Primitivas imediatas e integrais definidos

Páginas 78 a 83

1. Opção (D)

$\int 6x\ dx = 3\int 2x\ dx =$

$= 3x^2 + c, c \in \text{IR}$

2.

2.1 $\int \left(\frac{x^3}{3} + \frac{x^2}{2} - x + 1\right) dx =$

$= \frac{1}{3} \times \frac{x^4}{4} + \frac{1}{2} \times \frac{x^3}{3} - \frac{x^2}{2} + x + c =$

$= \frac{x^4}{12} + \frac{x^3}{6} - \frac{x^2}{2} + x + c, c \in \text{IR}$

2.2 $\int \frac{3}{x^3} dx =$

$= 3\int x^{-3} dx =$

$= 3 \times \frac{x^{-2}}{-2} + c =$

$= -\frac{3}{2x^2} + c, c \in \text{IR}$

2.3 $\int \frac{3x}{(2x^2+1)^3} dx =$

$= \frac{3}{4}\int (4x(2x^2+1)^{-3})\, dx =$

$= \frac{3}{4} \times \frac{(2x^2+1)^{-2}}{-2} + c =$

$= -\frac{3}{8(2x^2+1)^2} + c, c \in \text{IR}$

2.4 $\int x^2 (2x^3+2)^3 dx =$

$= \frac{1}{6}\int 6x^2(2x^3+2)^3 dx =$

$= \frac{1}{6} \times \frac{(2x^3+2)^4}{4} + c =$

$= \frac{(2x^3+2)^4}{24} + c, c \in \text{IR}$

2.5 $\int (x-1)(x^2-2x+3)^2 dx =$

$= \frac{1}{2}\int (2x-2)(x^2-2x+3)^2 dx =$

$= \frac{1}{2} \times \frac{(x^2-2x+3)^3}{3} + c =$

$= \frac{(x^2-2x+3)^3}{6} + c, c \in \text{IR}$

2.6 $\int \sqrt[3]{x} dx =$

$= \int x^{\frac{1}{3}} dx =$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c =$

$= \frac{3}{4} x \sqrt[3]{x} + c, c \in \text{IR}$

2.7 $\int \sqrt{2x+1} dx =$

$= \frac{1}{2}\int 2(2x+1)^{\frac{1}{2}} dx =$

$= \frac{1}{2} \times \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} + c =$

$= \frac{(2x+1)\sqrt{2x+1}}{3} + c, c \in \text{IR}$

2.8 $\int x\sqrt{x^2+2} dx =$

$= \frac{1}{2}\int 2x(x^2+2)^{\frac{1}{2}} dx =$

$= \frac{1}{2} \times \frac{(x^2+2)^{\frac{3}{2}}}{\frac{3}{2}} + c =$

$= \frac{(x^2+2)\sqrt{x^2+2}}{3} + c, c \in \text{IR}$

3.

3.1 $\int \frac{2}{x-1} dx =$

$= 2\int \frac{1}{x-1} dx =$

$= 2\ln|x-1| + c, c \in \text{IR}$

3.2 $\int \frac{x+1}{x^2+2x-1} dx =$

$= \frac{1}{2}\int \frac{2x+2}{x^2+2x-1} dx =$

$= \frac{1}{2}\ln|x^2+2x-1| + c, c \in \text{IR}$

3.3 $\int \frac{4x^2}{1-2x^3} dx =$

$= -\frac{4}{6}\int \frac{-6x^2}{1-2x^3} dx =$

$= -\frac{2}{3}\ln|1-2x^3| + c, c \in \text{IR}$

3.4 $\int \left(\frac{2}{x-3} + \frac{1}{1-x}\right) dx =$

$= 2\int \frac{1}{x-3} dx - \int \frac{-1}{1-x} dx =$

$= 2\ln|x-3| - \ln|1-x| + c, c \in \text{IR}$

4.

4.1 $\int e^{-2x} dx =$

$= -\frac{1}{2}\int -2e^{-2x} dx =$

$= -\frac{1}{2}e^{-2x} + c, c \in \text{IR}$

4.2 $\int e^{\frac{x}{3}} dx = 3\int \frac{1}{3} e^{\frac{x}{3}} dx =$

$= 3e^{\frac{x}{3}} + c, c \in \text{IR}$

4.3 $\int xe^{x^2} dx = \frac{1}{2}\int 2xe^{x^2} dx =$

$= \frac{1}{2}e^{x^2} + c, c \in \text{IR}$

4.4 $\int (2e^x - 3x^2) dx = 2e^x - x^3 + c, c \in \text{IR}$

5.

5.1 $\int \cos(3x)\, dx =$

$= \frac{1}{3}\int 3\cos(3x)\, dx =$

$= \frac{1}{3}\text{sen}(3x) + c, c \in \text{IR}$

5.2 $\int x\cos(x^2+2) dx =$

$= \frac{1}{2}\int 2x\cos(x^2+2)\, dx =$

$= \frac{1}{2}\text{sen}(x^2+2) + c, c \in \text{IR}$

5.3 $\int 3x^2 \text{sen}(x^3-1) dx =$

$= -\cos(x^3-1) + c, c \in \text{IR}$

5.4 $\int 2\cos\left(\frac{\pi}{2} - x\right) dx =$

$= 2\int \text{sen}\, x\, dx =$

$= -2\cos x + c, c \in \text{IR}$

6. Opção (B)

$G(x) = \int \text{sen}\, x\ dx =$

$= -\cos x + c, c \in \text{IR}$

$G(0) = 2 \Leftrightarrow -\cos 0 + c = 2$

$\Leftrightarrow -1 + c = 2$

$\Leftrightarrow c = 3$

Logo, $G(x) = -\cos x + 3$.

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7. Opção (C)

$\int_0^1 x^2\ dx = \left[\frac{x^3}{3}\right]_0^1 =$

$= \frac{1^3}{3} - \frac{0^3}{3} =$

$= \frac{1}{3} - 0 =$

$= \frac{1}{3}$

8. Opção (B)

$\int_{-1}^1 (1 - x^2)\ dx\ =$

$= \left[x - \frac{x^3}{3}\right]_{-1}^1 =$

$= \left(1 - \frac{1^3}{3}\right) - \left((-1) - \frac{(-1)^3}{3}\right) =$

$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) =$

$= \frac{2}{3} + \frac{2}{3} =$

$= \frac{4}{3}$

9.

9.1 $\int \frac{x^3}{\sqrt{x^4+2}} dx =$

$= \frac{1}{4} \int 4x^3 (x^4 + 2)^{-\frac{1}{2}} dx =$

$= \frac{1}{4} \times \frac{(x^4+2)^{\frac{1}{2}}}{\frac{1}{2}} + c =$

$= \frac{1}{2}\sqrt{x^4 + 2} + c, c \in \text{IR}$

9.2 $\int \frac{\text{sen}\, x}{1 - \cos x} dx =$

$= \ln|1 - \cos x| + c, c \in \text{IR}$

9.3 $\int \frac{2\cos x}{(2+\text{sen}\, x)^2} dx =$

$= 2 \int \cos x\, (2 + \text{sen}\, x)^{-2} dx =$

$= 2\frac{(2+\text{sen}\, x)^{-1}}{-1} + c =$

$= -\frac{2}{2+\text{sen}\, x} + c, c \in \text{IR}$

9.4 $\int \text{sen}\, x \cos^3 x\, dx =$

$= -\frac{(\cos x)^4}{4} + c =$

$= -\frac{\cos^4 x}{4} + c, c \in \text{IR}$

9.5 $\int \frac{e^x}{e^x + 1} dx =$

$= \ln(e^x + 1) + c, c \in \text{IR}$

9.6 $\int e^x \cos(e^x + 1)\, dx =$

$= \text{sen}(e^x + 1) + c, c \in \text{IR}$

10.

10.1 $F(x) = \int \left(-\frac{x}{e - x^2}\right) dx =$

$= \frac{1}{2} \int \frac{-2x}{e - x^2} dx =$

$= \frac{1}{2} \ln|e - x^2| + c, c \in \text{IR}$

Como o gráfico de F passa no ponto de coordenadas $(0, 2)$, temos:

$F(0) = 2 \Leftrightarrow \frac{1}{2}\ln|e - 0^2| + c = 2$

$\Leftrightarrow \frac{1}{2}\ln e + c = 2$

$\Leftrightarrow \frac{1}{2} + c = 2$

$\Leftrightarrow c = \frac{3}{2}$

Logo, $F(x) = \frac{1}{2}\ln|e - x^2| + \frac{3}{2}$.

10.2 $F(x) = \int \frac{1 + \cos x}{(x+\text{sen}\, x)^2} dx =$

$= \int (1 + \cos x)(x + \text{sen}\, x)^{-2} dx =$

$= \frac{(x+\text{sen}\, x)^{-1}}{-1} + c =$

$= -\frac{1}{x + \text{sen}\, x} + c, c \in \text{IR}$

Como o gráfico de F passa no ponto de coordenadas $\left(\pi, -\frac{1}{\pi}\right)$, temos:

$F(\pi) = -\frac{1}{\pi} \Leftrightarrow -\frac{1}{\pi + \text{sen}\, \pi} + c = -\frac{1}{\pi}$

$\Leftrightarrow -\frac{1}{\pi} + c = -\frac{1}{\pi}$

$\Leftrightarrow c = 0$

Logo, $F(x) = -\frac{1}{x + \text{sen}\, x}$.

11. Opção (A)

$F(x) = \int (\cos x - \text{sen}\, x)\ dx =$

$= \text{sen}\, x + \cos x + c, c \in \text{IR}$

Como o gráfico de F passa no ponto de coordenadas $(\pi, 0)$, temos:

$F(\pi) = 0 \Leftrightarrow \text{sen}\, \pi + \cos \pi + c = 0$

$\Leftrightarrow 0 - 1 + c = 0$

$\Leftrightarrow c = 1$

Assim, $F(x) = \text{sen}\, x + \cos x + 1$.

Logo, $F(0) = \text{sen}\, 0 + \cos 0 + 1 = 0 + 1 + 1 = 2$.

12. $f'(x) = \int (x^3 + 2x + 1) dx =$

$= \frac{x^4}{4} + x^2 + x + c, c \in \text{IR}$

$f'(1) = 2 \Leftrightarrow \frac{1}{4} + 1 + 1 + c = 2$

$\Leftrightarrow c = -\frac{1}{4}$

Assim, $f'(x) = \frac{x^4}{4} + x^2 + x - \frac{1}{4}$.

$f(x) = \int \left(\frac{x^4}{4} + x^2 + x - \frac{1}{4}\right) dx =$

$= \frac{1}{4} \times \frac{x^5}{5} + \frac{x^3}{3} + \frac{x^2}{2} - \frac{1}{4}x + k, k \in \text{IR}$

$f(0) = 1 \Leftrightarrow 0 + k = 1 \Leftrightarrow k = 1$

Logo, $f(x) = \frac{x^5}{20} + \frac{x^3}{3} + \frac{x^2}{2} - \frac{1}{4}x + 1$.

13.

13.1 $\int_0^7 f(x)\, dx = \int_0^3 f(x)\, dx + \int_3^7 f(x)\, dx =$

$= 5 + 3 =$

$= 8$

13.2 $\int_0^3 \big(f(x) + g(x)\big) dx =$

$= \int_0^3 f(x)\, dx + \int_0^3 g(x)\, dx =$

$= 5 + 2 =$

$= 7$

13.3 $\int_0^3 \big(2f(x) - 3g(x)\big) dx =$

$= 2\int_0^3 f(x)\, dx - 3\int_0^3 g(x)\, dx =$

$= 2 \times 5 - 3 \times 2 =$

$= 4$

14. Opção (A)

$\int_1^3 (2f(x) - 1)\ dx =$

$= \int_1^3 2f(x)\ dx - \int_1^3 1\, dx =$

$= 2\int_1^3 f(x)\ dx - \int_1^3 1\ dx =$
$= 2 \times 4 - [x]_1^3 =$
$= 8 - (3 - 1) =$
$= 8 - 2 =$
$= 6$

15. Opção (B)

$\int_0^5 f(x)dx = \int_0^2 f(x)dx + \int_2^5 f(x)dx$
$\Leftrightarrow 12 = 5 + \int_2^5 f(x)dx$
$\Leftrightarrow \int_2^5 f(x)dx = 12 - 5$
$\Leftrightarrow \int_2^5 f(x)dx = 7$
Logo, $\int_2^5 (f(x) + 3)dx =$
$= \int_2^5 f(x)dx + \int_2^5 3dx =$
$= 7 + [3x]_2^5 =$
$= 7 + (15 - 6) =$
$= 7 + 9 =$
$= 16$

16. Opção (A)

$\int_0^a x^2\ dx = \frac{8}{3} \Leftrightarrow \left[\frac{x^3}{3}\right]_0^a = \frac{8}{3}$
$\Leftrightarrow \frac{a^3}{3} - \frac{0^3}{3} = \frac{8}{3}$
$\Leftrightarrow \frac{a^3}{3} = \frac{8}{3}$
$\Leftrightarrow a^3 = 8$
$\Leftrightarrow a = 2$

17.

17.1 $\int_0^1 (4x^2 + 5x)\ dx =$
$= \left[\frac{4x^3}{3} + \frac{5x^2}{2}\right]_0^1 =$
$= \frac{4}{3} + \frac{5}{2} - (0 + 0) =$
$= \frac{8}{6} + \frac{15}{6} =$
$= \frac{23}{6}$

17.2 $\int_1^3 \frac{(x^2-1)}{x} dx =$
$= \int_1^3 \left(x - \frac{1}{x}\right) dx =$
$= \left[\frac{x^2}{2} - \ln x\right]_1^3 =$
$= \frac{9}{2} - \ln 3 - \left(\frac{1}{2} - \ln 1\right) =$
$= \frac{9}{2} - \ln 3 - \frac{1}{2} =$
$= 4 - \ln 3$

17.3 $\int_0^1 xe^{x^2} dx =$
$= \frac{1}{2}\left[e^{x^2}\right]_0^1 =$
$= \frac{1}{2}(e - 1) =$
$= \frac{e-1}{2}$

17.4 $\int_0^1 x\sqrt{x^2 + 1} dx =$
$= \frac{1}{2}\left[\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 =$
$= \frac{1}{3}\left[\sqrt{(x^2 + 1)^3}\right]_0^1 =$
$= \frac{1}{3}(\sqrt{8} - 1) =$
$= \frac{2\sqrt{2} - 1}{3}$

17.5 $\int_0^{\frac{\pi}{2}} \operatorname{sen} x \cos^2 x\ dx =$
$= -\left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}} =$
$= -\left(\frac{\cos^3\left(\frac{\pi}{2}\right)}{3} - \frac{\cos^3 0}{3}\right) =$
$= -\left(\frac{0}{3} - \frac{1}{3}\right) =$
$= \frac{1}{3}$

17.6 $\int_1^e \frac{\sqrt{2-\ln x}}{x} dx =$
$= -\int_1^e \left(-\frac{1}{x}(2 - \ln x)^{\frac{1}{2}}\right) dx =$
$= -\left[\frac{(2 - \ln x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^e =$
$= -\frac{2}{3}\left[\sqrt{(2 - \ln x)^3}\right]_1^e =$
$= -\frac{2}{3}\left(\sqrt{(2 - \ln e)^3} - \sqrt{(2 - \ln 1)^3}\right) =$
$= -\frac{2}{3}\left(1 - \sqrt{8}\right) =$
$= \frac{4\sqrt{2} - 2}{3}$

18. Opção (B)

Como o gráfico de f é uma reta que passa pela origem do referencial, então $f(x) = mx$.
Além disso, o gráfico de f passa no ponto de coordenadas $(2, 4)$, logo $m = \frac{4-0}{2-0} = 2$.
Assim, $f(x) = 2x$ e, portanto, $A = \int_0^3 (2x)\ dx =$
$= [x^2]_0^3 = 3^2 - 0^2 = 9 - 0 = 9.$

19. Opção (A)

$A = \int_{-1}^1 |x| dx =$
$= \int_{-1}^0 (-x)dx + \int_0^1 x\ dx =$
$= \left[-\frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^1 =$
$= \left(-\frac{0^2}{2} + \frac{1^2}{2}\right) + \left(\frac{1^2}{2} - \frac{0^2}{2}\right) =$
$= \frac{1}{2} + \frac{1}{2} =$
$= 1$

20.

20.1 $A = \int_0^3 (9 - x^2) dx =$
$= \left[9x - \frac{x^3}{3}\right]_0^3 =$
$= 27 - 9 - 0 =$
$= 18$

Cálculo auxiliar
$9 - x^2 = 0$
$\Leftrightarrow x = 3 \vee x = -3$

20.2 $A = \int_1^3 \left(1 + \frac{3}{x^2}\right) dx =$
$= \left[x - \frac{3}{x}\right]_1^3 =$
$= 3 - 1 - (1 - 3) =$
$= 4$

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20.3 $A = \int_0^1 (x^2 - 4x + 3)dx =$

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 =$

$= \left(\frac{1}{3} - 2 + 3 - 0\right) =$

$= \frac{4}{3}$

21. Seja $h(t)$a altura da árvore, em centímetros, t anos após ter sido plantada.

O aumento da altura da árvore durante os primeiros t anos é dado por $\int_0^t \frac{60}{\sqrt{x+16}}\,dx$.

Como, no momento em que foi plantada, a árvore tinha 10 cm de altura, tem-se:

$h(t) = 10 + \int_0^t \frac{60}{\sqrt{x+16}}\,dx$

Ao fim de nove anos:

$h(9) = 10 + \int_0^9 \frac{60}{\sqrt{x+16}}dx =$

$= 10 + 60\int_0^9 (x+16)^{-\frac{1}{2}}dx =$

$= 10 + \left[120\sqrt{x+16}\right]_0^9 =$

$= 10 + 120\left(\sqrt{25} - \sqrt{16}\right) =$

$= 10 + 120(5 - 4) =$

$= 130$

Assim, ao fim de nove anos, a árvore mede 130 cm.

22. Opção (C)

$\int_0^7 10e^{-0,5t}dt =$

$= \frac{10}{-0,5}\int_0^7 (-0,5e^{-0,5t})dt =$

$= -20[e^{-0,5t}]_0^7 =$

$= -20(e^{-3,5} - e^0) \approx$

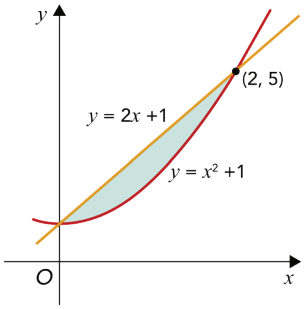
$\approx 19,4$ toneladas

23.

23.1 $x^2 + 1 = 2x + 1$

$\Leftrightarrow x^2 - 2x = 0$

$\Leftrightarrow x = 0 \vee x = 2$



Assim:

$A = \int_0^2 \left(2x + 1 - (x^2 + 1)\right)dx =$

$= \int_0^2 (2x - x^2)dx =$

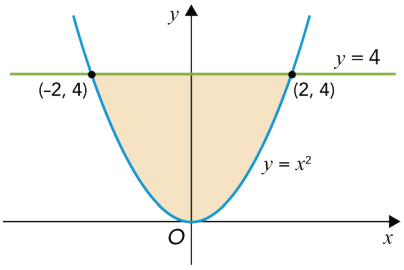
$= \left[x^2 - \frac{x^3}{3}\right]_0^2 =$

$= 4 - \frac{8}{3} - 0 =$

$= \frac{4}{3}$

23.2 $x^2 = 4$

$\Leftrightarrow x = 2 \vee x = -2$



Assim:

$A = \int_{-2}^2 (4 - x^2)\,dx =$

$= \left[4x - \frac{x^3}{3}\right]_{-2}^2 =$

$= 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right) =$

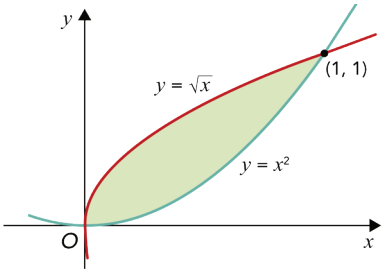
$= \frac{32}{3}$

23.3 $\sqrt{x} = x^2$

$\Rightarrow x = x^4$

$\Leftrightarrow x(x^3 - 1) = 0$

$\Leftrightarrow x = 0 \vee x = 1$



Assim:

$A = \int_0^1 \left(\sqrt{x} - x^2\right)dx =$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1 =$

$= \left[\frac{2}{3}\sqrt{x^3} - \frac{x^3}{3}\right]_0^1 =$

$= \frac{2}{3} - \frac{1}{3} - 0 =$

$= \frac{1}{3}$

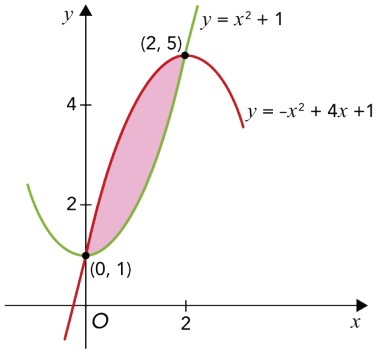
23.4 $x^2 + 1 = -x^2 + 4x + 1$

$\Leftrightarrow 2x^2 - 4x = 0$

$\Leftrightarrow 2x(x - 2) = 0$

$\Leftrightarrow 2x = 0 \vee x - 2 = 0$

$\Leftrightarrow x = 0 \vee x = 2$



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Assim:

$A = \int_0^2(-x^2+4x+1-(x^2+1))dx =$

$= \int_0^2(-2x^2+4x)dx =$

$= \left[-\frac{2x^3}{3}+2x^2\right]_0^2 =$

$= -\frac{16}{3}+8 =$

$= \frac{8}{3}$

24.

24.1 $\int_1^a(x+1)dx = 16$

$\Leftrightarrow \left[\frac{x^2}{2}+x\right]_1^a = 16$

$\Leftrightarrow \frac{a^2}{2}+a-\frac{1}{2}-1-16=0$

$\Leftrightarrow \frac{a^2}{2}+a-\frac{1}{2}-17=0$

$\Leftrightarrow \frac{a^2}{2}+a-\frac{35}{2}=0$

$\Leftrightarrow a^2+2a-35=0$

$\Leftrightarrow a = \frac{-2\pm\sqrt{4+140}}{2}$

$\Leftrightarrow a=-7 \vee a=5$

Como $a>1$, então $a=5$.

24.2 $\int_{-2}^1(3+ax)dx = 14$

$\Leftrightarrow \left[3x+\frac{ax^2}{2}\right]_{-2}^1 = 14$

$\Leftrightarrow 3+\frac{a}{2}-(-6+2a)=14$

$\Leftrightarrow 3+6-14+\frac{a}{2}-2a=0$

$\Leftrightarrow -5-\frac{3}{2}a=0$

$\Leftrightarrow -10-3a=0$

$\Leftrightarrow -3a=10$

$\Leftrightarrow a=-\frac{10}{3}$

O valor de a é $-\frac{10}{3}$.

24.3 $\int_0^2(a+x^3)dx = 18$

$\Leftrightarrow \left[ax+\frac{x^4}{4}\right]_0^2 = 18$

$\Leftrightarrow 2a+4-0=18$

$\Leftrightarrow 2a=14$

$\Leftrightarrow a=7$

O valor de a é 7.

24.4 $\int_0^1 \frac{ax}{(x^2+1)^2}dx = 2$

$\Leftrightarrow \frac{a}{2}\int_0^1 2x(x^2+1)^{-2}\,dx = 2$

$\Leftrightarrow \frac{a}{2}\left[\frac{(x^2+1)^{-1}}{-1}\right]_0^1 = 2$

$\Leftrightarrow -\frac{a}{2}\left[\frac{1}{x^2+1}\right]_0^1 = 2$

$\Leftrightarrow -\frac{a}{2}\left(\frac{1}{2}-1\right)=2$

$\Leftrightarrow -\frac{a}{2}\times\left(-\frac{1}{2}\right)=2$

$\Leftrightarrow \frac{a}{4}=2$

$\Leftrightarrow a=8$

O valor de a é 8.

25. Opção (D)

$A = \int_0^4 \sqrt{x}\,dx =$

$= \int_0^4 x^{\frac{1}{2}}\,dx =$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 =$

$= \left[\frac{2}{3}x\sqrt{x}\right]_0^4 =$

$= \frac{2}{3}\times 4\times\sqrt{4}-\frac{2}{3}\times 0 =$

$= \frac{16}{3}$

Pretende-se, assim, determinar o valor de k tal que $\int_0^k \sqrt{x}\,dx = \frac{8}{3}$.

$\int_0^k \sqrt{x}\,dx = \frac{8}{3} \Leftrightarrow \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^k = \frac{8}{3}$

$\Leftrightarrow \frac{2}{3}k^{\frac{3}{2}} = \frac{8}{3}$

$\Leftrightarrow k^{\frac{3}{2}} = 4$

$\Leftrightarrow k = 4^{\frac{2}{3}}$

$\Leftrightarrow k = \sqrt[3]{4^2}$

$\Leftrightarrow k = \sqrt[3]{16}$

O valor de k é $\sqrt[3]{16}$.

26. $f''(x) = \int(24x-6)\,dx =$

$= 12x^2-6x+c_1, c_1 \in \mathbb{R}$

$f''(0)=0 \Leftrightarrow c_1=0$

Logo, $f''(x)=12x^2-6x$.

$f'(x) = \int(12x^2-6x)dx =$

$= 4x^3-3x^2+c_2, c_2 \in \mathbb{R}$

$f'(0)=2 \Leftrightarrow c_2=2$

Logo, $f'(x)=4x^3-3x^2+2$.

$f(x) = \int(4x^3-3x^2+2)dx =$

$= x^4-x^3+2x+c_3, c_3 \in \mathbb{R}$

$f(0)=0 \Leftrightarrow c_3=0$

Logo, $f(x)=x^4-x^3+2x$.

27. $\int_{-1}^3 f(x)dx =$

$= \int_{-1}^0(-4x+1)dx + \int_0^1(e^x)dx + \int_1^3\left(\frac{2}{x+1}\right)dx =$

$= [-2x^2+x]_{-1}^0 + [e^x]_0^1 + [2\ln|x+1|]_1^3 =$

$= 0-(-3)+e-1+(2\ln 4-2\ln 2) =$

$= 2+e+2\left(\ln\frac{4}{2}\right) =$

$= 2+e+2\ln 2$

28.

28.1 Seja F uma primitiva de f. Então:

$F(x) = \int(x^3-4x+4)\,dx =$

$= \frac{x^4}{4}-2x^2+4x+c, c \in \mathbb{R}$

Como o gráfico de F passa pelo ponto $(1,1)$, tem-se $F(1)=1$.

Logo, $\frac{1}{4}-2+4+c=1 \Leftrightarrow c=-\frac{5}{4}$.

Portanto, $F(x)=\frac{x^4}{4}-2x^2+4x-\frac{5}{4}$.

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28.2 Para $a = 4$, a área da região sombreada é

$A = \int_0^4 f(x)\,dx$.

Assim, $A = \int_0^4 (x^3 - 4x + 4)\,dx =$

$= \left[\frac{x^4}{4} - 2x^2 + 4x\right]_0^4 =$

$= \left(\frac{4^4}{4} - 2 \times 4^2 + 4 \times 4\right) - 0 =$

$= 64 - 32 + 16 =$

$= 48$

Logo, $A = 48$.

28.3 Pretende-se determinar $a > 0$ para o qual a área da região sombreada é igual a $4a$.

Assim, $\int_0^a (x^3 - 4x + 4)\,dx = 4a$.

Logo, $\left[\frac{x^4}{4} - 2x^2 + 4x\right]_0^a = 4a$

$\Leftrightarrow \frac{a^4}{4} - 2a^2 + 4a = 4a$

$\Leftrightarrow \frac{a^4}{4} - 2a^2 = 0$

$\Leftrightarrow a^4 - 8a^2 = 0$

$\Leftrightarrow a^2(a^2 - 8) = 0$

$\Leftrightarrow a = 0 \vee a^2 = 8$

Como $a > 0$, $a = \sqrt{8} = 2\sqrt{2}$.

Logo, $a = 2\sqrt{2}$.

29.

29.1 O ponto A é o vértice da parábola que representa a função f, logo, $A(1, 1)$.

O ponto B é um ponto do gráfico de f, logo $B(x, f(x))$.

$m_{AB} = 2 \Leftrightarrow \frac{f(x) - 1}{x - 1} = 2$

$\Leftrightarrow \frac{(x-1)^2+1-1}{x-1} = 2$

$\Leftrightarrow (x - 1)^2 = 2(x - 1) \wedge x - 1 \neq 0$

$\Leftrightarrow x^2 - 4x + 3 = 0 \wedge x \neq 1$

$\Leftrightarrow x = \frac{4 \pm \sqrt{16-12}}{2} \wedge x \neq 1$

$\Leftrightarrow (x = 1 \vee x = 3) \wedge x \neq 1$

Logo, a abcissa do ponto B é 3.

$AB: y = 2x + b$

Como o ponto A pertence à reta AB:

$1 = 2 \times 1 + b \Leftrightarrow b = -1$

Logo, $AB: y = 2x - 1$.

Assim:

$A = \int_1^3 (2x - 1 - (x - 1)^2 - 1)dx =$

$= \int_1^3 (-x^2 + 4x - 3)dx =$

$= \left[-\frac{x^3}{3} + 2x^2 - 3x\right]_1^3 =$

$= (-9 + 18 - 9) - \left(-\frac{1}{3} + 2 - 3\right) =$

$= \frac{4}{3}$

29.2 $f(x) = x^3 - x + 1$

$f'(x) = 3x^2 - 1$

$f'(-1) = 2$

$t: y = 2x + b$

Como o ponto A pertence à reta t:

$1 = -2 + b \Leftrightarrow b = 3$

Logo, $t: y = 2x + 3$.

$f(x) = 2x + 3$

$\Leftrightarrow x^3 - x + 1 = 2x + 3$

$\Leftrightarrow x^3 - 3x - 2 = 0$

$\Leftrightarrow x = -1 \vee x = 2$

Cálculos auxiliares

	1	0	−3	−2
−1		−1	1	2
	1	−1	−2	0

$x^2 - x - 2 = 0$

$\Leftrightarrow x = \frac{1 \pm \sqrt{1+8}}{2}$

$\Leftrightarrow x = 2 \vee x = -1$

Assim:

$A = \int_{-1}^2 (2x + 3 - (x^3 - x + 1))dx =$

$= \int_{-1}^2 (-x^3 + 3x + 2)dx =$

$= \left[-\frac{x^4}{4} + \frac{3x^2}{2} + 2x\right]_{-1}^2 =$

$= \left(-\frac{2^4}{4} + 3 \times \frac{2^2}{2} + 2 \times 2\right) -$

$-\left(-\frac{(-1)^4}{4} + 3 \times \frac{(-1)^2}{2} + 2 \times (-1)\right) =$

$= 6 - \left(-\frac{3}{4}\right) =$

$= \frac{27}{4}$

30.

30.1 $\int f(x)dx =$

$= \int (-x^2 - 2x + 9)dx =$

$= -\frac{x^3}{3} - x^2 + 9x + c, c \in \mathbb{R}$

$F(1) = 6$

$\Leftrightarrow -\frac{1}{3} - 1 + 9 + c = 6$

$\Leftrightarrow c = -\frac{5}{3}$

Logo, $F(x) = -\frac{x^3}{3} - x^2 + 9x - \frac{5}{3}$.

30.2 $g(x) = h(x)$

$\Leftrightarrow -x^2 + 4x + 1 = x^2 - 2x + 1$

$\Leftrightarrow -2x^2 + 6x = 0$

$\Leftrightarrow 2x(-x + 3) = 0$

$\Leftrightarrow 2x = 0 \vee -x + 3 = 0$

$\Leftrightarrow x = 0 \vee x = 3$

Assim:

$A = \int_0^3 \left(g(x) - h(x)\right) dx =$

$= \int_0^3 (-x^2 + 4x + 1 - x^2 + 2x - 1)\,dx =$

$= \int_0^3 (-2x^2 + 6x)\,dx =$

$= \left[-\frac{2}{3}x^3 + 3x^2\right]_0^3 =$

$= -\frac{2}{3} \times 3^3 + 3 \times 3^2 - 0 - 0 =$

$= -\frac{2}{3} \times 27 + 3 \times 9 =$

$= -18 + 27 =$

$= 9$

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30.3 $f(x) = h(x)$

$\Leftrightarrow -x^2 - 2x + 9 = x^2 - 2x + 1$

$\Leftrightarrow -2x^2 + 8 = 0$

$\Leftrightarrow x^2 = 4$

$\Leftrightarrow x = 2 \vee x = -2$

Assim, a região sombreada situada no segundo quadrante está compreendida entre $x = -2$ e $x = 0$. A sua área é:

$A = \int_{-2}^{0}(f(x) - h(x))\,dx =$

$= \int_{-2}^{0}(-x^2 - 2x + 9 - (x^2 - 2x + 1))dx =$

$= \int_{-2}^{0}(-2x^2 + 8)dx =$

$= \left[-\frac{2}{3}x^3 + 8x\right]_{-2}^{0} =$

$= 0 - \left(-\frac{2}{3} \times (-8) + 8 \times (-2)\right) =$

$= \frac{32}{3}$

Pretende-se determinar o valor de k tal que:

$\int_{-2}^{k}(-2x^2 + 8)\,dx = \frac{32}{3} + 6k$

$\Leftrightarrow \left[-\frac{2}{3}x^3 + 8x\right]_{-2}^{k} = \frac{32}{3} + 6k$

$\Leftrightarrow \left(-\frac{2}{3}k^3 + 8k\right) - \left(-\frac{2}{3} \times (-8) + 8 \times (-2)\right) = \frac{32}{3} + 6k$

$\Leftrightarrow -\frac{2}{3}k^3 + 8k + \frac{32}{3} = \frac{32}{3} + 6k$

$\Leftrightarrow -\frac{2}{3}k^3 + 2k = 0$

$\Leftrightarrow k\left(-\frac{2}{3}k^2 + 2\right) = 0$

$\Leftrightarrow k = 0 \vee -\frac{2}{3}k^2 + 2 = 0$

$\Leftrightarrow k = 0 \vee k^2 = 3$

$\Leftrightarrow k = 0 \vee k = -\sqrt{3} \vee k = \sqrt{3}$

Como $-2 < k < 0$, então $k = -\sqrt{3}$.

30.4 $f(x) = g(x)$

$\Leftrightarrow -x^2 - 2x + 9 = -x^2 + 4x + 1$

$\Leftrightarrow -6x = -8$

$\Leftrightarrow x = \frac{4}{3}$

Assim:

$A = \int_{-2}^{\frac{4}{3}}(f(x) - h(x))dx + \int_{\frac{4}{3}}^{3}(g(x) - h(x))dx =$

$= \int_{-2}^{\frac{4}{3}}(-x^2 - 2x + 9 - (x^2 - 2x + 1))dx +$

$+ \int_{\frac{4}{3}}^{3}(-x^2 + 4x + 1 - (x^2 - 2x + 1))dx =$

$= \int_{-2}^{\frac{4}{3}}(-2x^2 + 8)\,dx + \int_{\frac{4}{3}}^{3}(-2x^2 + 6x)\,dx =$

$= \left[-\frac{2}{3}x^3 + 8x\right]_{-2}^{\frac{4}{3}} + \left[-\frac{2}{3}x^3 + 3x^2\right]_{\frac{4}{3}}^{3} =$

$= \left(-\frac{2}{3} \times \frac{64}{27} + 8 \times \frac{4}{3} - \left(-\frac{2}{3} \times (-8) + 8 \times (-2)\right)\right) +$

$+ \left(-\frac{2}{3} \times 27 + 3 \times 9 - \left(-\frac{2}{3} \times \frac{64}{27} + 3 \times \frac{16}{9}\right)\right) =$

$= \frac{736}{81} + \frac{32}{3} + \left(9 - \frac{304}{81}\right) =$

$= \frac{1600}{81} + \frac{425}{81} =$

$= 25$

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