{1 - i}{-1 - 2i} =$
$= \frac{1 - i}{-1 - 2i} \times \frac{-1 + 2i}{-1 + 2i} =$
$= \frac{-1 + 2i + i - 2i^2}{(-1)^2 - (2i)^2} =$
$= \frac{1 + 3i}{1 + 4} =$
$= \frac{1}{5} + \frac{3}{5}i$

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7.12 $\frac{\bar{w}}{1+i} = \frac{-2-i}{1+i} =$
$= \frac{-2-i}{1+i} \times \frac{1-i}{1-i} =$
$= \frac{-2+2i-i+i^2}{1^2-i^2} =$
$= \frac{-3+i}{1+1} =$
$= -\frac{3}{2} + \frac{1}{2}i$

8.

8.1 $(3+2i)+(1-i) = 3+2i+1-i =$
$= 4+i$

8.2 $(1+2i)-(4-3i) = 1+2i-4+3i =$
$= -3+5i$

8.3 $3(-2-i)-i(2+i) = -6-3i-2i-i^2 =$
$= -5-5i$

8.4 $(2+5i)(3+4i) = 6+8i+15i+20i^2 =$
$= -14+23i$

8.5 $(-1+5i)(-1-5i) = (-1)^2-(5i)^2 =$
$= 1+25 =$
$= 26$

8.6 $(3-i)+(2-i)(-1+2i) =$
$= 3-i-2+4i+i-2i^2 =$
$= 3+4i$

8.7 $\frac{2+5i}{3+4i} = \frac{2+5i}{3+4i} \times \frac{3-4i}{3-4i} =$
$= \frac{6-8i+15-20i^2}{3^2-(4i)^2} =$
$= \frac{26+7i}{9+16} =$
$= \frac{26}{25} + \frac{7}{25}i$

8.8 $\frac{-1+5i}{i} = \frac{-1+5i}{i} \times \frac{-i}{-i} =$
$= \frac{i-5i^2}{1} =$
$= 5+i$

8.9 $\frac{(2-i)(3+2i)}{1+2i} = \frac{6+4i-3i-2i^2}{1+2i} =$
$= \frac{8+i}{1+2i} =$
$= \frac{8+i}{1+2i} \times \frac{1-2i}{1-2i} =$
$= \frac{8-16i+i-2i^2}{1^2-(2i)^2} =$
$= \frac{10-15}{1+4} =$
$= 2-3i$

8.10 $\frac{2-i}{2+i} + \frac{2+i}{2-i} = \frac{(2-i)(2-i)+(2+i)(2+i)}{(2+i)(2-i)} =$
$= \frac{4-4i+i^2+4+4i+i^2}{2^2-i^2} =$
$= \frac{6}{5}$

8.11 $\frac{2-3i}{(1+i)^2} = \frac{2-3i}{1+2i+i^2} =$
$= \frac{2-3i}{2i} =$
$= \frac{2-3i}{2i} \times \frac{-2i}{-2i} =$
$= \frac{-4i+6i^2}{-4i^2} =$
$= \frac{-6-4i}{4} =$
$= -\frac{3}{2} - i$

8.12 $(1+2i)^{-3} = \frac{1}{(1+2i)^3} =$
$= \frac{1}{(1+2i)^2(1+2i)} =$
$= \frac{1}{(1+4i+4i^2)(1+2i)} =$
$= \frac{1}{(-3+4i)(1+2i)} =$
$= \frac{1}{-3-6i+4i+8i^2} =$
$= \frac{1}{-11-2i} =$
$= \frac{1}{-1-2i} \times \frac{-11+2i}{-1+2i} =$
$= \frac{-1+2i}{(-1)^2-(2i)^2} =$
$= \frac{-11+2i}{121+4} =$
$= -\frac{11}{125} + \frac{2}{125}i$

8.13 $i^{10}+i^{20}+i^{30}+i^{40} = i^2+i^0+i^2+i^0 =$
$= -1+1-1+1 =$
$= 0$

8.14 $\frac{1+i^4}{(1+i)^4} = \frac{1+i^0}{((1+i)^2)^2} =$
$= \frac{1+1}{(1+2i+i^2)^2} =$
$= \frac{2}{(2i)^2} =$
$= \frac{2}{-4} =$
$= -\frac{1}{2}$

8.15 $\frac{(2-i)^2-3i^{17}}{i^{23}-2} = \frac{4-4i+i^2-3i^1}{i^3-2} =$
$= \frac{3-7i}{-2-i} =$
$= \frac{3-7i}{-2-i} \times \frac{-2+i}{-2+i} =$
$= \frac{-6+3i+14-7i^2}{(-2)^2-i^2} =$
$= \frac{1+17}{4+1} =$
$= \frac{1}{5} + \frac{17}{5}i$

8.16 $\frac{2i^{38}-3i^{47}+i^{56}}{i^5-3i^{26}} = \frac{2i^2-3i^3+i^0}{i^1-3i^2} =$
$= \frac{-2+3i+1}{i+3} =$
$= \frac{-1+3i}{3+i} =$
$= \frac{-1+3i}{3+i} \times \frac{3-i}{3-i} =$
$= \frac{-3+i+9i-3i^2}{3^2-i^2} =$
$= \frac{10i}{10} =$
$= i$

9. $z = \frac{2-3ki}{3-4i} =$
$= \frac{2-3ki}{3-4i} \times \frac{3+4i}{3+4i} =$
$= \frac{6+8i-9k-12ki^2}{3^2-(4i)^2} =$
$= \frac{6+12k}{25} + \frac{8-9k}{25}i$

9.1 Para que z seja um imaginário puro:
$\text{Re}(z) = 0 \wedge \text{Im}(z) \neq 0$
$\Leftrightarrow \frac{6+12k}{25} = 0 \wedge \frac{8-9k}{25} \neq 0$
$\Leftrightarrow 6+12k = 0 \wedge 8-9k \neq 0$
$\Leftrightarrow k = -\frac{1}{2}$
Logo, o valor de k é $-\frac{1}{2}$.

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9.2 Para que z seja um número real:

$\text{Im}(z) = 0 \Leftrightarrow \frac{8-9k}{25} = 0$

$\Leftrightarrow 8 - 9k = 0$

$\Leftrightarrow k = \frac{8}{9}$

Logo, o valor de k é $\frac{8}{9}$.

9.3 Para que z pertença à bissetriz dos quadrantes ímpares:

$\text{Re}(z) = \text{Im}(z)$

$\Leftrightarrow \frac{6+12k}{25} = \frac{8-9k}{25}$

$\Leftrightarrow 6 + 12k = 8 - 9k$

$\Leftrightarrow 12k + 9k = 8 - 6$

$\Leftrightarrow 21k = 2$

$\Leftrightarrow k = \frac{2}{21}$

Logo, o valor de k é $\frac{2}{21}$.

10.

10.1 $(a-3) + 4bi = 2b + (3a-5)i$

$\Leftrightarrow \begin{cases} a - 3 = 2b \\ 4b = 3a - 5 \end{cases} \Leftrightarrow \begin{cases} a = 2b + 3 \\ 4b = 3(2b+3) - 5 \end{cases}$

$\Leftrightarrow \begin{cases} \\ 4b = 6b + 9 - 5 \end{cases} \Leftrightarrow \begin{cases} \\ -2b = 4 \end{cases}$

$\Leftrightarrow \begin{cases} a = 2 \times (-2) + 3 \\ b = -2 \end{cases} \Leftrightarrow \begin{cases} a = -1 \\ b = -2 \end{cases}$

Assim, o valor de a é -1 e o valor de b é -2.

10.2 $\frac{a+5bi}{1+bi} = 2 - 3i$

$\Leftrightarrow a + 5bi = (2-3i)(1+bi)$

$\Leftrightarrow a + 5bi = 2 + 2bi - 3i - 3bi^2$

$\Leftrightarrow a + 5bi = 2 + 3b + (2b-3)i$

$\Leftrightarrow \begin{cases} a = 2 + 3b \\ 5b = 2b - 3 \end{cases} \Leftrightarrow \begin{cases} \\ 3b = -3 \end{cases}$

$\Leftrightarrow \begin{cases} a = 2 + 3 \times (-1) \\ b = -1 \end{cases} \Leftrightarrow \begin{cases} a = -1 \\ b = -1 \end{cases}$

Assim, o valor de a é -1 e o valor de b é -1.

11.

11.1 $\frac{k+i}{2-i} = \frac{k+i}{2-i} \times \frac{2+i}{2+i} =$

$= \frac{2k + ki + 2i + i^2}{2^2 - i^2} =$

$= \frac{2k-1}{5} + \frac{2+k}{5}i$

Para que $\frac{k+i}{2-i}$ represente um imaginário puro:

$\frac{2k-1}{5} = 0 \wedge \frac{2+k}{5} \neq 0$

$\Leftrightarrow 2k - 1 = 0 \wedge 2 + k \neq 0$

$\Leftrightarrow k = \frac{1}{2}$

Então, o valor de k é $\frac{1}{2}$.

11.2 $\frac{1+i}{2-ki} = \frac{1+i}{2-ki} \times \frac{2+ki}{2+ki} =$

$= \frac{2 + ki + 2i + ki^2}{4 + k^2} =$

$= \frac{2-k}{4+k^2} + \frac{k+2}{4+k^2}i$

Para que $\frac{1+i}{2-ki}$ represente um número imaginário puro:

$\frac{2-k}{4+k^2} = 0 \wedge \frac{k+2}{4+k^2} \neq 0$

$\Leftrightarrow 2 - k = 0 \wedge k + 2 \neq 0$

$\Leftrightarrow k = 2$

Então, o valor de k é 2.

11.3 $2ki \times \frac{1+ki}{2-i} = \frac{2ki + 2k^2i^2}{2-i} =$

$= \frac{-2k^2 + 2k}{2-i} \times \frac{2+i}{2+i} =$

$= \frac{-4k^2 - 2k^2i + 4ki + 2ki^2}{4+1} =$

$= \frac{-4k^2 - 2k}{5} + \frac{-2k^2 + 4k}{5}i$

Para que $2ki \times \frac{1+ki}{2-i}$ represente um número imaginário puro:

$\frac{-4k^2 - 2k}{5} = 0 \wedge \frac{-2k^2 + 4k}{5} \neq 0$

$\Leftrightarrow -4k^2 - 2k = 0 \wedge -2k^2 + 4k \neq 0$

$\Leftrightarrow -2k(2k+1) = 0 \wedge 2k(-k+2) \neq 0$

$\Leftrightarrow \left(k = 0 \vee k = -\frac{1}{2}\right) \wedge (k \neq 0 \wedge k \neq 2)$

$\Leftrightarrow k = -\frac{1}{2}$

Então, o valor de k é $-\frac{1}{2}$.

11.4 $\frac{2+ki}{k+i} = \frac{2+ki}{k+i} \times \frac{k-i}{k-i} =$

$= \frac{2k - 2i + k^2i - ki^2}{k^2 + 1} =$

$= \frac{3k}{k^2+1} + \frac{k^2-2}{k^2+1}i$

Para que $\frac{2+ki}{k+i}$ represente um número imaginário puro:

$\frac{3k}{k^2+1} = 0 \wedge \frac{k^2-2}{k^2+1} \neq 0$

$\Leftrightarrow 3k = 0 \wedge k^2 - 2 \neq 0$

$\Leftrightarrow k = 0 \wedge k \neq \sqrt{2} \wedge k \neq -\sqrt{2}$

$\Leftrightarrow k = 0$

Então, o valor de k é 0.

12. Seja $z = a + bi, a, b \in \mathbb{R}$.

$\text{Im}(z) = \text{Re}(z) - 1 \Leftrightarrow b = a - 1$

Então, $z = a + (a-1)i$.

$z^2 = (a + (a-1)i)^2 =$

$= a^2 + 2a(a-1)i + (a-1)^2i^2 =$

$= a^2 - (a-1)^2 + 2a(a-1)i$

Para que $z^2 \in \mathbb{R}$:

$2a(a-1) = 0 \Leftrightarrow 2a = 0 \vee a - 1 = 0$

$\Leftrightarrow a = 0 \vee a = 1$

Se $a = 0$, então $b = -1$ e se $a = 1$, então $b = 0$.

Logo, $z = -i$ ou $z = 1$.

13. $z = \frac{2-i}{1+ki} =$

$= \frac{2-i}{1+ki} \times \frac{1-ki}{1-ki} =$

$= \frac{2 - 2ki - i + ki^2}{1^2 - (ki)^2} =$

$= \frac{2-k}{1+k^2} + \frac{-2k-1}{1+k^2}i$

$|z| = 1 \Leftrightarrow \sqrt{\left(\frac{2-k}{1+k^2}\right)^2 + \left(\frac{-2k-1}{1+k^2}\right)^2} = 1$

$\Leftrightarrow \frac{4 - 4k + k^2}{(1+k^2)^2} + \frac{4k^2 + 4k + 1}{(1+k^2)^2} = 1$

$\Leftrightarrow \frac{5k^2 + 5}{(1+k^2)^2} = 1$

$\Leftrightarrow \frac{5(1+k^2)}{(1+k^2)^2} = 1$

$\Leftrightarrow \frac{5}{1+k^2} = 1$

$\Leftrightarrow 5 = 1 + k^2$

$\Leftrightarrow k^2 = 4$

$\Leftrightarrow k = 2 \vee k = -2$

Logo, os valores de k são 2 e -2.

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14.

14.1 $z+2i=\frac{3}{i} \Leftrightarrow z=-2i+\frac{3}{i}\times\frac{-i}{-i}$
$\Leftrightarrow z=-2i+(-3i)$
$\Leftrightarrow z=-5i$
C. S. $=\{-5i\}$

14.2 $\frac{1}{z}=2+i \Leftrightarrow z=\frac{1}{2+i} \wedge z\neq 0$
$\Leftrightarrow z=\frac{1}{2+i}\times\frac{2-i}{2-i} \wedge z\neq 0$
$\Leftrightarrow z=\frac{2-i}{4+1} \wedge z\neq 0$
$\Leftrightarrow z=\frac{2}{5}-\frac{1}{5}i$
C. S. $=\left\{\frac{2}{5}-\frac{1}{5}i\right\}$

14.3 $(z+i)^2=2iz \Leftrightarrow z^2+2iz+i^2=2iz$
$\Leftrightarrow z^2-1=0$
$\Leftrightarrow z=1 \vee z=-1$
C. S. $=\{-1,1\}$

14.4 $\frac{1+z^2}{i+z}=i \Leftrightarrow 1+z^2=i^2+iz \wedge z\neq -i$
$\Leftrightarrow z^2-iz+2=0 \wedge z\neq -i$
$\Leftrightarrow z=\frac{i\pm\sqrt{-1-8}}{2} \wedge z\neq -i$
$\Leftrightarrow z=\frac{i\pm\sqrt{-9}}{2} \wedge z\neq -i$
$\Leftrightarrow z=\frac{i\pm 3i}{2} \wedge z\neq -i$
$\Leftrightarrow (z=2i \vee z=-i) \wedge z\neq -i$
$\Leftrightarrow z=2i$
C. S. $=\{2i\}$

15. Para qualquer $n\in\mathbb{N}$:
$2i^{4n-3}-i=2i^{4n}\times i^{-3}-i=$
$=2(i^4)^n\times\frac{1}{i^3}-i=$
$=2\times 1^n\times\frac{1}{-i}-i=$
$=2\times\frac{1}{-i}\times\frac{i}{i}-i=$
$=2\times\frac{i}{-i^2}-i=$
$=2i-i=$
$=i$

16.

16.1 $z_1=1+2i$
$z_2=-iz_1=$
$=-i(1+2i)=$
$=-i-2i^2=$
$=2-i$
$|z_1|=\sqrt{1^2+2^2}=\sqrt{5}=\overline{OA}$
$|z_2|=\sqrt{2^2+(-1)^2}=\sqrt{5}=\overline{OB}$
$A(1,2)$
$B(2,-1)$
$\overline{AB}=\sqrt{(1-2)^2+(2+1)^2}=\sqrt{1+9}=\sqrt{10}$
$\overline{AB}^2=\left(\sqrt{10}\right)^2=10=$
$=5+5=$
$=\left(\sqrt{5}\right)^2+\left(\sqrt{5}\right)^2=$
$=\overline{OA}^2+\overline{OB}^2$
Logo, pelo teorema de Pitágoras, o triângulo $[ABO]$ é retângulo em O.

16.2 $A_{[ABO]}=\frac{\overline{OA}\times\overline{OB}}{2}=\frac{\sqrt{5}\times\sqrt{5}}{2}=\frac{5}{2}$ u.a.

17.

17.1 $\bar{z}+3z=2-i \underbrace{\Leftrightarrow}_{z=a+b} a-bi+3(a+bi)=2-i$
$\Leftrightarrow a-bi+3a+3bi=2-i$
$\Leftrightarrow 4a+2bi=2-i$
$\Leftrightarrow 4a=2 \wedge 2b=-1$
$\Leftrightarrow a=\frac{1}{2} \wedge b=-\frac{1}{2}$
$\Leftrightarrow z=\frac{1}{2}-\frac{1}{2}i$
C. S. $=\left\{\frac{1}{2}-\frac{1}{2}i\right\}$

17.2 $z^2+(1-i)z+(-6+2i)=0$
$\Leftrightarrow z=\frac{-1+i\pm\sqrt{(1-i)^2-4\times(-6+2i)}}{2}$
$\Leftrightarrow z=\frac{-1+i\pm\sqrt{1-2i+i^2+24-8i}}{2}$
$\Leftrightarrow z=\frac{-1+i\pm\sqrt{25-10i+i^2}}{2}$
$\Leftrightarrow z=\frac{-1+i\pm\sqrt{(5-i)^2}}{2}$
$\Leftrightarrow z=\frac{-1+i+5-i}{2} \vee z=\frac{-1+i-5+i}{2}$
$\Leftrightarrow z=2 \vee z=-3+i$
C. S. $=\{2,-3+i\}$

18. Seja $z=a+bi$ e seja $w=c+di, a,b,c,d\in\mathbb{R}$.
$z+w=3 \Leftrightarrow a+bi+c+di=3$
$\Leftrightarrow (a+c)+(b+d)i=3$
$\Leftrightarrow a+c=3 \wedge b+d=0$
$\Leftrightarrow a=3-c \wedge b=-d$
Assim:
$\frac{z}{w}=i \Leftrightarrow \frac{a+bi}{c+di}=i$
$\Leftrightarrow \frac{3-c-di}{c+di}=i$
$\Leftrightarrow \frac{(3-c)-di}{c+di}\times\frac{c-di}{c-di}=i$
$\Leftrightarrow \frac{3c-c^2-3di+cdi-cdi+d^2i^2}{c^2+d^2}=i$
$\Leftrightarrow \frac{3c-c^2-d^2}{c^2+d^2}+\frac{-3d}{c^2+d^2}i=i$
$\Leftrightarrow \frac{3c}{c^2+d^2}-1=0 \wedge -\frac{3d}{c^2+d^2}=1$
$\Leftrightarrow 3c=c^2+d^2 \wedge -\frac{3d}{3c}=1 \wedge c^2+d^2\neq 0$
$\Leftrightarrow d=-c \wedge 3c=2c^2 \wedge c\neq 0 \wedge d\neq 0$
$\Leftrightarrow c=\frac{3}{2} \wedge d=-\frac{3}{2}$
Logo, $a=3-\frac{3}{2}=\frac{3}{2}$ e $b=\frac{3}{2}$, ou seja, $z=\frac{3}{2}+\frac{3}{2}i$ e $w=\frac{3}{2}-\frac{3}{2}i$.

19. Como $a+bi$ é solução da equação $z^2+cz+d=0$, então:
$(a+bi)^2+c(a+bi)+d=0$
$\Leftrightarrow a^2+2abi+(bi)^2+ca+cbi+d=0$
$\Leftrightarrow a^2+2abi-b^2+ca+cbi+d=0$
$\Leftrightarrow (a^2-b^2+ca+d)+(2ab+cb)i=0$
$\Leftrightarrow \begin{cases} a^2-b^2+ca+d=0 \\ 2ab+cb=0 \end{cases} \Leftrightarrow \begin{cases} a^2-b^2+ca+d=0 \\ b(2a+c)=0 \end{cases}$
$\underbrace{\Leftrightarrow}_{b\neq 0} \begin{cases} a^2-b^2+(-2a)a+d=0 \\ 2a+c=0 \end{cases}$
$\Leftrightarrow \begin{cases} a^2-b^2-2a^2+d=0 \\ \end{cases} \Leftrightarrow \begin{cases} -a^2-b^2=-d \\ \end{cases}$
$\Leftrightarrow \begin{cases} a^2+b^2=d \\ 2a+c=0 \end{cases}$, como queríamos demonstrar.

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2. Números complexos na forma trigonométrica

Páginas 6 a 10

20.

20.1 $\rho = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$

$\operatorname{tg}\theta = \sqrt{3} \wedge \theta \in 1.^{\circ}\text{ Q}$. Logo, $\theta = \frac{\pi}{3}$, por exemplo.

Então, $1 + \sqrt{3}i = 2e^{i\frac{\pi}{3}}$.

20.2 $\rho = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2+2} = 2$

$\operatorname{tg}\theta = -1 \wedge \theta \in 4.^{\circ}\text{ Q}$. Logo, $\theta = -\frac{\pi}{4}$, por exemplo.

Então, $\sqrt{2} - \sqrt{2}i = 2e^{i\left(-\frac{\pi}{4}\right)}$.

20.3 $\rho = \sqrt{(-6)^2 + (2\sqrt{3})^2} = \sqrt{36+12} = \sqrt{48} = 4\sqrt{3}$

$\operatorname{tg}\theta = -\frac{\sqrt{3}}{3} \wedge \theta \in 2.^{\circ}\text{ Q}$. Logo, $\theta = \frac{5\pi}{6}$, por exemplo.

Então, $-6 + 2\sqrt{3}i = 4\sqrt{3}e^{i\frac{5\pi}{6}}$.

20.4 $\rho = \sqrt{(-\sqrt{6})^2 + (-\sqrt{2})^2} = \sqrt{6+2} = \sqrt{8} = 2\sqrt{2}$

$\operatorname{tg}\theta = \frac{1}{\sqrt{3}} \wedge \theta \in 3.^{\circ}\text{ Q}$. Logo, $\theta = -\frac{5\pi}{6}$, por exemplo.

Então, $-\sqrt{6} - \sqrt{2}i = 2\sqrt{2}e^{i\left(-\frac{5\pi}{6}\right)}$.

20.5 $3i = 3e^{i\frac{\pi}{2}}$

20.6 $-2i = 2e^{i\left(-\frac{\pi}{2}\right)}$

20.7 $\rho = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\operatorname{tg}\theta = 1 \wedge \theta \in 1.^{\circ}\text{ Q}$. Logo, $\theta = \frac{\pi}{4}$, por exemplo.

Então, $2 + 2i = 2\sqrt{2}e^{i\frac{\pi}{4}}$.

20.8 $\rho = \sqrt{(-\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{3+3} = \sqrt{6}$

$\operatorname{tg}\theta = -1 \wedge \theta \in 2.^{\circ}\text{ Q}$. Logo, $\theta = \frac{3\pi}{4}$, por exemplo.

Então, $-\sqrt{3} + \sqrt{3}i = \sqrt{6}e^{i\frac{3\pi}{4}}$.

20.9 $\rho = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$\operatorname{tg}\theta = -\frac{\sqrt{3}}{3} \wedge \theta \in 4.^{\circ}\text{ Q}$. Logo, $\theta = -\frac{\pi}{6}$, por exemplo.

Então, $3 - \sqrt{3}i = 2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}$.

21.

21.1 $4e^{i\frac{\pi}{3}} = 4\left(\cos\frac{\pi}{3} + i\operatorname{sen}\frac{\pi}{3}\right) =$
$= 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) =$
$= 2 + 2\sqrt{3}i$

21.2 $2e^{i\frac{\pi}{2}} = 2i$

21.3 $2e^{i\frac{3\pi}{4}} = 2\left(\cos\frac{3\pi}{4} + i\operatorname{sen}\frac{3\pi}{4}\right) =$
$= 2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) =$
$= -\sqrt{2} + \sqrt{2}i$

21.4 $3e^{i\frac{5\pi}{6}} = 3\left(\cos\frac{5\pi}{6} + i\operatorname{sen}\frac{5\pi}{6}\right) =$
$= 3\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) =$
$= -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

21.5 $\sqrt{3}e^{i\frac{7\pi}{4}} = \sqrt{3}\left(\cos\frac{7\pi}{4} + i\operatorname{sen}\frac{7\pi}{4}\right) =$
$= \sqrt{3}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) =$
$= \frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{2}i$

21.6 $2e^{i\frac{4\pi}{3}} = 2\left(\cos\frac{4\pi}{3} + i\operatorname{sen}\frac{4\pi}{3}\right) =$
$= 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) =$
$= -1 - \sqrt{3}i$

21.7 $\frac{1}{2}e^{i\left(-\frac{\pi}{2}\right)} = -\frac{1}{2}i$

21.8 $2e^{i\left(-\frac{5\pi}{4}\right)} = 2\left(\cos\left(-\frac{5\pi}{4}\right) + i\operatorname{sen}\left(-\frac{5\pi}{4}\right)\right) =$
$= 2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) =$
$= -\sqrt{2} + \sqrt{2}i$

21.9 $7e^{i\pi} = -7$

22. Seja *A* o afixo do número complexo apresentado no exercício 21, *B* o afixo do seu conjugado, *C* o afixo do seu simétrico e *D* o afixo do seu inverso.

22.1 Conjugado: $4e^{i\left(-\frac{\pi}{3}\right)}$

Simétrico: $4e^{i\left(\pi+\frac{\pi}{3}\right)} = 4e^{i\frac{4\pi}{3}}$

Inverso: $\frac{1}{4e^{i\frac{\pi}{3}}} = \frac{1}{4}e^{i\left(-\frac{\pi}{3}\right)}$

22.2 Conjugado: $2e^{i\left(-\frac{\pi}{2}\right)}$

Simétrico: $2e^{i\left(\pi+\frac{\pi}{2}\right)} = 2e^{i\frac{3\pi}{2}}$

Inverso: $\frac{1}{2e^{i\frac{\pi}{2}}} = \frac{1}{2}e^{i\left(-\frac{\pi}{2}\right)}$

22.3 Conjugado: $2e^{i\left(-\frac{3\pi}{4}\right)}$

Simétrico: $2e^{i\left(\pi+\frac{3\pi}{4}\right)} = 2e^{i\frac{7\pi}{4}}$

Inverso: $\frac{1}{2e^{i\frac{3\pi}{4}}} = \frac{1}{2}e^{i\left(-\frac{3\pi}{4}\right)}$

22.4 Conjugado: $3e^{i\left(-\frac{5\pi}{6}\right)}$

Simétrico: $3e^{i\left(\pi+\frac{5\pi}{6}\right)} = 3e^{i\frac{11\pi}{6}}$

Inverso: $\frac{1}{3e^{i\frac{5\pi}{6}}} = \frac{1}{3}e^{i\left(-\frac{5\pi}{6}\right)}$

22.5 Conjugado: $\sqrt{3}e^{i\left(-\frac{7\pi}{4}\right)}$

Simétrico: $\sqrt{3}e^{i\left(\pi+\frac{7\pi}{4}\right)} = \sqrt{3}e^{i\frac{11\pi}{4}} = \sqrt{3}e^{i\frac{3\pi}{4}}$

Inverso: $\frac{1}{\sqrt{3}e^{i\frac{7\pi}{4}}} = \frac{\sqrt{3}}{3}e^{i\left(-\frac{7\pi}{4}\right)}$

22.6 Conjugado: $2e^{i\left(-\frac{4\pi}{3}\right)}$

Simétrico: $2e^{i\left(\pi+\frac{4\pi}{3}\right)} = 2e^{i\frac{7\pi}{3}} = 2e^{i\frac{\pi}{3}}$

Inverso: $\frac{1}{2e^{i\frac{4\pi}{3}}} = \frac{1}{2}e^{i\left(-\frac{4\pi}{3}\right)}$

22.7 Conjugado: $\frac{1}{2}e^{i\frac{\pi}{2}}$

Simétrico: $\frac{1}{2}e^{i\left(-\frac{\pi}{2}+\pi\right)} = \frac{1}{2}e^{i\frac{\pi}{2}}$

Inverso: $\frac{1}{\frac{1}{2}e^{i\left(-\frac{\pi}{2}\right)}} = 2e^{i\frac{\pi}{2}}$

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22.8 Conjugado: $2e^{i\frac{5\pi}{4}}$

Simétrico: $2e^{i\left(-\frac{5\pi}{4}+\pi\right)} = 2e^{i\left(-\frac{\pi}{4}\right)}$

Inverso: $\frac{1}{2e^{i\left(-\frac{5\pi}{4}\right)}} = \frac{1}{2}e^{i\frac{5\pi}{4}}$

22.9 Conjugado: $7e^{i(-\pi)}$

Simétrico: $7e^{i2\pi}$

Inverso: $\frac{1}{7e^{i\pi}} = \frac{1}{7}e^{i(-\pi)}$

23.

23.1 $(-2i)^4 = (-2)^4 \times i^4 =$
$= 16 \times i^0 =$
$= 16 \times 1 =$
$= 16$

Logo, $-2i$ é uma raiz quarta de 16.

23.2 $(1+i)^6 = ((1+i)^2)^3 =$
$= (1+2i+i^2)^3 =$
$= (2i)^3 =$
$= 2^3 \times i^3 =$
$= -8i$

Logo, $1+i$ é uma raiz sexta de $-8i$.

23.3 $(2+i)^3 = (2+i)^2(2+i) =$
$= (4+4i+i^2)(2+i) =$
$= (3+4i)(2+i) =$
$= 6+3i+8i+4i^2 =$
$= 2+11i$

Logo, $2+i$ é uma raiz cúbica de $2+11i$.

23.4 $\left(\sqrt[4]{2}e^{i\frac{7\pi}{12}}\right)^4 = \left(\sqrt[4]{2}\right)^4\left(e^{i\frac{7\pi}{12}}\right)^4 =$
$= 2e^{i\frac{7\pi}{3}} =$
$= 2\left(\cos\frac{7\pi}{3} + i\,\text{sen}\frac{7\pi}{3}\right) =$
$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) =$
$= 1+\sqrt{3}i$

Logo, $\sqrt[4]{2}e^{i\frac{7\pi}{12}}$ é uma raiz quarta de $1+\sqrt{3}i$.

24.

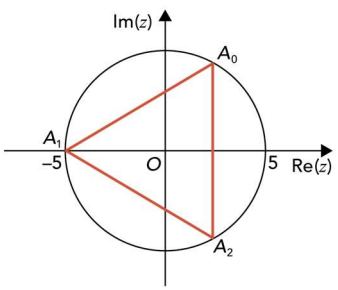
24.1 $z^3 = -125$
$\Leftrightarrow z^3 = 125e^{i\pi}$
$\Leftrightarrow z_k = \sqrt[3]{125}e^{i\left(\frac{\pi}{3}+\frac{2k\pi}{3}\right)}, k \in \{0,1,2\}$
$\Leftrightarrow z_k = 5e^{i\left(\frac{\pi}{3}+\frac{2k\pi}{3}\right)}, k \in \{0,1,2\}$

Se $k=0$, então $z_0 = 5e^{i\frac{\pi}{3}}$.

Se $k=1$, então $z_1 = 5e^{i\pi}$.

Se $k=2$, então $z_2 = 5e^{i\frac{5\pi}{3}}$.

Sendo A_0, A_1 e A_2 os afixos de z_0, z_1 e z_2, respetivamente:



24.2 $z^4 = 256i$
$\Leftrightarrow z^4 = 256e^{i\frac{\pi}{2}}$
$\Leftrightarrow z_k = \sqrt[4]{256}e^{i\left(\frac{\pi}{8}+\frac{2k\pi}{4}\right)}, k \in \{0,1,2,3\}$
$\Leftrightarrow z_k = 4e^{i\left(\frac{\pi}{8}+\frac{k\pi}{2}\right)}, k \in \{0,1,2,3\}$

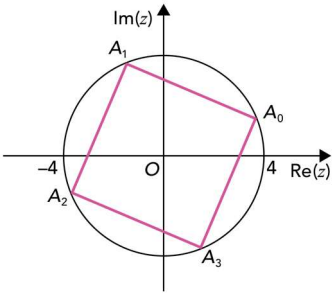
Se $k=0$, então $z_0 = 4e^{i\frac{\pi}{8}}$.

Se $k=1$, então $z_1 = 4e^{i\frac{5\pi}{8}}$.

Se $k=2$, então $z_2 = 4e^{i\frac{9\pi}{8}}$.

Se $k=3$, então $z_3 = 4e^{i\frac{13\pi}{8}}$.

Sendo A_0, A_1, A_2 e A_3 os afixos de z_0, z_1, z_2 e z_3, respetivamente:



24.3 $z^5 = 243$
$\Leftrightarrow z^5 = 243e^{i0}$
$\Leftrightarrow z_k = \sqrt[5]{243}e^{i\left(0+\frac{2k\pi}{5}\right)}, k \in \{0,1,2,3,4\}$
$\Leftrightarrow z_k = 3e^{i\frac{2k\pi}{5}}, k \in \{0,1,2,3,4\}$

Se $k=0$, então $z_0 = 3e^{i0}$.

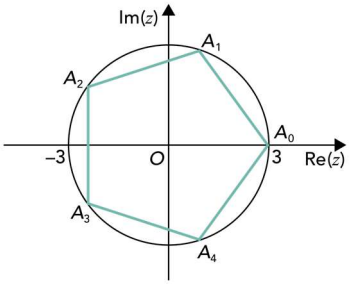
Se $k=1$, então $z_1 = 3e^{i\frac{2\pi}{5}}$.

Se $k=2$, então $z_2 = 3e^{i\frac{4\pi}{5}}$.

Se $k=3$, então $z_3 = 3e^{i\frac{6\pi}{5}}$.

Se $k=4$, então $z_4 = 3e^{i\frac{8\pi}{5}}$.

Sendo A_0, A_1, A_2, A_3 e A_4 os afixos de z_0, z_1, z_2, z_3 e z_4, respetivamente:



24.4 $z^6 = -64i$
$\Leftrightarrow z^6 = 64e^{i\frac{3\pi}{2}}$
$\Leftrightarrow z_k = \sqrt[6]{64}e^{i\left(\frac{3\pi}{12}+\frac{2k\pi}{6}\right)}, k \in \{0,1,2,3,4,5\}$
$\Leftrightarrow z_k = 2e^{i\left(\frac{\pi}{4}+\frac{k\pi}{3}\right)}, k \in \{0,1,2,3,4,5\}$

Se $k=0$, então $z_0 = 2e^{i\frac{\pi}{4}}$.

Se $k=1$, então $z_1 = 2e^{i\frac{7\pi}{12}}$.

Se $k=2$, então $z_2 = 2e^{i\frac{11\pi}{12}}$.

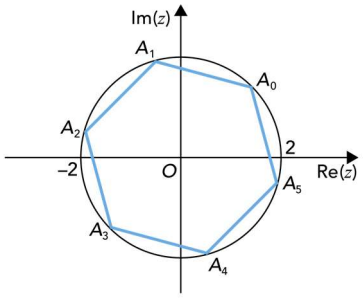
Se $k=3$, então $z_3 = 2e^{i\frac{5\pi}{4}}$.

Se $k=4$, então $z_4 = 2e^{i\frac{19\pi}{12}}$.

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Se $k = 5$, então $z_5 = 2e^{i\frac{23\pi}{12}}$.

Sendo A_0, A_1, A_2, A_3, A_4 e A_5 os afixos de z_0, z_1, z_2, z_3, z_4 e z_5, respetivamente:



24.5 $z^3 = \sqrt{2} + \sqrt{2}i$

$\Leftrightarrow z^3 = 2e^{i\frac{\pi}{4}}$

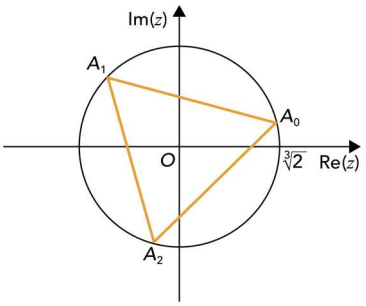
$\Leftrightarrow z_k = \sqrt[3]{2}e^{i\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)}, k \in \{0, 1, 2\}$

Se $k = 0$, então $z_0 = \sqrt[3]{2}e^{i\frac{\pi}{12}}$.

Se $k = 1$, então $z_1 = \sqrt[3]{2}e^{i\frac{3\pi}{4}}$.

Se $k = 2$, então $z_2 = \sqrt[3]{2}e^{i\frac{17\pi}{12}}$.

Sendo A_0, A_1 e A_2 os afixos de z_0, z_1 e z_2, respetivamente:



24.6 $z^4 = -1 + \sqrt{3}i$

$\Leftrightarrow z^4 = 2e^{i\frac{2\pi}{3}}$

$\Leftrightarrow z_k = \sqrt[4]{2}e^{i\left(\frac{2\pi}{12} + \frac{2k\pi}{4}\right)}, k \in \{0, 1, 2, 3\}$

$\Leftrightarrow z_k = \sqrt[4]{2}e^{i\left(\frac{\pi}{6} + \frac{k\pi}{2}\right)}, k \in \{0, 1, 2, 3\}$

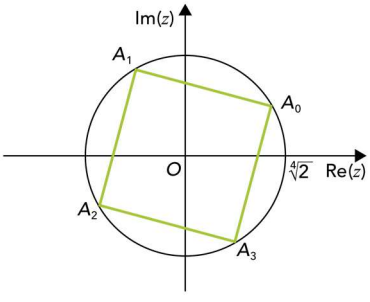
Se $k = 0$, então $z_0 = \sqrt[4]{2}e^{i\frac{\pi}{6}}$.

Se $k = 1$, então $z_1 = \sqrt[4]{2}e^{i\frac{2\pi}{3}}$.

Se $k = 2$, então $z_2 = \sqrt[4]{2}e^{i\frac{7\pi}{6}}$.

Se $k = 3$, então $z_3 = \sqrt[4]{2}e^{i\frac{5\pi}{3}}$.

Sendo A_0, A_1, A_2 e A_3 os afixos de z_0, z_1, z_2 e z_3, respetivamente:



25.

25.1 $z = z_0^6 \Leftrightarrow z = \left(2e^{i\frac{2\pi}{3}}\right)^6$

$\Leftrightarrow z = 2^6 e^{i4\pi}$

$\Leftrightarrow z = 64$

$z_0 = 2e^{i\frac{2\pi}{3}}$

$z_1 = 2e^{i\left(\frac{2\pi}{3} + \frac{2\pi}{6}\right)} = 2e^{i\pi}$

$z_2 = 2e^{i\left(\pi + \frac{2\pi}{6}\right)} = 2e^{i\frac{4\pi}{3}}$

$z_3 = 2e^{i\left(\frac{4\pi}{3} + \frac{2\pi}{6}\right)} = 2e^{i\frac{5\pi}{3}}$

$z_4 = 2e^{i\left(\frac{5\pi}{3} + \frac{2\pi}{6}\right)} = 2e^{i0}$

$z_5 = 2e^{i\frac{\pi}{3}}$

25.2 Os afixos das raízes sextas de z são os vértices de um hexágono regular inscrito numa circunferência de raio 2 com centro na origem do referencial. Por se tratar de um hexágono regular, a medida do raio da circunferência é igual à medida do lado do hexágono. Assim, $P = 6 \times 2 = 12$ u.c.

26. $z = 2e^{i\frac{\pi}{12}}$

As restantes raízes de ordem oito de w são:

$2e^{i\left(\frac{\pi}{12} + \frac{2\pi}{8}\right)} = 2e^{i\frac{\pi}{3}}$

$2e^{i\left(\frac{\pi}{12} + \frac{4\pi}{8}\right)} = 2e^{i\frac{7\pi}{12}}$

$2e^{i\left(\frac{\pi}{12} + \frac{6\pi}{8}\right)} = 2e^{i\frac{5\pi}{6}}$

$2e^{i\left(\frac{\pi}{12} + \frac{8\pi}{8}\right)} = 2e^{i\frac{13\pi}{12}}$

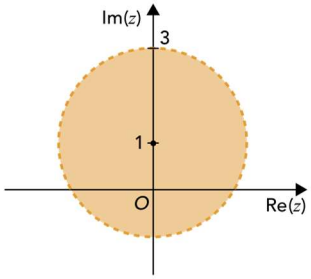
$2e^{i\left(\frac{\pi}{12} + \frac{10\pi}{8}\right)} = 2e^{i\frac{4\pi}{3}}$

$2e^{i\left(\frac{\pi}{12} + \frac{12\pi}{8}\right)} = 2e^{i\frac{19\pi}{12}}$

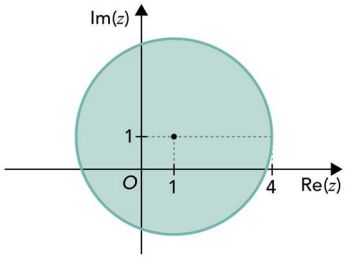
$2e^{i\left(\frac{\pi}{12} + \frac{14\pi}{8}\right)} = 2e^{i\frac{11\pi}{6}}$

27.

27.1 $|z - i| < 2$

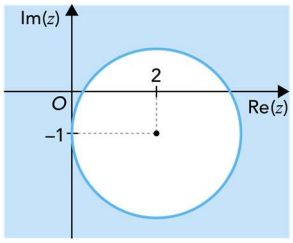


27.2 $|z - (1 + i)| \leq 3$

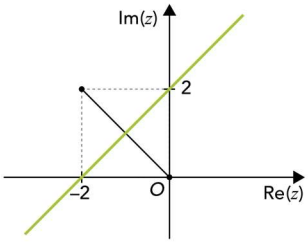


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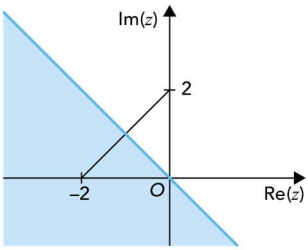
27.3 $|z-2+i| \geq 2 \Leftrightarrow |z-(2-i)| \geq 2$



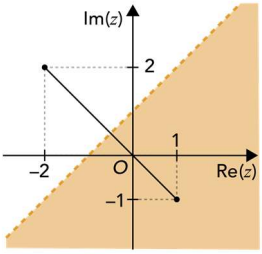
27.4 $|z| = |z+2-2i| \Leftrightarrow |z| = |z-(-2+2i)|$



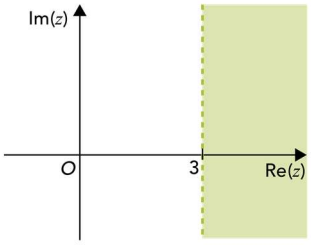
27.5 $|z-2i| \geq |z+2|$



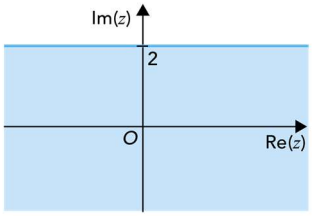
27.6 $|z-1+i| < |z+2-2i|$
$\Leftrightarrow |z-(1-i)| < |z-(-2+2i)|$



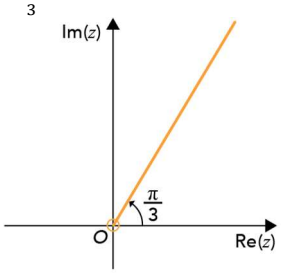
27.7 $\text{Re}(z) > 3$



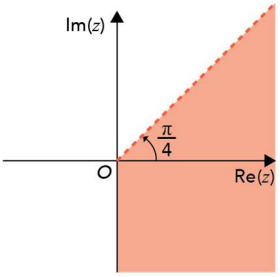
27.8 $\text{Im}(z) \leq 2$



27.9 $\arg(z) = \frac{\pi}{3}$



27.10 $-\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{4}$



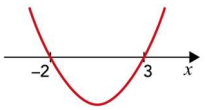
28. $x^2 + (2k-1)x + \frac{25}{4} = 0$

Para que as soluções desta equação sejam não reais, tem-se que:

$(2k-1)^2 - 4 \times 1 \times \frac{25}{4} < 0$
$\Leftrightarrow 4k^2 - 4k + 1 - 25 < 0$
$\Leftrightarrow 4k^2 - 4k - 24 < 0$

Cálculo auxiliar

$4k^2 - 4k - 24 = 0$
$\Leftrightarrow k^2 - k - 6 = 0$
$\Leftrightarrow k = \frac{1 \pm \sqrt{1+24}}{2}$
$\Leftrightarrow k = \frac{1 \pm 5}{2}$
$\Leftrightarrow k = 3 \vee k = -2$



Logo, $4k^2 - 4k - 24 < 0 \Leftrightarrow k > -2 \wedge k < 3$.
Assim, os valores de k pedidos são os que pertencem ao intervalo $]-2, 3[$.

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29. $z = e^{i\left(\frac{\pi}{2}+\alpha\right)}$

29.1 $z \times \bar{z} = e^{i\left(\frac{\pi}{2}+\alpha\right)} \times e^{-i\left(\frac{\pi}{2}+\alpha\right)} = e^{i0}$

29.2 $z \times \frac{1}{\bar{z}} = \frac{e^{i\left(\frac{\pi}{2}+\alpha\right)}}{e^{-i\left(\frac{\pi}{2}+\alpha\right)}} = e^{2i\left(\frac{\pi}{2}+\alpha\right)} = e^{i(\pi+2\alpha)}$

29.3 $-z^3 = -\left(e^{i\left(\frac{\pi}{2}+\alpha\right)}\right)^3 =$

$= -e^{3i\left(\frac{\pi}{2}+\alpha\right)} =$

$= -e^{i\left(\frac{3\pi}{2}+3\alpha\right)} =$

$= e^{i\left(\pi+\frac{3\pi}{2}+3\alpha\right.} =$

$= e^{i\left(\frac{5\pi}{2}+3\alpha\right.} =$

$= e^{i\left(\frac{\pi}{2}+3\alpha\right.}$

29.4 $\bar{z} \times \frac{i}{z^2} = e^{-i\left(\frac{\pi}{2}+\alpha\right)} \times \frac{e^{i\frac{\pi}{2}}}{\left(e^{i\left(\frac{\pi}{2}+\alpha\right)}\right)^2} =$

$= e^{i\left(-\frac{\pi}{2}-\alpha\right)} \times \frac{e^{i\frac{\pi}{2}}}{e^{i(\pi+2\alpha)}} =$

$= e^{i\left(-\frac{\pi}{2}-\alpha\right)} \times e^{i\left(-\frac{\pi}{2}-2\alpha\right)} =$

$= e^{i(-\pi-3\alpha)}$

30. $z_1 = 2e^{i\frac{2\pi}{3}}$

$|z_1| = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = 2$

$\operatorname{tg}\theta_1 = -\sqrt{3} \wedge \theta_1 \in 2.^{\circ}\,\mathrm{Q}.$

Logo, $\theta_1 = \frac{2\pi}{3}$, por exemplo.

$z_2 = 2e^{i\left(-\frac{\pi}{6}\right)}$

$|z_2| = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = \sqrt{3+1} = 2$

$\operatorname{tg}\theta_2 = -\frac{\sqrt{3}}{3} \wedge \theta_2 \in 4.^{\circ}\,\mathrm{Q}.$

Logo, $\theta_2 = -\frac{\pi}{6}$, por exemplo.

30.1 $z_1 z_2 = 2e^{i\frac{2\pi}{3}} \times 2e^{i\left(-\frac{\pi}{6}\right)} =$

$= 4e^{i\left(\frac{2\pi}{3}-\frac{\pi}{6}\right)} =$

$= 4e^{i\frac{\pi}{2}}$

30.2 $\frac{z_1}{z_2} = \frac{2e^{i\frac{2\pi}{3}}}{2e^{i\left(-\frac{\pi}{6}\right)}} = e^{i\left(\frac{2\pi}{3}+\frac{\pi}{6}\right)} = e^{i\frac{5\pi}{6}}$

30.3 $\frac{z_2}{z_3} = \frac{2e^{i\left(-\frac{\pi}{6}\right)}}{\sqrt{2}e^{i\frac{5\pi}{4}}} =$

$= \sqrt{2}e^{i\left(-\frac{\pi}{6}-\frac{5\pi}{4}\right)} =$

$= \sqrt{2}e^{i\left(-\frac{17\pi}{12}\right)} =$

$= \sqrt{2}e^{i\frac{7\pi}{12}}$

30.4 $z_2 z_3 = 2e^{i\left(-\frac{\pi}{6}\right)} \times \sqrt{2}e^{i\frac{5\pi}{4}} =$

$= 2\sqrt{2}e^{i\left(-\frac{\pi}{6}+\frac{5\pi}{4}\right)} =$

$= 2\sqrt{2}e^{i\frac{13\pi}{12}}$

30.5 $\frac{1}{z_3} = \frac{1}{\sqrt{2}e^{i\left(-\frac{5\pi}{4}\right)}} = \frac{\sqrt{2}}{2}e^{i\frac{5\pi}{4}}$

30.6 $z_1^2 = \left(2e^{i\frac{2\pi}{3}}\right)^2 = 4e^{i\frac{4\pi}{3}}$

30.7 $z_2^3 = \left(2e^{i\left(-\frac{\pi}{6}\right)}\right)^3 = 8e^{i\left(-\frac{\pi}{2}\right)}$

30.8 $z_3^4 = \left(\sqrt{2}e^{i\frac{5\pi}{4}}\right)^4 = 4e^{i5\pi} = 4e^{i\pi}$

30.9 $\frac{2z_1}{z_2 z_3} = \frac{2\times 2e^{i\frac{2\pi}{3}}}{2\sqrt{2}e^{i\frac{13\pi}{12}}} =$

$= \sqrt{2}e^{i\left(\frac{2\pi}{3}-\frac{13\pi}{12}\right)} =$

$= \sqrt{2}e^{i\left(-\frac{5\pi}{12}\right)}$

30.10 $z_1\bar{z_2} = 2e^{i\frac{2\pi}{3}} \times 2e^{i\frac{\pi}{6}} =$

$= 4e^{i\left(\frac{2\pi}{3}+\frac{\pi}{6}\right)} =$

$= 4e^{i\frac{5\pi}{6}}$

30.11 $\frac{\bar{z_1}z_3}{z_2} = \frac{2e^{i\left(-\frac{2\pi}{3}\right)} \times \sqrt{2}e^{i\frac{5\pi}{4}}}{2e^{i\left(-\frac{\pi}{6}\right)}} =$

$= \frac{2\sqrt{2}e^{i\left(-\frac{2\pi}{3}+\frac{5\pi}{4}\right)}}{2e^{i\left(-\frac{\pi}{6}\right)}} =$

$= \frac{\sqrt{2}e^{i\frac{7\pi}{12}}}{e^{i\left(-\frac{\pi}{6}\right)}} =$

$= \sqrt{2}e^{i\left(\frac{7\pi}{12}+\frac{\pi}{6}\right)} =$

$= \sqrt{2}e^{i\frac{3\pi}{4}}$

30.12 $\frac{(z_1)^2\bar{z_3}}{z_3} = \frac{4e^{i\frac{4\pi}{3}} \times \sqrt{2}e^{i\left(-\frac{5\pi}{4}\right)}}{\sqrt{2}e^{i\frac{5\pi}{4}}} =$

$= \frac{4e^{i\left(\frac{4\pi}{3}-\frac{5\pi}{4}\right)}}{e^{i\frac{5\pi}{4}}} =$

$= \frac{4e^{i\frac{\pi}{12}}}{e^{i\frac{5\pi}{4}}} =$

$= 4e^{i\left(\frac{\pi}{12}-\frac{5\pi}{4}\right)} =$

$= 4e^{i\left(-\frac{7\pi}{6}\right)} =$

$= 4e^{i\frac{5\pi}{6}}$

31.

31.1 Seja $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.

$|z| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

$\operatorname{tg}\theta = 1 \wedge \theta \in 1.^{\circ}\,\mathrm{Q}.$

Logo, $\theta = \frac{\pi}{4}$, por exemplo.

Assim, $z = e^{i\frac{\pi}{4}}$ e então:

$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{28} = \left(e^{i\frac{\pi}{4}}\right)^{28} = e^{i7\pi} = -1$

31.2 Seja $z = \frac{\sqrt{3}}{2} - \frac{i}{2}$.

$|z| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = 1$

$\operatorname{tg}\theta = -\frac{\sqrt{3}}{3} \wedge \theta \in 4.^{\circ}\,\mathrm{Q}.$

Logo, $\theta = -\frac{\pi}{6}$, por exemplo.

Assim, $z = e^{i\left(-\frac{\pi}{6}\right)}$ e então:

$\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{56} = \left(e^{i\left(-\frac{\pi}{6}\right)}\right)^{56} =$

$= e^{i\left(-\frac{28\pi}{3}\right)} =$

$= e^{i\left(-\frac{4\pi}{3}\right)} =$

$= e^{i\frac{2\pi}{3}} =$

$= \cos\frac{2\pi}{3} + i\,\operatorname{sen}\frac{2\pi}{3} =$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

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32. $w = \frac{13\, z_1 \times (-z_2)}{(\overline{z_2})^2 \times i^{24n+3}} + 2 - 9i =$

$= \frac{13(2-3i^{2025}) \times (-3+2i)}{(3+2i)^2 \times i^{24n+3}} + 2 - 9i =$

$= \frac{13(2-3i) \times (-3+2i)}{(3+2i)^2 \times i^3} + 2 - 9i =$

$= \frac{13(-6+4i+9i-6i^2)}{(9+12i+4i^2) \times (-i)} + 2 - 9i =$

$= \frac{13 \times 13i}{(5+12i) \times (-i)} + 2 - 9i =$

$= \frac{169i}{-5i-12^{\,2}} + 2 - 9i =$

$= \frac{169i}{12-5i} + 2 - 9i =$

$= \frac{169i}{12-5i} \times \frac{12+5i}{12+5i} + 2 - 9i =$

$= 169 \times \frac{12i+5i^2}{12^2-(5i)^2} + 2 - 9i =$

$= 169 \times \frac{-5+12i}{144+25} + 2 - 9i =$

$= -5 + 12i + 2 - 9i =$

$= -3 + 3i$

$|w| = \sqrt{(-3)^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$\text{tg}\,\theta = -1 \wedge \theta \in 2.^{\circ}\,Q$

Logo, $\theta = \frac{3\pi}{4}$, por exemplo.

Assim, $w = 3\sqrt{2}e^{i\frac{3\pi}{4}}$.

33.

33.1 $(\cos\theta + i\,\text{sen}\,\theta)^4 = \left(e^{i\theta}\right)^4 = e^{i4\theta}$

33.2 $\left(2(\cos\theta - i\,\text{sen}\,\theta)\right)^3 =$

$= \left(2\overline{(\cos\theta + i\,\text{sen}\,\theta)}\right)^3 =$

$= 2^3\left(e^{i(-\theta)}\right)^3 =$

$= 8e^{i(-3\theta)}$

33.3 $\left(3(\cos\theta + i\,\text{sen}\,\theta)\right)^{-2} =$

$= 3^{-2}\left(e^{i\theta}\right)^{-2} =$

$= \frac{1}{9}e^{i(-2\theta)}$

33.4 $\frac{(\cos\theta + i\,\text{sen}\,\theta)^4}{-\cos\theta - i\,\text{sen}\,\theta} = \frac{e^{i4\theta}}{-e^{i\theta}} = \frac{e^{i4\theta}}{e^{i(\pi+\theta)}}e^{i(3\theta-\pi)}$

33.5 $\frac{\text{sen}\,\theta + i\cos\theta}{\cos(2\theta) + i\,\text{sen}(2\theta)} =$

$= \frac{\cos\left(\frac{\pi}{2}-\theta\right) + i\,\text{sen}\left(\frac{\pi}{2}-\theta\right)}{\cos(2\theta) + i\,\text{sen}(2\theta)} =$

$= \frac{e^{i\left(\frac{\pi}{2}-\theta\right)}}{e^{i2\theta}} =$

$= e^{i\left(\frac{\pi}{2}-3\theta\right)}$

33.6 $\frac{(\cos(4\theta) + i\,\text{sen}(4\theta))^3}{(\text{sen}\,\theta - i\cos\theta)^2} =$

$= \frac{\left(e^{i4\theta}\right)^3}{\left(\cos\left(\frac{\pi}{2}-\theta\right) - i\,\text{sen}\left(\frac{\pi}{2}-\theta\right)\right)^2} =$

$= \frac{e^{i12\theta}}{\left(e^{i\left(-\frac{\pi}{2}+\theta\right)}\right)^2} =$

$= \frac{e^{i12\theta}}{e^{i(-\pi+2\theta)}} =$

$= e^{i(\pi+10\theta)}$

34. $z = 6e^{i\left(-\frac{2\pi}{3}\right)}$

$\frac{z}{-\bar{z}} = \frac{6e^{i\left(-\frac{2\pi}{3}\right)}}{-6e^{i\frac{2\pi}{3}}} =$

$= -e^{i\left(-\frac{4\pi}{3}\right)} =$

$= e^{i\left(\pi-\frac{4\pi}{3}\right)} =$

$= e^{i\left(-\frac{\pi}{3}\right)}$

$w^2 = e^{i\left(-\frac{\pi}{3}\right)}$

$\Leftrightarrow z^2 = e^{i\left(-\frac{\pi}{3}\right)}$

$\Leftrightarrow z_k = \sqrt{1}e^{i\left(-\frac{\pi}{6}+\frac{2k\pi}{2}\right)}, k \in \{0,1\}$

$\Leftrightarrow z_k = e^{i\left(-\frac{\pi}{6}+k\pi\right)}, k \in \{0,1\}$

Se $k = 0$, então $w = e^{i\left(-\frac{\pi}{6}\right)} =$

$= \cos\left(-\frac{\pi}{6}\right) + i\,\text{sen}\left(-\frac{\pi}{6}\right) =$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}i$

Se $k = 1$, então $w = e^{i\frac{5\pi}{6}} =$

$= \cos\left(\frac{5\pi}{6}\right) + i\,\text{sen}\left(\frac{5\pi}{6}\right) =$

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

Assim, as raízes quadradas de $\frac{z}{-\bar{z}}$ são $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ e $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$.

35. $z^3 = -64$

$\Leftrightarrow z^3 = 64e^{i\pi}$

$\Leftrightarrow z_k = \sqrt[3]{64}e^{i\left(\frac{\pi}{3}+\frac{2k\pi}{3}\right)}, k \in \{0,1,2\}$

$\Leftrightarrow z_k = 4e^{i\left(\frac{\pi}{3}+\frac{2k\pi}{3}\right)}, k \in \{0,1,2\}$

Assim, as raízes cúbicas de -64 são:

$z_0 = 4e^{i\frac{\pi}{3}} =$

$= 4\left(\cos\frac{\pi}{3} + i\,\text{sen}\,\frac{\pi}{3}\right) =$

$= 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) =$

$= 2 + 2\sqrt{3}i$

$z_1 = 4e^{i\pi} = -4$

$z_2 = 4e^{i\frac{5\pi}{3}} =$

$= 4\left(\cos\frac{5\pi}{3} + i\,\text{sen}\,\frac{5\pi}{3}\right) =$

$= 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) =$

$= 2 - 2\sqrt{3}i$

Logo, $A(2, 2\sqrt{3})$, $B(-4, 0)$ e $C\left(2, -2\sqrt{3}\right)$.

Note-se que os pontos A e C são simétricos em relação ao eixo real e que o ponto B pertence a esse eixo.

Seja D o ponto de interseção da reta AC com o eixo real. Então, $\overline{AC} = 4\sqrt{3}$ e $\overline{BD} = 6$.

Logo, $A_{[ABC]} = \frac{\overline{AC} \times \overline{BD}}{2} = \frac{4\sqrt{3} \times 6}{2} = 12\sqrt{3}$ u.a.

36.

36.1 $z^5 + 1 = 0$

$\Leftrightarrow z^5 = -1$

$\Leftrightarrow z^5 = e^{i\pi}$

$\Leftrightarrow z_k = \sqrt[5]{1}e^{i\left(\frac{\pi}{5}+\frac{2k\pi}{5}\right)}, k \in \{0,1,2,3,4\}$

$\Leftrightarrow z_k = e^{i\left(\frac{\pi}{5}+\frac{2k\pi}{5}\right)}, k \in \{0,1,2,3,4\}$

C. S. $= \left\{e^{i\frac{\pi}{5}}, e^{i\frac{3\pi}{5}}, e^{i\pi}, e^{i\frac{7\pi}{5}}, e^{i\frac{9\pi}{5}}\right\}$

36.2 $z^6 - i = 0$

$\Leftrightarrow z^6 = i$

$\Leftrightarrow z^6 = e^{i\frac{\pi}{2}}$

$\Leftrightarrow z_k = \sqrt[6]{1}e^{i\left(\frac{\pi}{12}+\frac{2k\pi}{6}\right)}, k \in \{0,1,2,3,4,5\}$

$\Leftrightarrow z_k = e^{i\left(\frac{\pi}{12}+\frac{k\pi}{3}\right)}, k \in \{0,1,2,3,4,5\}$

C. S. $= \left\{e^{i\frac{\pi}{12}}, e^{i\frac{5\pi}{12}}, e^{i\frac{3\pi}{4}}, e^{i\frac{13\pi}{12}}, e^{i\frac{17\pi}{12}}, e^{i\frac{7\pi}{4}}\right\}$

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36.3 $z^4 + 81 = 0$

$\Leftrightarrow z^4 = -81$

$\Leftrightarrow z^4 = 81e^{\pi i}$

$\Leftrightarrow z_k = \sqrt[4]{81}e^{i\left(\frac{\pi}{4} + \frac{2k\pi}{4}\right)}\ k \in \{0, 1, 2, 3\}$

$\Leftrightarrow z_k = 3e^{i\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)}\ k \in \{0, 1, 2, 3\}$

C. S. $= \left\{3e^{\frac{\pi}{4}i}, 3e^{\frac{3\pi}{4}i}, 3e^{\frac{5\pi}{4}i}, 3e^{\frac{7\pi}{4}i}\right\}$

36.4 $z^4 + 16i \times e^{i\frac{\pi}{4}} = 0$

$\Leftrightarrow z^4 + 16e^{i\frac{\pi}{2}} \times e^{i\frac{\pi}{4}} = 0$

$\Leftrightarrow z^4 + 16e^{i\frac{3\pi}{4}} = 0$

$\Leftrightarrow z^4 = -16e^{i\frac{3\pi}{4}}$

$\Leftrightarrow z^4 = 16e^{i\frac{7\pi}{4}}$

$\Leftrightarrow z_k = \sqrt[4]{16}e^{i\left(\frac{7\pi}{16} + \frac{2k\pi}{4}\right)}, k \in \{0, 1, 2, 3\}$

$\Leftrightarrow z_k = 2e^{i\left(\frac{7\pi}{16} + \frac{k\pi}{2}\right)}, k \in \{0, 1, 2, 3\}$

C. S. $= \left\{2e^{i\frac{7\pi}{16}}, 2e^{i\frac{15\pi}{16}}, 2e^{i\frac{23\pi}{16}}, 2e^{i\frac{31\pi}{16}}\right\}$

36.5 $z^2 - 1 + \sqrt{3}i = 0$

$\Leftrightarrow z^2 = 1 - \sqrt{3}i$

$\Leftrightarrow z^2 = 2e^{i\left(-\frac{\pi}{3}\right)}$

$\Leftrightarrow z = \sqrt{2}e^{i\left(-\frac{\pi}{6} + \frac{2k\pi}{2}\right)}, k \in \{0, 1\}$

$\Leftrightarrow z = \sqrt{2}e^{i\left(-\frac{\pi}{6} + k\pi\right)}, k \in \{0, 1\}$

Cálculos auxiliares

$\left|1 - \sqrt{3}i\right| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1 + 3} = 2$

$\text{tg}\,\alpha = -\sqrt{3} \wedge \alpha \in 4.^{\circ}\,Q$

Logo, $\alpha = -\frac{\pi}{3}$, por exemplo.

C. S. $= \left\{\sqrt{2}e^{i\left(-\frac{\pi}{6}\right)}, \sqrt{2}e^{i\frac{5\pi}{6}}\right\}$

36.6 $z^4 - 27z = 0$

$\Leftrightarrow z(z^3 - 27) = 0$

$\Leftrightarrow z = 0 \vee z^3 = 27$

$\Leftrightarrow z = 0 \vee z^3 = 27e^{i0}$

$\Leftrightarrow z = 0 \vee z_k = \sqrt[3]{27}\,e^{i\left(\frac{0}{3} + \frac{2k\pi}{3}\right)}, k \in \{0, 1, 2\}$

$\Leftrightarrow z = 0 \vee z_k = 3e^{i\frac{2k\pi}{3}}, k \in \{0, 1, 2\}$

C. S. $= \{0, 3e^{i0}, 3e^{i\frac{2\pi}{3}}, 3e^{i\frac{4\pi}{3}}\}$

36.7 $(iz)^4 + \sqrt{3} - i = 0$

$\Leftrightarrow z^4 = -\sqrt{3} + i$

$\Leftrightarrow z^4 = 2e^{i\frac{5\pi}{6}}$

$\Leftrightarrow z_k = \sqrt[4]{2}e^{i\left(\frac{5\pi}{24} + \frac{2k\pi}{4}\right)}, k \in \{0, 1, 2, 3\}$

$\Leftrightarrow z_k = \sqrt[4]{2}e^{i\left(\frac{5\pi}{24} + \frac{k\pi}{2}\right)}, k \in \{0, 1, 2, 3\}$

Cálculos auxiliares

$\left|-\sqrt{3} + i\right| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = 2$

$\text{tg}\,\alpha = -\frac{\sqrt{3}}{3} \wedge \alpha \in 2.^{\circ}\,Q$

Logo, $\alpha = \frac{5\pi}{6}$, por exemplo.

C. S. $= \left\{\sqrt[4]{2}e^{\frac{5\pi}{24}i}, \sqrt[4]{2}e^{\frac{17\pi}{24}i}, \sqrt[4]{2}e^{\frac{29\pi}{24}i}, \sqrt[4]{2}e^{\frac{41\pi}{24}i}\right\}$

36.8 $(2z)^5 - 16 = 16\sqrt{3}i$

$\Leftrightarrow 32z^5 = 16 + 16\sqrt{3}i$

$\Leftrightarrow z^5 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\Leftrightarrow z^5 = e^{i\frac{\pi}{3}}$

$\Leftrightarrow \rho^5 e^{i5\theta} = e^{i\frac{\pi}{3}}$

$\Leftrightarrow z_k = \sqrt[5]{1}e^{i\left(\frac{\pi}{15} + \frac{2k\pi}{5}\right)}, k \in \{0, 1, 2, 3, 4\}$

$\Leftrightarrow z_k = e^{i\left(\frac{\pi}{15} + \frac{2k\pi}{5}\right)}, k \in \{0, 1, 2, 3, 4\}$

C. S. $= \left\{e^{i\frac{\pi}{15}}, e^{i\frac{7\pi}{15}}, e^{i\frac{13\pi}{15}}, e^{i\frac{19\pi}{15}}, e^{i\frac{5\pi}{3}}\right\}$

36.9 $z^2 = -i\bar{z}$

$\Leftrightarrow \left(re^{\theta i}\right)^2 = e^{i\frac{3\pi}{2}} \times re^{i(-\theta)}$

$\Leftrightarrow r^2 e^{i2\theta} = re^{i\left(\frac{3\pi}{2} - \theta\right)}$

$\Leftrightarrow r^2 = r \wedge 2\theta = \frac{3\pi}{2} - \theta + 2k\pi, k \in \{0, 1, 2\}$

$\Leftrightarrow r = 0 \vee \left(r = 1 \wedge 3\theta = \frac{3\pi}{2} + 2k\pi, k \in \{0, 1, 2\}\right)$

$\Leftrightarrow r = 0 \vee \left(r = 1 \wedge \theta = \frac{\pi}{2} + \frac{2k\pi}{3}, k \in \{0, 1, 2\}\right)$

C. S. $= \left\{0, e^{i\frac{\pi}{2}}, e^{i\frac{7\pi}{6}}, e^{i\frac{11}{6}}\right\}$

36.10 $z^3\bar{z} + \sqrt{2}i - \sqrt{2} = 0$

$\Leftrightarrow \left(\rho e^{i\theta}\right)^3 \times \rho e^{i(-\theta)} = \sqrt{2} - \sqrt{2}i$

$\Leftrightarrow \rho^3 e^{i3\theta} \times \rho e^{i(-\theta)} = 2e^{i\left(-\frac{\pi}{4}\right)}$

$\Leftrightarrow \rho^4 e^{i2\theta} = 2e^{i\left(-\frac{\pi}{4}\right)}$

$\Leftrightarrow \rho^4 = 2 \wedge 2\theta = -\frac{\pi}{4} + 2k\pi, k \in \{0, 1\}$

$\Leftrightarrow \rho = \sqrt[4]{2} \wedge \theta = -\frac{\pi}{8} + k\pi, k \in \{0, 1\}$

Cálculos auxiliares

$\left|\sqrt{2} - \sqrt{2}i\right| = \sqrt{\left(\sqrt{2}\right)^2 + (-\sqrt{2})^2} = \sqrt{2 + 2} = 2$

$\text{tg}\,\alpha = -1 \wedge \alpha \in 4.^{\circ}\,Q$

Logo, $\alpha = -\frac{\pi}{4}$, por exemplo.

C. S. $= \left\{\sqrt[4]{2}e^{i\left(-\frac{\pi}{8}\right)}, \sqrt[4]{2}e^{i\frac{7\pi}{8}}\right\}$

37. $z_1 = 2\sqrt{3} - 2i$

$|z_1| = \sqrt{\left(2\sqrt{3}\right)^2 + (-2)^2} =$

$= \sqrt{12 + 4} =$

$= 4$

$\text{tg}\,\theta = -\frac{\sqrt{3}}{3} \wedge \theta \in 4.^{\circ}\,Q$

Logo, $\theta = -\frac{\pi}{6}$, por exemplo.

Assim, $z_1 = 4e^{i\left(-\frac{\pi}{6}\right)}$.

$z_2 = 4e^{i\frac{\pi}{30}}$

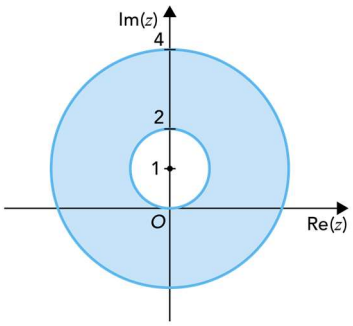
$\frac{\pi}{30} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{5} = \frac{2\pi}{10}$

Logo, z_1 e z_2 são raízes de índice 10 do mesmo número complexo, ou seja, $n = 10$.

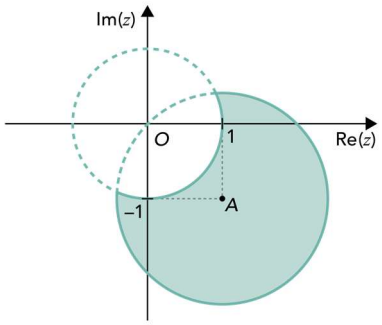
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38.

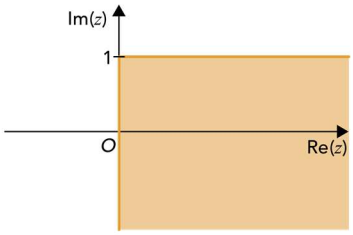
38.1 $1 \leq |z - i| \leq 3$



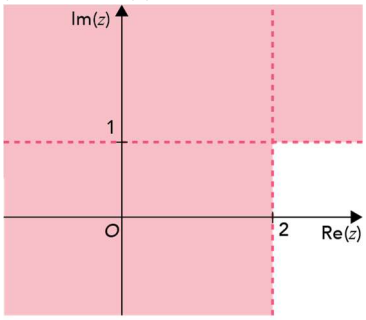
38.2 $|z| \geq 1 \wedge |z - 1 + i| \leq 2$



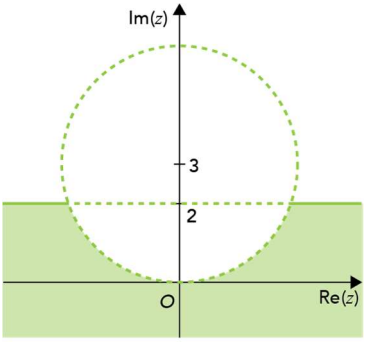
38.3 $-\frac{\pi}{2} \leq \arg(z - i) \leq 0$



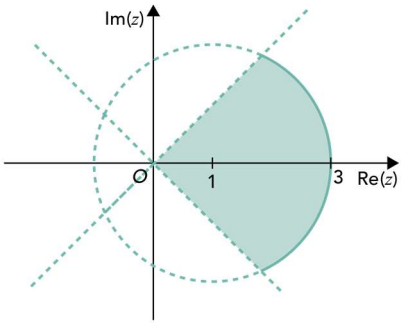
38.4 $\text{Re}(z) < 2 \vee \text{Im}(z) > 1$



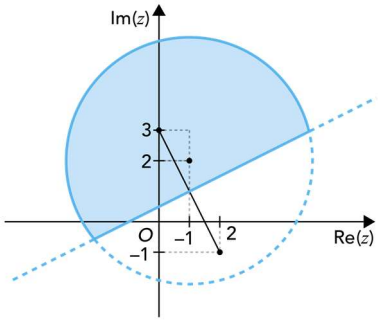
38.5 $|z - 3i| > 3 \wedge \text{Im}(z) \leq 2$



38.6 $-\frac{\pi}{4} < \arg(z) < \frac{\pi}{4} \wedge |z - 1| \leq 2$



38.7 $|z - 2 + i| \geq |z - 3i| \wedge |z - 1 - 2i| \leq 4$



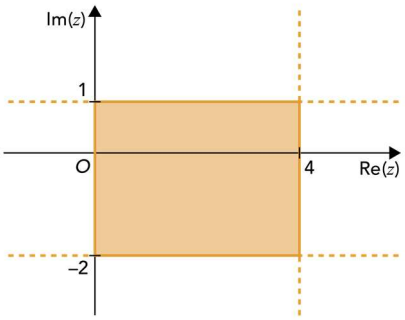
38.8 $-1 \leq \text{Re}(z - 1) \leq 3 \wedge -1 \leq \text{Im}(z - 2 + i) \leq 2$

$\Leftrightarrow -1 \leq \text{Re}(x + yi - 1) \leq 3 \wedge$

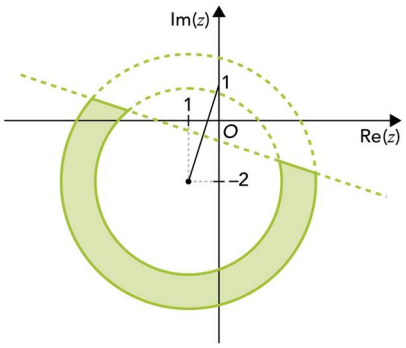
$\wedge -1 \leq \text{Im}(x + yi - 2 + i) \leq 2$

$\Leftrightarrow -1 \leq x - 1 \leq 3 \wedge -1 \leq y + 1 \leq 2$

$\Leftrightarrow 0 \leq x \leq 4 \wedge -2 \leq y \leq 1$

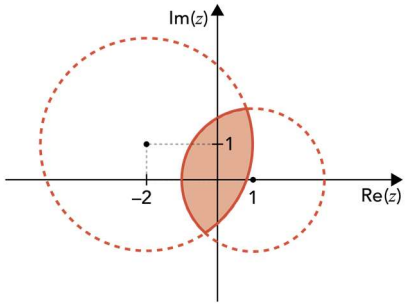


38.9 $|i - z| \geq |z + 1 + 2i| \wedge 3 \leq |z + 1 + 2i| \leq 4$



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38.10 $2|z-1| \leq 4 \wedge |z+2-i| \leq 3$



39.

39.1 $-\frac{\pi}{4} < \arg(z) \leq \pi$

39.2 $\text{Re}(z) < 1 \wedge \text{Im}(z) \leq 1$

39.3 $|z-1| \geq 1 \wedge |z-(1+i)| \leq 3$

39.4 $|z+i| \geq 1 \vee |z-(1+i)| \geq 2$

40.

$$\frac{\text{sen}\left(\alpha-\frac{3\pi}{2}\right)-\cos\left(\frac{\pi}{2}+\alpha\right)i}{-\cos\alpha+i\,\text{sen}\,\alpha} =$$

$$= \frac{-\text{sen}\left(\frac{3\pi}{2}-\alpha\right)+i\,\text{sen}\,\alpha}{\cos(\pi-\alpha)+i\,\text{sen}(\pi-\alpha)} =$$

$$= \frac{\text{sen}\left(\frac{\pi}{2}-\alpha\right)+i\,\text{sen}\,\alpha}{\cos(\pi-\alpha)+i\,\text{sen}(\pi-\alpha)} =$$

$$= \frac{\cos\alpha+i\,\text{sen}\,\alpha}{\cos(\pi-\alpha)+i\,\text{sen}(\pi-\alpha)} =$$

$$= \frac{e^{\alpha i}}{e^{(\pi-\alpha)i}} =$$

$$= e^{i(\alpha-\pi+\alpha)} =$$

$$= e^{i(2\alpha-\pi)}$$

41.

41.1 $2|z| = |z-i|$

$\Leftrightarrow 2|x+yi| = |x+yi-i|$

$\Leftrightarrow 2\sqrt{x^2+y^2} = \sqrt{x^2+(y-1)^2}$

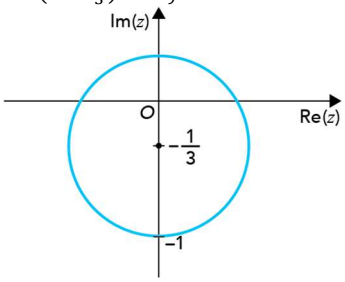
$\Leftrightarrow 4(x^2+y^2) = x^2+y^2-2y+1$

$\Leftrightarrow 3x^2+3y^2+2y = 1$

$\Leftrightarrow x^2+y^2+\frac{2}{3}y = \frac{1}{3}$

$\Leftrightarrow x^2+y^2+\frac{2}{3}y+\frac{1}{9} = \frac{1}{3}+\frac{1}{9}$

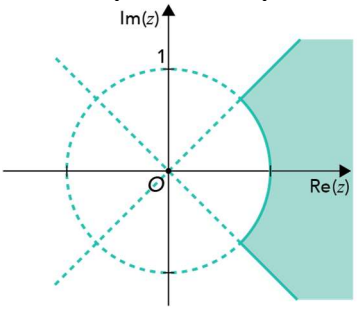
$\Leftrightarrow x^2+\left(y+\frac{1}{3}\right)^2 = \frac{4}{9}$



41.2 $\left|\frac{1}{z}\right| \leq 1 \wedge -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$

$\Leftrightarrow \frac{1}{|z|} \leq 1 \wedge -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$

$\Leftrightarrow |z| \geq 1 \wedge -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$



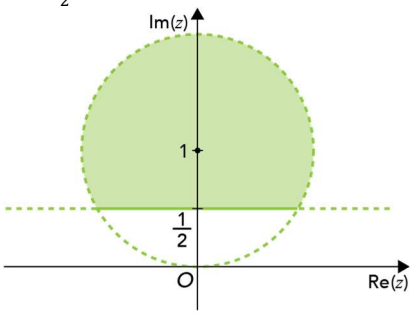
41.3 $\text{Im}(z-\bar{z}) \geq 1 \wedge |z-i| < 1$

$\Leftrightarrow \text{Im}\big((x+yi)-(x-yi)\big) \geq 1 \wedge |z-i| < 1$

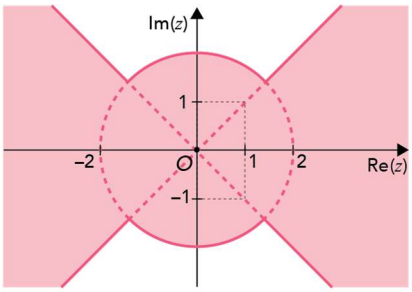
$\Leftrightarrow \text{Im}(2yi) \geq 1 \wedge |z-i| < 1$

$\Leftrightarrow 2y \geq 1 \wedge |z-i| < 1$

$\Leftrightarrow y \geq \frac{1}{2} \wedge |z-i| < 1$



41.4



42.

42.1 $\left(-\frac{3\pi}{4} \leq \arg(z) \leq -\frac{\pi}{4} \vee \frac{\pi}{4} \leq \arg(z) \leq \frac{3\pi}{4}\right) \wedge$

$\wedge\ -1 \leq \text{Re}(z) \leq 1$

42.2 $|z-(2+i)| \leq 2 \wedge \text{Re}(z) \geq 0 \wedge |z-(2+2i)| \leq |z|$

42.3 $|z-2| \leq 2 \vee \left(|z| \leq 4 \wedge \frac{3\pi}{4} \leq \arg(z) \leq \frac{5\pi}{4}\right)$

42.4 $|z| \leq 2 \wedge \left(0 \leq \arg(z) \leq \frac{\pi}{3} \vee \frac{2\pi}{3} \leq \arg(z) \leq \pi \vee\right.$

$\left. \vee \frac{4\pi}{3} \leq \arg(z) \leq \frac{5\pi}{3}\right)$

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Exercícios globais

Páginas 11 a 14

43. Opção (A)

$z = \rho e^{i\frac{\pi}{9}}$

$-i\bar{z} = e^{i\left(-\frac{\pi}{2}\right)} \times \rho e^{i\left(-\frac{\pi}{9}\right)} =$

$= \rho e^{i\left(-\frac{\pi}{2}-\frac{\pi}{9}\right)} =$

$= \rho e^{i\left(-\frac{11\pi}{18}\right)}$

Assim, um argumento de $-i\bar{z}$ é $-\frac{11\pi}{18}$.

44. $|z_1| = \sqrt{1^2+\sqrt{3}^2} = \sqrt{1+3} = 2$

$\text{tg}\,\theta = \sqrt{3} \wedge \theta \in 1.^{o}\,Q$. Logo, $\theta = \frac{\pi}{3}$, por exemplo.

Assim, $z_1 = 2e^{i\frac{\pi}{3}}$.

z_1, z_2, z_3 e z_4 são as raízes quartas de um mesmo número complexo.

Então:

$z_2 = 2e^{i\left(\frac{\pi}{2}+\frac{\pi}{3}\right)} = 2e^{i\frac{5\pi}{6}}$, na forma trigonométrica, e $z_2 = i\left(1+\sqrt{3}i\right) = -\sqrt{3}+i$, na forma algébrica.

$z_3 = 2e^{i\left(\frac{\pi}{2}+\frac{5\pi}{6}\right)} = 2e^{i\frac{4\pi}{3}}$, na forma trigonométrica, e $z_3 = i\left(-\sqrt{3}+i\right) = -1-\sqrt{3}i$, na forma algébrica.

$z_4 = 2e^{i\left(\frac{\pi}{2}+\frac{4\pi}{3}\right)} = 2e^{i\frac{11\pi}{6}}$, na forma trigonométrica, e $z_4 = i\left(-1-\sqrt{3}i\right) = \sqrt{3}-i$, na forma algébrica.

45. Opção (B)

Se $z_1 = a+bi$, então $\text{Re}(z_1) = a$ e $\text{Im}(z_1) = b$.

$iz_1 = i(a+bi) = ai+bi^2 = -b+ai$

$\text{Re}(iz_1) = -b \neq \text{Im}(z_1)$. Logo, a opção (A) é falsa.

$\text{Im}(iz_1) = a = \text{Re}(z_1)$ Logo, a opção (B) é verdadeira.

$z_1 \times z_2 = (a+bi)(c+di) =$

$= ac+adi+bci+bdi^2 =$

$= (ac-bd)+(ad+bc)i$

Se, por exemplo, $a = 0$ e $b, c, d \neq 0$, então:

$\text{Re}(z_1) \times \text{Re}(z_2) = a \times c \neq$

$\neq ac-bd = \text{Re}(z_1 \times z_2)$. Logo, a opção (C) é falsa.

$\frac{z_1}{z_2} = \frac{a+bi}{c+di} =$

$= \frac{a+bi}{c+di} \times \frac{c-di}{c-di} =$

$= \frac{ac-adi+bci-bd^{\,2}}{c^2+d^2} =$

$= \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$

Se, por exemplo, $a = 0$ e $b, c, d \neq 0$, então:

$\frac{\text{Im}(z_1)}{\text{Im}(z_2)} = \frac{b}{d} \neq \frac{bc-ad}{c^2+d^2} = \text{Im}\left(\frac{z_1}{z_2}\right)$. Logo, a opção (D) é falsa.

46. Opção (A)

$z = i^{2017}+2i^{2018}-3i^{2019}+4i^{2020} =$

$= i^1+2i^2-3i^3+4i^0 =$

$= i-2+3i+4 =$

$= 2+4i$

$\frac{1}{z} = \frac{1}{2+4i} =$

$= \frac{1}{2+4i} \times \frac{2-4i}{2-4i} =$

$= \frac{2-4i}{4+16} =$

$= \frac{1}{10} - \frac{1}{5}i$

47. Opção (A)

$z = 3 \times i^8 \times i^{-3} + 2 \times i^{28n+3} \times i^9 =$

$= 3 \times 1 \times i + 2 \times i^3 \times i =$

$= 3i + 2 \times (-i) \times i =$

$= 3i + 2 =$

$= 2+3i$

O afixo de z pertence ao primeiro quadrante.

48. Opção (C)

$\frac{i+i^2-i^3+ai^4}{i-2i^2-i^3-i^4} = bi$

$\Leftrightarrow \frac{i-1+i+a}{i+2+i-1} = bi$

$\Leftrightarrow \frac{a-1+2i}{1+2i} = bi$

$\Leftrightarrow a-1+2i = bi(1+2i)$

$\Leftrightarrow a-1+2i = bi+2bi^2$

$\Leftrightarrow a-1+2i = -2b+bi$

$\Leftrightarrow \begin{cases} a-1 = -2b \\ 2 = b \end{cases}$

$\Leftrightarrow \begin{cases} a-1 = -4 \\ b = 2 \end{cases}$

$\Leftrightarrow \begin{cases} a = -3 \\ b = 2 \end{cases}$

49. Opção (D)

$z = \rho e^{i\theta}$, com $\theta \in 2.^{o}\,Q$.

$w = -\bar{z} \times i^{2027} =$

$= -\rho e^{i(-\theta)} \times i^3 =$

$= -\rho e^{i(-\theta)} \times (-i) =$

$= \rho e^{i(-\theta)} \times i =$

$= \rho e^{i(-\theta)} \times e^{i\frac{\pi}{2}} =$

$= \rho e^{i\left(\frac{\pi}{2}-\theta\right)}$

Como $\theta \in 2.^{o}\,Q$, então:

$\frac{\pi}{2} < \theta < \pi \Leftrightarrow -\frac{\pi}{2} > -\theta > -\pi$

$\Leftrightarrow 0 > \frac{\pi}{2} - \theta > -\frac{\pi}{2}$

Logo, o afixo de w pertence ao quarto quadrante.

50. Opção (D)

z é um número complexo cujo afixo está no segundo quadrante e pertence à bissetriz dos quadrantes pares, logo $z = -a+ai$, com $a \in \mathbb{R}^+$.

$\frac{z}{\bar{z}} + i^{35} + i = \frac{-a+ai}{-a-ai} + i^3 + i =$

$= \frac{-a+ai}{-a-ai} \times \frac{-a+ai}{-a+ai} - i + i =$

$= \frac{a^2-2a^2i+a^2i^2}{a^2-a^2i^2} =$

$= \frac{-2a^2i}{2a^2} =$

$= -i$

Assim, o afixo de $\frac{z}{\bar{z}} + i^{35} + i$ pode ser o de z_4.

51. Opção (D)

As circunferências têm centro no ponto *A*, que é o afixo de 1 + 2*i*.

A circunferência menor é tangente ao eixo imaginário, pelo que o seu raio é 1.

A circunferência maior é tangente ao eixo real, pelo que o seu raio é 2.

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Assim, a coroa circular de centro *A* e raios 1 e 2 é definida, em $\mathbb{C}$, por:
$1 \leq |z-(1+2i)| \leq 2 \Leftrightarrow 1 \leq |z-1-2i| \leq 2$
A semirreta *BC* tem origem em *B* e é paralela à bissetriz dos quadrantes ímpares, logo uma condição que a define é $\arg(z-2i) = \frac{\pi}{4}$.
A semirreta *BD* tem origem em *B* e é perpendicular à semirreta *BC*, logo uma condição que a define é $\arg(z-2i) = -\frac{\pi}{4}$.
Então, a condição que define a região sombreada é:
$1 \leq |z-1-2i| \leq 2 \wedge -\frac{\pi}{4} \leq \arg(z-2i) \leq \frac{\pi}{4}$

52.

52.1 $\frac{\bar{w}}{-i \times w} = \frac{\sqrt{12}e^{i\left(-\frac{3\pi}{8}\right)}}{e^{i\left(-\frac{\pi}{2}\right)} \times \sqrt{12}e^{i\frac{3\pi}{8}}} =$
$= \frac{e^{i\left(-\frac{3\pi}{8}\right)}}{e^{i\left(-\frac{\pi}{2}+\frac{3\pi}{8}\right)}} =$
$= \frac{e^{i\left(-\frac{3\pi}{8}\right)}}{e^{i\left(-\frac{\pi}{8}\right)}} =$
$= e^{i\left(-\frac{3\pi}{8}+\frac{\pi}{8}\right)} =$
$= e^{i\left(-\frac{\pi}{4}\right)} =$
$= \cos\left(-\frac{\pi}{4}\right) + i\,\text{sen}\left(-\frac{\pi}{4}\right) =$
$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

52.2 $|z|^{-2} \times z^4 = w^8$
$\Leftrightarrow \rho^{-2} \times \left(\rho e^{i\theta}\right)^4 = \left(\sqrt{12}e^{i\frac{3\pi}{8}}\right)^8$
$\Leftrightarrow \rho^{-2} \times \rho^4 e^{i4\theta} = 12^4 e^{i3\pi}$
$\Leftrightarrow \rho^2 e^{i4\theta} = 12^4 e^{i3\pi}$
$\Leftrightarrow \rho^2 = 12^4 \wedge 4\theta = 3\pi + 2k\pi, k \in \{0,1,2,3\}$
$\Leftrightarrow \rho = 12^2 \wedge \theta = \frac{3\pi}{4} + \frac{k\pi}{2}, k \in \{0,1,2,3\}$
$\Leftrightarrow \rho = 144 \wedge \theta = \frac{3\pi}{4} + \frac{k\pi}{2}, k \in \{0,1,2,3\}$
C. S. $= \left\{144e^{i\frac{3\pi}{4}}, 144e^{i\frac{5\pi}{4}}, 144e^{i\frac{7\pi}{4}}, 144e^{i\frac{\pi}{4}}\right\}$

53. $z_1 = \frac{\sqrt{3}+\sqrt{3}i}{\frac{\sqrt{6}}{9}e^{i\left(-\frac{\pi}{3}\right)}} = \frac{\sqrt{6}e^{i\frac{\pi}{4}}}{\frac{\sqrt{6}}{9}e^{i\left(-\frac{\pi}{3}\right)}} = 9e^{i\frac{7\pi}{12}}$

Cálculo auxiliar

$\rho = \sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2} = \sqrt{3+3} = \sqrt{6}$

$z_2 = \left(\sqrt{3}e^{i\frac{3\pi}{16}}\right)^4 = \left(\sqrt{3}\right)^4 e^{4\times i\frac{3\pi}{16}} = 9e^{i\frac{3\pi}{4}}$
Como as imagens geométricas dos complexos z_1 e z_2 são vértices consecutivos de um polígono regular de n lados, com centro na origem do referencial, então:
$\frac{3\pi}{4} - \frac{7\pi}{12} = \frac{2\pi}{n} \Leftrightarrow \frac{\pi}{6} = \frac{2\pi}{n} \Leftrightarrow n = 12$

54.

54.1 Opção (D)
$z_1 = 2 - i^{13} = 2 - i^1 = 2 - i$
$|z_1| = \sqrt{2^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$
O afixo do número complexo apresentado na opção (A) pertence à bissetriz dos quadrantes pares, mas $|2-2i| = \sqrt{2^2+(-2)^2} = \sqrt{8} > |z_1|$. Logo, a opção (A) não corresponde a w.
O afixo do número complexo apresentado na opção (B) pertence à bissetriz dos quadrantes pares e
$\left|\frac{3}{2} - \frac{3}{2}i\right| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} =$
$= \sqrt{\frac{9}{4}+\frac{9}{4}} =$
$= \sqrt{\frac{9}{2}} < |z_1|$
No entanto, $\frac{3}{2} - \frac{3}{2}i + \left(\overline{\frac{3}{2} - \frac{3}{2}i}\right) =$
$= \frac{3}{2} - \frac{3}{2}i + \frac{3}{2} + \frac{3}{2}i = 3 > 2$
Logo, a opção (B) não corresponde a w.
O afixo do número complexo apresentado na opção (C) não pertence à bissetriz dos quadrantes pares, logo a opção (C) não corresponde a w.
O afixo do número complexo apresentado na opção (D) pertence à bissetriz dos quadrantes pares e
$\left|\frac{1}{2} - \frac{1}{2}i\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} =$
$= \sqrt{\frac{1}{4}+\frac{1}{4}} =$
$= \sqrt{\frac{1}{2}} < |z_1|$
$\frac{1}{2} - \frac{1}{2}i + \left(\overline{\frac{1}{2} + \frac{1}{2}i}\right) = \frac{1}{2} - \frac{1}{2}i + \frac{1}{2} + \frac{1}{2}i = 1 < 2$
Logo, a opção (D) pode ser w.

54.2 $z_1 = 2 - i^{13} = 2 - i^1 = 2 - i$
$z = \frac{\overline{z_1} \times z_2 + i^{4n+3}}{(z_2)^3} =$
$= \frac{(2+i) \times (-1+i) + i^3}{(-1+i)^3} =$
$= \frac{(-2+2i-i+i^2) - i}{(-1+i)^2(-1+i)} =$
$= \frac{-3+i-i}{(1-2i+i^2)(-1+i)} =$
$= \frac{-3}{-2i(-1+i)} =$
$= \frac{-3}{2i-2i^2} =$
$= \frac{-3}{2+2i} =$
$= \frac{-3}{2+2i} \times \frac{2-2i}{2-2i} =$
$= \frac{-6+6i}{2^2-(2i)^2} =$
$= \frac{-6}{8} + \frac{6}{8}i =$
$= -\frac{3}{4} + \frac{3}{4}i$
$|z| = \sqrt{\left(-\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2} =$
$= \sqrt{\frac{9}{16}+\frac{9}{16}} =$
$= \sqrt{\frac{18}{16}} =$
$= \frac{3\sqrt{2}}{4}$
$\text{tg}\,\theta = -1 \wedge \theta \in 2.^{\circ}\,\text{Q}$. Logo, $\theta = \frac{3\pi}{4}$, por exemplo.
Então, $z = \frac{3\sqrt{2}}{4}e^{i\frac{3\pi}{4}}$.

55.

55.1 $z_1 = e^{i\frac{5\pi}{6}} =$
$= \cos\frac{5\pi}{6} + i\,\text{sen}\frac{5\pi}{6} =$
$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

Substituindo na equação:

$\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2-1+e^{i\frac{\pi}{3}}=0$

$\Leftrightarrow \frac{3}{4}-\frac{\sqrt{3}}{2}i+\frac{1}{4}i^2-1+\cos\frac{\pi}{3}+i\,\text{sen}\frac{\pi}{3}=0$

$\Leftrightarrow \frac{3}{4}-\frac{\sqrt{3}}{2}i-\frac{1}{4}-1+\frac{1}{2}+\frac{\sqrt{3}}{2}i=0$

$\Leftrightarrow \frac{1}{2}-\frac{1}{2}=0$

$\Leftrightarrow 0=0$ Proposição verdadeira

Logo, z_1 é solução da equação.

$z_2=e^{i\frac{11\pi}{6}}=\cos\frac{11\pi}{6}+i\,\text{sen}\frac{11\pi}{6}=\frac{\sqrt{3}}{2}-\frac{1}{2}i$

Substituindo na equação:

$\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2-1+e^{i\frac{\pi}{3}}=0$

$\Leftrightarrow \frac{3}{4}-\frac{\sqrt{3}}{2}i+\frac{1}{4}i^2-1+\cos\frac{\pi}{3}+i\,\text{sen}\frac{\pi}{3}=0$

$\Leftrightarrow \frac{3}{4}-\frac{\sqrt{3}}{2}i-\frac{1}{4}-1+\frac{1}{2}+\frac{\sqrt{3}}{2}i=0$

$\Leftrightarrow \frac{1}{2}-\frac{1}{2}=0$

$\Leftrightarrow 0=0$ Proposição verdadeira

Logo, z_2 é solução da equação.

Outro processo

$z^2-1+e^{i\frac{\pi}{3}}=0$

$\Leftrightarrow z^2-1+\cos\frac{\pi}{3}+i\,\text{sen}\frac{\pi}{3}=0$

$\Leftrightarrow z^2-1+\frac{1}{2}+\frac{\sqrt{3}}{2}i=0$

$\Leftrightarrow z^2-\frac{1}{2}+\frac{\sqrt{3}}{2}i=0$

$\Leftrightarrow z^2=\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Cálculo auxiliar

$\left|\frac{1}{2}-\frac{\sqrt{3}}{2}i\right|=\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$

$\text{tg}\,\theta=-\sqrt{3}\wedge\theta\in 4.^{o}\,Q$

Logo, $\theta=-\frac{\pi}{3}$, por exemplo.

Assim:

$z^2=\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$\Leftrightarrow \left(\rho e^{i\theta}\right)^2=e^{i\left(-\frac{\pi}{3}\right)}$

$\Leftrightarrow \rho^2e^{i2\theta}=e^{i\left(-\frac{\pi}{3}\right)}$

$\Leftrightarrow \rho^2=1\wedge 2\theta=-\frac{\pi}{3}+2k\pi,k\in\{0,1\}$

$\Leftrightarrow \rho=1\wedge\theta=-\frac{\pi}{6}+k\pi,k\in\{0,1\}$

Se $k=0$, $z=e^{i\left(-\frac{\pi}{6}\right)}=e^{i\frac{11\pi}{6}}=z_2$.

Se $k=1$, $z=e^{i\frac{5\pi}{6}}=z_1$.

55.2 $z_1=e^{i\frac{5\pi}{6}}=$

$=\cos\frac{5\pi}{6}+i\,\text{sen}\frac{5\pi}{6}=$

$=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$z_2=e^{i\frac{11\pi}{6}}=$

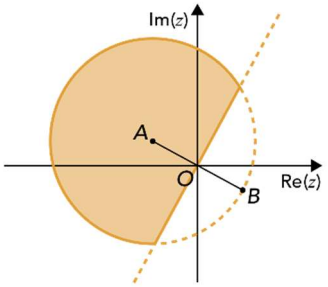
$=\cos\frac{11\pi}{6}+i\,\text{sen}\frac{11\pi}{6}=$

$=\frac{\sqrt{3}}{2}-\frac{1}{2}i$

Assim, $|z-z_1|\leq|z-z_2|\wedge|z-z_1|\leq 2$

$\Leftrightarrow \left|z-\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right|\leq\left|z-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\right|\wedge$

$\wedge\left|z-\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right|\leq 2$



A é o afixo de z_1 e B é o afixo de z_2.

56.

56.1 $z_1^3-z_1^2+2z_1+4=$

$=\left(1+\sqrt{3}i\right)^3-\left(1+\sqrt{3}i\right)^2+2\left(1+\sqrt{3}i\right)+4=$

$=\left(1+2\sqrt{3}i+3i^2\right)\left(1+\sqrt{3}i\right)-\left(1+2\sqrt{3}i+3i^2\right)+$

$+2+2\sqrt{3}i+4=$

$=\left(-2+2\sqrt{3}i\right)\left(1+\sqrt{3}i\right)+2-2\sqrt{3}i+6+2\sqrt{3}i=$

$=-2-2\sqrt{3}i+2\sqrt{3}i+6i^2+8=$

$=-2-6+8=$

$=0$

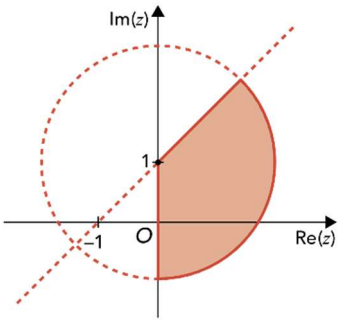
Logo, z_1 é raiz do polinómio z^3-z^2+2z+4.

56.2 $|z_1|=\sqrt{1^2+\left(\sqrt{3}\right)^2}=2$

Logo:

$|z-i|\leq|z_1|\wedge-\frac{\pi}{2}\leq\arg(z-i)\leq\frac{\pi}{4}$

$\Leftrightarrow |z-i|\leq 2\wedge-\frac{\pi}{2}\leq\arg(z-i)\leq\frac{\pi}{4}$



$A_R=\frac{\frac{3\pi}{4}}{2}\times 2^2=\frac{3\pi}{2}$ u.a.

56.3 $|z_1|=\sqrt{1^2+\left(\sqrt{3}\right)^2}=2$

$\text{tg}\,\theta_1=\sqrt{3}\wedge\theta_1\in 1.^{o}\,Q$

Logo, $\theta_1=\frac{\pi}{3}$, por exemplo.

$(z_1)^n=\left(2e^{i\frac{\pi}{3}}\right)^n=2^ne^{i\frac{n\pi}{3}}$

Para que $(z_1)^n$ seja um número real negativo:

$\frac{n\pi}{3}=\pi+2k\pi,k\in\mathbb{Z}\Leftrightarrow n=3+6k,k\in\mathbb{Z}$

O menor valor de n natural, nestas condições, obtém-se quando $k=0$, tendo-se, então, $n=3$.

57.

57.1 $(z_1)^3=\left(2e^{i\frac{7\pi}{12}}\right)^3=$

$=2^3e^{i\frac{7\pi}{4}}=$

$=8\left(\cos\frac{7\pi}{4}+i\,\text{sen}\frac{7\pi}{4}\right)=$

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$= 8\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) =$

$= 4\sqrt{2} - 4\sqrt{2}i$

Logo:

$2k - 2ki = 4\sqrt{2} - 4\sqrt{2}i$

$\Leftrightarrow 2k = 4\sqrt{2}$

$\Leftrightarrow k = 2\sqrt{2}$

57.2 $z = (z_1)^6 =$

$= \left(2e^{i\frac{7\pi}{12}}\right)^6 =$

$= 2^6 e^{i\frac{7\pi}{2}} =$

$= 64e^{i\left(-\frac{\pi}{2}\right)} =$

$= -64i$

As restantes raízes sextas de z são:

$2e^{i\left(\frac{7\pi}{12}+\frac{2\pi}{6}\right)} = 2e^{i\frac{11\pi}{12}}$

$2e^{i\left(\frac{7\pi}{12}+\frac{4\pi}{6}\right)} = 2e^{i\frac{5\pi}{4}}$

$2e^{i\left(\frac{7\pi}{12}+\frac{6\pi}{6}\right)} = 2e^{i\frac{19\pi}{12}}$

$2e^{i\left(\frac{7\pi}{12}+\frac{8\pi}{6}\right)} = 2e^{i\frac{23\pi}{12}}$

$2e^{i\left(\frac{7\pi}{12}+\frac{10\pi}{6}\right)} = 2e^{i\frac{9\pi}{4}} = 2e^{i\frac{\pi}{4}}$

57.3 $\frac{(z_1)^2 \times i^{26}}{(z_1)^n} = \frac{\left(2e^{i\frac{7\pi}{12}}\right)^2 \times i^2}{\left(2e^{i\frac{7\pi}{12}}\right)^n} =$

$= \frac{4e^{i\frac{7\pi}{6}} \times (-1)}{2^n e^{i\frac{7n\pi}{12}}} =$

$= \frac{4e^{i\left(\pi+\frac{7\pi}{6}\right)}}{2^n e^{i\frac{7n\pi}{12}}} =$

$= \frac{4e^{i\frac{13\pi}{6}}}{2^n e^{i\frac{7n\pi}{12}}} =$

$= \frac{4e^{i\frac{\pi}{6}}}{2^n e^{i\frac{7n\pi}{12}}} =$

$= \frac{4}{2^n} \times e^{\left(\frac{\pi}{6}-\frac{7n\pi}{12}\right)i}$

Para que o número complexo $\frac{(z_1)^2 \times i^{26}}{(z_1)^n}$ seja um imaginário puro, então:

$\frac{\pi}{6} - \frac{7n\pi}{12} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\Leftrightarrow 2 - 7n = 6 + 12k, k \in \mathbb{Z}$

$\Leftrightarrow -7n = 4 + 12k, k \in \mathbb{Z}$

$\Leftrightarrow n = -\frac{4}{7} - \frac{12}{7}k, k \in \mathbb{Z}$

O menor valor de n natural obtém-se quando $k = -5$ e é $n = 8$.

58.

58.1 $\frac{z_1 \times z_2 - 2\sqrt{6}}{(z_1)^3} = \frac{(3-\sqrt{3}i)e^{i\frac{5\pi}{12}} - 2\sqrt{6}}{(3-\sqrt{3}i)^3}$

Cálculos auxiliares

$|z_1| = \sqrt{3^2 + \left(\sqrt{3}\right)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$\text{tg}\,\theta_1 = -\frac{\sqrt{3}}{3} \wedge \theta_1 \in 4.^{\circ}\,Q$

Logo, $\theta_1 = -\frac{\pi}{6}$, por exemplo.

$= \frac{2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)} \times e^{i\frac{5\pi}{12}} - 2\sqrt{6}}{\left(2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}\right)^3} =$

$= \frac{2\sqrt{3}e^{i\frac{\pi}{4}} - 2\sqrt{6}}{2^3 \times 3\sqrt{3} \times e^{i\left(-\frac{\pi}{2}\right)}} =$

$= \frac{\left(\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right) - \sqrt{2}}{-4 \times 3i} =$

$= \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i - \sqrt{2}}{-12i} =$

$= \frac{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}{-12i} =$

$= \frac{\sqrt{2} - \sqrt{2}i}{24i} \times \frac{-i}{-i} =$

$= \frac{-\sqrt{2}i + \sqrt{2}i^2}{24} =$

$= -\frac{\sqrt{2}}{24} - \frac{\sqrt{2}}{24}i$

58.2 $z_1 = 3 - \sqrt{3}i = 2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}$

$z_2 = z_3 = 2\sqrt{3}e^{i\frac{\pi}{3}}$

$\frac{\pi}{3} - \left(-\frac{\pi}{6}\right) = \frac{2\pi}{n} \Leftrightarrow \frac{3\pi}{6} = \frac{2\pi}{n}$

$\Leftrightarrow n = \frac{12\pi}{3\pi}$

$\Leftrightarrow n = 4$

O valor de n é 4.

58.3 $z^4 = -9\overline{z_2}$

$\Leftrightarrow \left(re^{i\theta}\right)^4 = -9e^{i\left(-\frac{5\pi}{12}\right)}$

$\Leftrightarrow r^4 e^{i4\theta} = 9e^{i\frac{7\pi}{12}}$

$\Leftrightarrow r^4 = 9 \wedge 4\theta = \frac{7\pi}{12} + 2k\pi, k \in \{0,1,2,3\}$

$\Leftrightarrow r = \sqrt{3} \wedge \theta = \frac{7\pi}{48} + \frac{k\pi}{2}, k \in \{0,1,2,3\}$

C.S. $= \left\{\sqrt{3}e^{i\frac{7\pi}{48}}, \sqrt{3}e^{i\frac{31\pi}{48}}, \sqrt{3}e^{i\frac{55\pi}{48}}, \sqrt{3}e^{i\frac{79\pi}{48}}\right\}$

58.4 $w = i^{2019} \times e^{i2\theta} =$

$= i^{4\times 504} \times e^{i2\theta} =$

$= i^3 \times e^{i2\theta} =$

$= -i \times e^{i2\theta} =$

$= e^{i\frac{3\pi}{2}} \times e^{i2\theta} =$

$= e^{i\left(\frac{3\pi}{2}+2\theta\right)}$

$\frac{w}{z_3} = \frac{e^{i\left(\frac{3\pi}{2}+2\theta\right)}}{2\sqrt{3}e^{i\frac{\pi}{3}}} = \frac{\sqrt{3}}{6}e^{i\left(2\theta+\frac{7\pi}{6}\right)}$

Para que $\frac{w}{z_3}$ seja um imaginário puro:

$2\theta + \frac{7\pi}{6} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\Leftrightarrow 2\theta = -\frac{2\pi}{3} + k\pi, k \in \mathbb{Z}$

$\Leftrightarrow \theta = -\frac{\pi}{3} + \frac{k\pi}{2}, k \in \mathbb{Z}$

Como $\theta \in]-\pi, \pi[$, então $\theta = -\frac{\pi}{3}$ (para $k = 0$) ou $\theta = \frac{\pi}{6}$ (para $k = 1$) ou $\theta = \frac{2\pi}{3}$ (para $k = 2$) ou $\theta = -\frac{5\pi}{6}$ (para $k = -1$).

59.

59.1 $\frac{1}{(\overline{z_1})^2} = \frac{1}{\left(3+\sqrt{3}i\right)^2} =$

$= \frac{1}{9 + 6\sqrt{3}i + 3i^2} =$

$= \frac{1}{6 + 6\sqrt{3}i} =$

$= \frac{1}{6} \times \frac{1}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} =$

$= \frac{1}{6} \times \frac{1-\sqrt{3}i}{1+3} =$

$= \frac{1}{24}\left(1 - \sqrt{3}i\right) =$

$= \frac{1}{24} - \frac{\sqrt{3}}{24}i$

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59.2 $z_1 = 3 - \sqrt{3}i$

Cálculos auxiliares
$\rho_1 = \sqrt{9+3} = 2\sqrt{3}$
$\text{tg}\,\theta_1 = -\frac{\sqrt{3}}{3} \wedge \theta_1 \in 4.^{\circ}\,Q$
Logo, $\theta_1 = -\frac{\pi}{6}$, por exemplo.

$z \times z_1 = e^{i\theta} \times 2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)} =$
$= 2\sqrt{3}e^{i\left(\theta - \frac{\pi}{6}\right)}$

Assim, $z_1 = 2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}$.

Para que $z \times z_1$ seja um imaginário puro:
$\theta - \frac{\pi}{6} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow \theta = \frac{2\pi}{3} + k\pi, k \in \mathbb{Z}$.

Os valores de θ são, então, $\frac{2\pi}{3}$ (para $k = 0$) e $\frac{5\pi}{3}$ (para $k = 1$).

59.3 De acordo com a alínea anterior, $z_1 = 2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}$.

Logo, $(z_1)^4 = \left(2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}\right)^4 =$
$= 16 \times 9e^{i\left(-\frac{4\pi}{6}\right)} =$
$= 144e^{i\left(-\frac{2\pi}{3}\right)}$

O polígono cujos vértices são as imagens geométricas, no plano complexo, das raízes quartas de $(z_1)^4$ é um quadrado inscrito numa circunferência centrada na origem do referencial e de raio $\sqrt[4]{|(z_1)^4|} = |z_1| = 2\sqrt{3}$.

Como as diagonais de um quadrado formam um ângulo reto, pelo teorema de Pitágoras, sendo x o comprimento do lado do quadrado, tem-se:
$x^2 = \left(2\sqrt{3}\right)^2 + \left(2\sqrt{3}\right)^2 \Leftrightarrow x^2 = 12 + 12 \Leftrightarrow x^2 = 24$

Logo, a área do polígono cujos vértices são as imagens geométricas, no plano complexo, das raízes quartas de z_1 é 24 u.a.

59.4 Opção (A)

A condição $|z - z_1| \leq 3$ define o círculo de centro no ponto de coordenadas $(3, -\sqrt{3})$ e raio 3, pelo que se exclui a opção (C).

A condição $0 \leq \arg(z - z_1) \leq \frac{\pi}{2}$ define o ângulo cujo vértice é o ponto de coordenadas $(3, -\sqrt{3})$ e cujos lados são as semirretas com origem nesse ponto e que são paralelas às semirretas definidas por $\arg(z) = 0$ e $\arg(z) = \frac{\pi}{2}$, pelo que se excluem as opções (B) e (D).

60. Seja $z = a + bi, a, b \in \mathbb{R}$.

$\frac{z}{z-i} = \frac{a+bi}{a+bi-i} =$
$= \frac{a+bi}{a+(b-1)i} \times \frac{a-(b-1)i}{a-(b-1)i} =$
$= \frac{a^2 - a(b-1)i + abi - b(b-1)i^2}{a^2 + (b-1)^2} =$
$= \frac{(a^2+b^2-b) + (-ab+a+ab)i}{a^2 + (b-1)^2} =$
$= \frac{(a^2+b^2-b) + ai}{a^2 + (b-1)^2} =$
$= \frac{a^2 + b^2 - b}{a^2 + (b-1)^2} + \frac{a}{a^2 + (b-1)^2}i$

Como $\frac{z}{z-i}$ é um número real, não nulo, então:
$\frac{a}{a^2 + (b-1)^2} = 0 \wedge \frac{a^2 + b^2 - b}{a^2 + (b-1)^2} \neq 0$
$\Leftrightarrow a = 0 \wedge b^2 - b \neq 0$
$\Leftrightarrow a = 0 \wedge b(b-1) \neq 0$
$\Leftrightarrow a = 0 \wedge b \neq 0 \wedge b \neq 1$

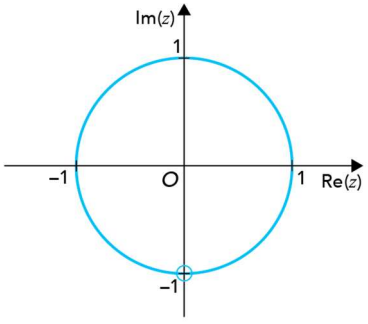
Logo $z = bi$, com $b \neq 0$ e $b \neq 1$, ou seja, z é um número imaginário puro.

61. Seja $z = x + yi$, com $1 - iz \neq 0 \Leftrightarrow z \neq \frac{1}{i} \Leftrightarrow z \neq -i$.

Tem-se que:
$\frac{z-i}{1-iz} = \frac{x+yi-i}{1-i(x+yi)} =$
$= \frac{x+(y-1)i}{1-xi-yi^2} =$
$= \frac{x+(y-1)i}{(1+y)-xi} =$
$= \frac{x+(y-1)i}{(1+y)-xi} \times \frac{(1+y)+xi}{(1+y)+xi} =$
$= \frac{x(1+y) + x^2i + (y-1)(y+1)i + x(y-1)i^2}{(1+y)^2 + x^2} =$
$= \frac{x + xy + x^2i + (y^2-1)i - x(y-1)}{x^2 + (y+1)^2} =$
$= \frac{x + xy - xy + x + x^2i + (y^2-1)i}{x^2 + (y+1)^2} =$
$= \frac{2x}{x^2 + (y+1)^2} + \frac{x^2 + y^2 - 1}{x^2 + (y+1)^2}i$

Para que este seja um número real:
$\frac{x^2 + y^2 - 1}{x^2 + (y+1)^2} = 0 \Leftrightarrow x^2 + y^2 - 1 = 0 \Leftrightarrow x^2 + y^2 = 1$

Os números complexos para os quais a expressão apresentada corresponde a um número real são da forma $z = x + yi$, com $x^2 + y^2 = 1$ e $z \neq -i$.



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