10.2. $(x-3)^2+y^2+(z+2)^2=9 \wedge x=0 \iff y^2+(z+2)^2=0 \wedge x=0 \iff y=0 \wedge z+2=0 \wedge x=0$

Como o único ponto de interseção da superfície esférica com o plano yOz é o ponto de coordendas $(0\,,\,0\,,\,-2)$, a superfície esférica é tangente ao plano coordenado yOz.

11.1. $x=-2+4 \iff x=2$

$x=-2-4 \iff x=-6$

11.2. $y=1+4 \iff y=5$

$y=1-4 \iff y=-3$

12.1. $r=|-2-(-1)|=1$

$(x+2)^2+(y-4)^2+(z+3)^2 \leqslant 1$

12.2. $r=|4-2|=2$

$(x+2)^2+(y-4)^2+(z+3)^2 \leqslant 4$

12.3. $r=|-3-(-6)|=3$

$(x+2)^2+(y-4)^2+(z+3)^2 \leqslant 9$

Unidade 6

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1.1. **a)** Por exemplo, $\overrightarrow{AG}$

b) $\overrightarrow{DC}$

c) $\overrightarrow{GD}$

d) $\overrightarrow{EB}$

1.2. **a)** $\|\vec{u}\|^2=2^2+2^2 \iff \|\vec{u}\|=\sqrt{8}$

b) $\|\vec{v}\|=2$

c) $\|\vec{w}\|=3$

d) $\|\overrightarrow{GH}\|^2=4^2+2^2 \iff \|\overrightarrow{GH}\|=\sqrt{20} \iff \|\overrightarrow{GH}\|=2\sqrt{5}$

2.1. **a)** Por exemplo, $\overrightarrow{CE}$

b) Por exemplo, $\overrightarrow{BC}$

c) $\overrightarrow{FD}$

d) Por exemplo, $\overrightarrow{GC}$

2.2. **a)** $\|\vec{u}\|^2=1^2+1^2 \iff \|\vec{u}\|=\sqrt{2}$

b) $\|\vec{w}\|=2$

c) $\|\overrightarrow{DG}\|^2=3^2+1^2 \iff \|\overrightarrow{DG}\|=\sqrt{10}$

d) $\|\overrightarrow{BH}\|^2=2^2+2^2 \iff \|\overrightarrow{BH}\|=\sqrt{8}$; $\|\overrightarrow{BG}\|^2=\|\overrightarrow{BH}\|^2+1^2 \iff \|\overrightarrow{BG}\|^2=(\sqrt{8})^2+1 \iff \|\overrightarrow{BG}\|^2=9 \iff \|\overrightarrow{BG}\|=3$

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3.1. Falsa

3.2. Verdadeira

3.3. Falsa

3.4. Falsa

3.5. Verdadeira

3.6. Verdadeira

4. Opção **(D)**

5.1. $C+\vec{w}=G$

5.2. $C+(-\vec{u})=B$

5.3. $H+(-\vec{v})=L$

5.4. $D + \vec{w} = E$

5.5. $H + \vec{v} = J$

6.1. $A + \overrightarrow{HJ} = E$

6.2. $E + \left(-\overrightarrow{AB}\right) = F$

6.3. $H + \overrightarrow{IC} = B$

6.4. $K + \left(-\overrightarrow{FE}\right) = L$

6.5. $L + \overrightarrow{KC} = B$

6.6. $B + \overrightarrow{BC} = C$

7.1. **a)** $A + \vec{w} = B$
b) $G + \vec{v} = F$
c) $K + \left(-\vec{u}\right) = C$
d) $B + \left(-\vec{w}\right) = A$

7.2. Opção **(B)**

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8.1. **a)** $E + \vec{v} = D$
b) $B + \left(-\vec{u}\right) = A$
c) $A + \left(-\vec{v}\right) = F$

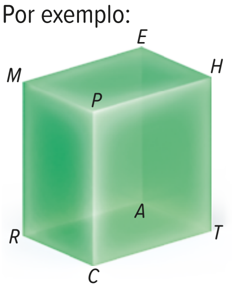
8.2. A imagem de C pela reflexão de eixo AD é B e $B + \vec{u} = E$.

9.1. **a)** $\left(H - \vec{u}\right) + \vec{v} = I + \vec{v} = C$
b) $\left(C - \vec{v}\right) + \vec{u} = I + \vec{u} = H$

9.2. **a)** $J + \vec{v} = D$
b) $B - \vec{u} = C$

9.3. Opção **(A)**

10. Por exemplo:



11. $\left(A + \vec{u}\right) - \vec{v} = B$

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1.1. $\overrightarrow{DE}$

1.2. $\overrightarrow{AC}$

1.3. $\overrightarrow{DB}$

1.4. $\overrightarrow{EA}$

2.1. $\overrightarrow{AB} + \overrightarrow{GL} = \overrightarrow{AB} + \overrightarrow{BG} = \overrightarrow{AG}$

2.2. $\overrightarrow{BD} - \overrightarrow{SI} = \overrightarrow{BD} + \overrightarrow{IS} = \overrightarrow{BD} + \overrightarrow{DN} = \overrightarrow{BN}$

2.3. $\overrightarrow{JE} + \overrightarrow{RT} = \overrightarrow{TO} + \overrightarrow{RT} = \overrightarrow{RT} + \overrightarrow{TO} = \overrightarrow{RO}$

2.4. $\overrightarrow{HI} + \overrightarrow{PS} = \overrightarrow{ST} + \overrightarrow{PS} = \overrightarrow{PS} + \overrightarrow{ST} = \overrightarrow{PT}$

2.5. $\overrightarrow{KC} + \overrightarrow{JT} = \overrightarrow{KC} + \overrightarrow{CM} = \overrightarrow{KM}$

2.6. $\overrightarrow{ND} - \overrightarrow{QL} = \overrightarrow{ND} + \overrightarrow{LQ} = \overrightarrow{ND} + \overrightarrow{DI} = \overrightarrow{NI}$

2.7. $\left(\overrightarrow{FL} + \overrightarrow{QM}\right) - \overrightarrow{RT} = \left(\overrightarrow{FL} + \overrightarrow{LH}\right) + \overrightarrow{TR} = \overrightarrow{FH} + \overrightarrow{HF} = \overrightarrow{FF}$

2.8. $\overrightarrow{DI} - \left(\overrightarrow{MH} + \overrightarrow{AB}\right) = \overrightarrow{DI} - \left(\overrightarrow{MH} + \overrightarrow{HI}\right) = \overrightarrow{DI} - \overrightarrow{MI} = \overrightarrow{DI} + \overrightarrow{IM} = \overrightarrow{DM}$

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3.1. $\vec{u}+\vec{v}=\overrightarrow{CE}$

3.2. $\vec{u}-\vec{w}=\overrightarrow{CB}$

3.3. $\vec{w}-\vec{v}=\overrightarrow{CA}$

4.1. $\vec{v}+\vec{u}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

4.2. $\vec{v}-\vec{w}=\overrightarrow{AB}+\overrightarrow{BF}=\overrightarrow{AF}$

4.3. $-\vec{w}+\vec{u}=\overrightarrow{BF}+\overrightarrow{FE}=\overrightarrow{BE}$

5. $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EF}+\overrightarrow{FA}$

$=\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}-\overrightarrow{AB}-\overrightarrow{BC}-\overrightarrow{CD}$

$=\left(\overrightarrow{AB}-\overrightarrow{AB}\right)+\left(\overrightarrow{BC}-\overrightarrow{BC}\right)+\left(\overrightarrow{CD}-\overrightarrow{CD}\right)=\vec{0}+\vec{0}+\vec{0}=\vec{0}$

6.1. $I-\overrightarrow{TO}=N$

6.2. $F+\overrightarrow{FP}=P$

6.3. $R-\overrightarrow{JR}=J$

6.4. $Q-\overrightarrow{DS}=B$

6.5. $B+3\,\overrightarrow{BC}=E$

6.6. $E+2\overrightarrow{HL}=M$

6.7. $I-\frac{1}{2}\overrightarrow{GS}=C$

6.8. $J+\frac{1}{3}\overrightarrow{ET}=O$

7.1. **a)** $-\vec{u}=\overrightarrow{EB}$

b) $2\vec{v}=\overrightarrow{HD}$

c) $-2\vec{u}=\overrightarrow{HB}$

d) $-3\vec{v}=\overrightarrow{BH}$

e) $\frac{4}{3}\vec{u}=\overrightarrow{BF}$

f) $-\frac{5}{2}\vec{v}=\overrightarrow{AF}$

g) $-\frac{5}{3}\vec{u}=\overrightarrow{GB}$

h) $\frac{1}{2}\vec{v}=\overrightarrow{HG}$

7.2. **a)** $\left\|\frac{2}{3}\vec{u}\right\|=\frac{2}{3}\|\vec{u}\|=\frac{2}{3}\times 3=2$

b) $\|-3\vec{v}\|=|-3|\|\vec{v}\|=3\times 2=6$

c) $\left\|-\frac{1}{2}\vec{u}\right\|\left|-\frac{1}{2}\right|\|\vec{u}\|=\frac{1}{2}\times 3=\frac{3}{2}$

d) $\|\vec{u}+\vec{v}\|=\left\|\overrightarrow{BC}\right\|=1$

e) $\|\vec{u}-\vec{v}\|=\|\vec{u}+(-\vec{v})\|=\left\|\overrightarrow{BG}\right\|=5$

f) $\|-\vec{u}+2\vec{v}\|=\left\|\overrightarrow{EB}+\overrightarrow{HD}\right\|=\left\|\overrightarrow{HA}\right\|=7$

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8.1. $\vec{w}=-3\vec{u}$

8.2. $\vec{u}=\frac{1}{2}\vec{v}$

8.3. $\vec{w}=-\frac{3}{2}\vec{v}$

9.1. **a)** $2\vec{u}=\overrightarrow{EA}$

b) $-\frac{1}{3}\vec{v}=\overrightarrow{PA}$

c) $\frac{3}{2}\vec{u}=\overrightarrow{EB}$

d) $\frac{5}{3}\vec{v}=\overrightarrow{AT}$

e) $-\frac{5}{2}\vec{u}=\overrightarrow{AF}$

f) $\frac{2}{3}\vec{v}=\overrightarrow{RT}$

9.2. $\overrightarrow{AE}=-2\vec{u}$, então: $\left\|\overrightarrow{AE}\right\|=\|-2\vec{u}\|=2\|\vec{u}\|=2\times\frac{5}{2}=5$

10.1. $\vec{u}+\vec{v}=\overrightarrow{JD}$ (por exemplo)

10.2. $M+2\vec{v}=I$

10.3. $C-2\vec{u}=K$

10.4. $\vec{v}-\vec{u}=\overrightarrow{HD}+\overrightarrow{DI}=\overrightarrow{HI}$ (por exemplo)

10.5. $P+\left(\overrightarrow{ON}+\vec{v}\right)=P+\left(\overrightarrow{PO}+\overrightarrow{OQ}\right)=P+\overrightarrow{PQ}=Q$

10.6. $\overrightarrow{JL}+\vec{u}=\overrightarrow{JL}+\overrightarrow{LG}=\overrightarrow{JG}$ (por exemplo)

10.7. $G+\left(\overrightarrow{LJ}+\vec{v}\right)=G+\left(\overrightarrow{GH}+\overrightarrow{HD}\right)=G+\overrightarrow{GD}=D$

10.8. $\frac{1}{2}\overrightarrow{KC}-\vec{u}=\overrightarrow{KF}-\vec{u}=\vec{u}-\vec{u}=\vec{0}=\overrightarrow{KK}$ (por exemplo)

10.9. $N+2\left(\vec{v}+\vec{u}\right)=N+2\left(\overrightarrow{JI}+\overrightarrow{ID}\right)=N+2\overrightarrow{JD}=N+\overrightarrow{ND}=D$

10.10. $B+\left(-2\vec{u}+\overrightarrow{KM}\right)=B+\left(\overrightarrow{CK}+\overrightarrow{KM}\right)=B+\overrightarrow{CM}=B+\overrightarrow{BJ}=J$

11. $\overrightarrow{MA}=-\frac{1}{2}\overrightarrow{AC}=-\frac{1}{2}\left(\frac{2}{3}\overrightarrow{AB}\right)=-\frac{1}{3}\overrightarrow{AB}$

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1.1. $\vec{u}=3\vec{i}+(-1)\vec{j}$, logo $\vec{u}(3\,,\,-1)$

1.2. $\vec{v}=-2\,\vec{i}+0\,\vec{j}$, logo $\vec{v}(-2\,,\,0)$

1.3. $\vec{w}=1\,\vec{i}+3\,\vec{j}$, logo $\vec{w}(1,\,3)$

1.4. $\vec{t}=-2\,\vec{i}+(-3)\,\vec{j}$, logo $\vec{t}(-2\,,\,-3)$

1.5. $\vec{a}=0\,\vec{i}+(-3)\,\vec{j}$, logo $\vec{a}(0\,,\,-3)$

1.6. $\vec{b}=(-4)\,\vec{i}+1\,\vec{j}$, logo $\vec{w}(-4\,,\,1)$

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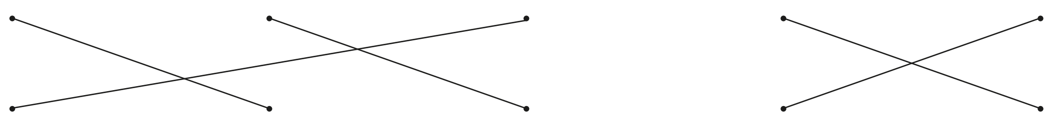
2.1. $\vec{e}$ **2.2.** $\vec{w}$ **2.3.** $\vec{a}$ **2.4.** $\vec{d}$ **2.5.** $\vec{b}$ **2.6.** $\vec{f}$

3.1. $\vec{u}=1\,\vec{i}+(-1)\,\vec{j}+0\,\vec{k}$; $\vec{u}\,(1,\,-1,\,0)$

3.2. $\vec{v}=-1\,\vec{i}+0\,\vec{j}+1\,\vec{k}$; $\vec{u}\,(-1\,,\,0\,,\,1)$

4.

$\vec{a}=-3\vec{i}+\vec{j}-\vec{k}$ $\vec{b}=2\vec{j}-3\vec{k}$ $\vec{c}=-\vec{i}-3\vec{j}+\vec{k}$ $\vec{d}=2\vec{i}-3\vec{j}$ $\vec{e}=3\vec{i}-\vec{j}+\vec{k}$



$(-1,\,-3\,,\,1)$ $(-3\,,\,1,\,-1)$ $(0\,,\,2\,,\,-3)$ $(3\,,\,-1,\,1)$ $(2\,,\,-3\,,\,0)$

5.1. $\overrightarrow{AB}(0\,,\,1,\,-3)$

5.2. $\overrightarrow{AE}(-3\,,\,1,\,0)$

5.3. $\overrightarrow{BE}(-3\,,\,0\,,\,3)$

5.4. $\overrightarrow{DA}(3\,,\,-2\,,\,1)$

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1.1. $2\vec{u}=2\,(-2\,,\,3)=(-4\,,\,6)$

1.2. $\vec{v}-\vec{w}=(0\,,\,-4)-(2\,,\,-4)=(0-2\,,\,-4+4)=(-2\,,\,0)$

1.3. $\vec{v}+\vec{t}=(0\,,\,-4)+(5\,,\,0)=(0+5\,,\,-4+0)=(5\,,\,-4)$

1.4. $\frac{1}{2}\vec{w}+t=\frac{1}{2}(2\,,\,-4)+(5\,,\,0)=(1,\,-2)+(5\,,\,0)=(6\,,\,-2)$

1.5. $\frac{1}{2}\vec{v}+\vec{w}=\frac{1}{2}(0\,,\,-4)+(2\,,\,-4)=(0\,,\,-2)+(2\,,\,-4)=(2\,,\,-6)$

1.6. $3\vec{u}+\vec{w}=3\,(-2\,,\,3)+(2\,,\,-4)=(-6\,,\,9)+(2\,,\,-4)=(-4\,,\,5)$

2.1. $\vec{w}=-\frac{1}{2}(-2\,,\,2\,,\,-4)+(-2\,,\,3\,,\,1)=(1\,,\,-1\,,\,2)+(-2\,,\,3\,,\,1)=(-1\,,\,2\,,\,3)$

2.2. $\vec{t}=2\vec{u}-\vec{v}+2\vec{i}=2(-2\,,\,3\,,\,1)-(-2\,,\,2\,,\,-4)+(2\,,\,0\,,\,0)=(-4\,,\,6\,,\,2)-(-2\,,\,2\,,\,-4)+(2\,,\,0\,,\,0)=$

$=(-2\,,\,4\,,\,6)+(2\,,\,0\,,\,0)=(0\,,\,4\,,\,6)$

3. Opção **(D)**

$(8\,,\,-12)=-4(-2\,,\,3)$

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4. Pretende-se mostrar que, sendo $\vec{u}(-5\,,\,5\,,\,2)$ e $\vec{v}(-2\,;\,2\,;\,0{,}8)$, $\exists\,\lambda\in\mathbb{R}:\ \vec{v}=\lambda\vec{u}$

$\vec{v}=\lambda\vec{u}\Leftrightarrow(-2\,;\,2\,;\,0{,}8)=\lambda(-5\,,\,5\,,\,2)\Leftrightarrow(-2\,;\,2\,;\,0{,}8)=(-5\lambda\,,\,5\lambda\,,\,2\lambda)\Leftrightarrow$

$\Leftrightarrow\begin{cases}-5\lambda=-2\\5\lambda=2\\2\lambda=0{,}8\end{cases}\Leftrightarrow\begin{cases}\lambda=0{,}4\\\lambda=0{,}4\\\lambda=0{,}4\end{cases}$

$\vec{u}$ e $\vec{v}$ são colineares, atendendo a que $\vec{v}=0{,}4\vec{u}$.

5.1. Se $\vec{u}$ e $\vec{v}$ fossem colineares, então existiria um $k\in\mathbb{R}$, tal que $k\vec{u}=\vec{v}$.

$-6k=-2\wedge\frac{3}{2}k=1\wedge-2k=a\Leftrightarrow k=\frac{1}{3}\wedge k=\frac{2}{3}\wedge k=-\frac{a}{2}$ (Impossível)

5.2. **a)** $-6k=-2\Leftrightarrow k=\frac{1}{3}$

$\frac{1}{3}\times\frac{3}{2}=\frac{1}{2}$

Se $b\neq\frac{1}{2}$, por exemplo, $b=5$, os vetores são não colineares.

b) Se $b=\frac{1}{2}$, os vetores são colineares, pois $\frac{1}{3}\vec{u}=\vec{w}$.

6. $\vec{u}(1\,,\,3)$ e $\vec{v}(-2\,,\,1)$

$\vec{u}+\vec{v}=(1\,,\,3)+(-2\,,\,1)=(-1\,,\,4)$

$\vec{w}$ e $\vec{u}+\vec{v}$ são colineares, pelo que: $\vec{w}=k(-1\,,\,4)=(-k\,,\,4k)$

Sabe-se que $-k+4k=6$, logo $k=2$.

Assim, $\vec{w}(-2\,,\,8)$.

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1.1. **a)** $\overrightarrow{PT}=T-P=(2\,,\,-5)-(-2\,,\,7)=(4\,,\,-12)$

b) $\overrightarrow{RT}=T-R=(2\,,\,-5)-(-4\,,\,0)=(6\,,\,-5)$

c) $\overrightarrow{PR}=R-P=(-4\,,\,0)-(-2\,,\,7)=(-2\,,\,-7)$

$\vec{v}-\overrightarrow{PR}=(3\,,\,1)-(-2\,,\,-7)=(5\,,\,8)$

d) $\overrightarrow{TR}=R-T=(-4\,,\,0)-(2\,,\,-5)=(-6\,,\,5)$

$2\vec{u}=2(-1\,,\,2)=(-2\,,\,4)$

$2\vec{u}+\overrightarrow{TR}=(-2\,,\,4)+(-6\,,\,5)=(-8\,,\,9)$

2.1. Seja $A(x\,,\,0)$. $\overrightarrow{PA}=A-P=(x\,,\,0)-(-2\,,\,3)=(x+2\,,\,-3)$, pelo que: $x+2=4\wedge-3=-3\Leftrightarrow x=2$

O ponto A tem coordenadas $(2\,,\,0)$.

2.2. Seja $B(0\,,\,y)$. $\overrightarrow{PB}=B-P=(0\,,\,y)-(-2\,,\,3)=(2\,,\,y-3)$, pelo que: $2=2\wedge y-3=1\Leftrightarrow y=4$

O ponto B tem coordenadas $(0\,,\,4)$.

2.3. Seja $C(x\,,\,y)$. $\overrightarrow{PC}=C-P=(x+2\,,\,y-3)$

Como $(x+2\,,\,y-3)=(-1\,,\,5)$, conclui-se que $C(-3\,,\,8)$.

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3. $\overrightarrow{AE} = E - A = (-2\,,\,-2\,,\,5) - (-2\,,\,-2\,,\,0) = (0\,,\,0\,,\,5)$

$\overrightarrow{DE} = E - D = (-2\,,\,-2\,,\,5) - (-2\,,\,2\,,\,0) = (0\,,\,-4\,,\,5)$

4. $C = B + \overrightarrow{AD} = (5\,,\,-1) + \left(1\,,\,\frac{7}{2}\right) = \left(6\,,\,\frac{5}{2}\right)$

$D = A + \overrightarrow{AD} = (-2\,,\,1) + \left(1\,,\,\frac{7}{2}\right) = \left(-1\,,\,\frac{9}{2}\right)$

5. $\overrightarrow{BC} = C - B = (2\,,\,0\,,\,-1)$;

$P + \overrightarrow{BC} = (0\,,\,1\,,\,1) + (2\,,\,0\,,\,-1) = (2\,,\,1\,,\,0) = A$

6. $C = M + \overrightarrow{BM} = \left(2\,,\,\frac{1}{2}\right) + \left(2-3\,,\,\frac{1}{2}+2\right) = (1\,,\,3)$

$S = A + \frac{1}{3}\overrightarrow{AC} = (-5\,,\,0) + \frac{1}{3}(1+5\,,3-0) = (-3\,,\,1)$

$\overrightarrow{SM} = M - S = \left(2\,,\,\frac{1}{2}\right) - (-3\,,\,1) = \left(5\,,\,-\frac{1}{2}\right)$

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1.1. $\|\vec{u}\| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

1.2. $\|\vec{v}\| = \sqrt{3^2 + (-2)^2} = \sqrt{13}$

1.3. $\overrightarrow{AB} = B - A = (1\,,\,-3)$

$\|\overrightarrow{AB}\| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

2.1. $\|\overrightarrow{PA}\| = \sqrt{3}$. A distância de $A(2\,,\,-1)$ ao eixo Oy é 2. Então: $\|\overrightarrow{PA}\| \geqslant 2$ e $\sqrt{3} < 2$

2.2. $\|\overrightarrow{AP}\| = 2$. $P(0\,,\,y)$ e $\overrightarrow{AP} = P - A = (0-2\,,\,y+1)$.

$\|\overrightarrow{AP}\| = 2 \Leftrightarrow \sqrt{(-2)^2 + (y+1)^2} = 2 \Leftrightarrow 4 + (y+1)^2 = 4 \Leftrightarrow y = -1$.

Solução: $P(0\,,\,-1)$

2.3. $\|\overrightarrow{AP}\| = \sqrt{8}$. $P(0\,,\,y)$ e $\overrightarrow{AP} = P - A = (0-2\,,\,y+1)$.

$\|\overrightarrow{AP}\| = \sqrt{8} \Leftrightarrow \sqrt{(-2)^2 + (y+1)^2} = \sqrt{8} \Leftrightarrow 4 + (y+1)^2 = 8 \Leftrightarrow (y+1)^2 = 4 \Leftrightarrow$

$\Leftrightarrow y+1 = 2 \vee y+1 = -2 \Leftrightarrow y = 1 \vee y = -3$.

Soluções: $P(0\,,\,1)$ e $(0\,,\,-3)$

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3.1. $\vec{w} = k\vec{u}$, $k > 0$ e $\|\vec{w}\| = \|k\vec{u}\| = |k|\,\|\vec{u}\| = 2$. Como $\|\vec{u}\| = \sqrt{(-3)^2 + 4^2} = 5$, tem-se: $|k| \times 5 = 2$

$|k| \times 5 = 2 \Leftrightarrow |k| = \frac{2}{5} \Leftrightarrow k = \frac{2}{5} \vee \cancel{k = -\frac{2}{5}}$. Assim, $\vec{w} = \frac{2}{5}\vec{u} = \frac{2}{5}(-3\,,\,4) = \left(-\frac{6}{5}\,,\,\frac{8}{5}\right)$.

3.2. $\vec{w} = k\vec{v}$, $k < 0$ e $\|\vec{w}\| = \|k\vec{v}\| = |k|\,\|\vec{v}\| = \sqrt{10}$. Como $\|\vec{v}\| = \sqrt{2^2 + 1^2} = \sqrt{5}$, tem-se: $|k| \times \sqrt{5} = \sqrt{10}$

$|k| \times \sqrt{5} = \sqrt{10} \Leftrightarrow |k| = \sqrt{2} \Leftrightarrow k = -\sqrt{2} \vee \cancel{k = \sqrt{2}}$. Assim, $\vec{w} = -\sqrt{2}\,\vec{v} = -\sqrt{2}\,(2\,,\,1) = (-2\sqrt{2}\,,\,-\sqrt{2})$.

4.1. **a)** $A(1\,,\,-1\,,\,0)$

b) $B(1\,,\,1\,,\,-1)$

c) $C(0\,,\,1\,,\,1)$

4.2. **a)** $\overrightarrow{AB} = B - A = (0\,,\,2\,,\,-1)$

$\|\overrightarrow{AB}\| = \sqrt{0^2 + 2^2 + (-1)^2} = \sqrt{5}$

b) $\overrightarrow{AC} = C - A = (-1\,,\,2\,,\,1)$

$\|\overrightarrow{AC}\| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$

c) $\overrightarrow{BC} = C - B = (-1\,,\,0\,,\,2)$

$\|\overrightarrow{BC}\| = \sqrt{(-1)^2 + 0^2 + 2^2} = \sqrt{5}$

5. $M\left(\frac{0+6}{2}, \frac{-2+2}{2}\right)=(3, 0)$ e $\overrightarrow{AM}=M-A=(5, -1)$

$\|\overrightarrow{AG}\|=\frac{2}{3}\|\overrightarrow{AM}\|$, ou seja, $\|\overrightarrow{AG}\|=\frac{2}{3}\sqrt{25+1}=\frac{2\sqrt{26}}{3}$.

$G=A+\frac{2}{3}\overrightarrow{AM}$. Então: $G=(-2, 1)+\frac{2}{3}(5, -1)=(-2, 1)+\left(\frac{10}{3}, -\frac{2}{3}\right)=\left(\frac{4}{3}, \frac{1}{3}\right)$

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1.1. **a)** $(8, -1)=(4, 1)+k\left(3, -\frac{3}{2}\right) \Leftrightarrow (8, -1)=\left(4+3k, 1-\frac{3}{2}k\right) \Leftrightarrow$

$\Leftrightarrow 4+3k=8 \wedge 1-\frac{3}{2}k=-1 \Leftrightarrow k=\frac{4}{3} \wedge k=\frac{4}{3}$

$k=\frac{4}{3}$

b) $(2, 2)=(4, 1)+k\left(3, -\frac{3}{2}\right) \Leftrightarrow (2, 2)=\left(4+3k, 1-\frac{3}{2}k\right) \Leftrightarrow 4+3k=2 \wedge 1-\frac{3}{2}k=2 \Leftrightarrow k=-\frac{2}{3} \wedge k=-\frac{2}{3}$ $k=-\frac{2}{3}$

1.2. $(-1, 3)=(4, 1)+k\left(3, -\frac{3}{2}\right) \Leftrightarrow (-1, 3)=\left(4+3k, 1-\frac{3}{2}k\right)$

$\Leftrightarrow 4+3k=-1 \wedge 1-\frac{3}{2}k=3 \Leftrightarrow k=-\frac{5}{3} \wedge k=-\frac{4}{3} \longrightarrow$ Impossível, logo o ponto D não pertence a r.

2.1. $(4, y)=(-1, 2)+k(3, 1) \Leftrightarrow (4, y)=(-1+3k, 2+k) \Leftrightarrow -1+3k=4 \wedge 2+k=y \Leftrightarrow k=\frac{5}{3} \wedge y=\frac{11}{3}$

O ponto é o de coordenadas $\left(4, \frac{11}{3}\right)$.

2.2. $(x, 0)=(-1, 2)+k(3, 1) \Leftrightarrow (x, 0)=(-1+3k, 2+k) \Leftrightarrow -1+3k=x \wedge 2+k=0 \Leftrightarrow x=-7 \wedge k=-2$

O ponto é o de coordenadas $(-7, 0)$.

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3. $A(x, 0)$ e $B(0, y)$. Os pontos A e B pertencem à reta r, pelo que:

$(x, 0)=(-1, 2)+k(2, 2) \Leftrightarrow -1+2k=x \wedge 2+2k=0 \Leftrightarrow x=-3 \wedge k=-1$, logo $A(-3, 0)$.

$(0, y)=(-1, 2)+k(2, 2) \Leftrightarrow -1+2k=0 \wedge 2+2k=y \Leftrightarrow k=\frac{1}{2} \wedge y=3$, logo $B(0, 3)$.

$M\left(-\frac{3}{2}, \frac{3}{2}\right)$ e $\overrightarrow{OM}=M-O=\left(-\frac{3}{2}, \frac{3}{2}\right)$. Uma equação vetorial da reta OM é, por exemplo:

$(x, y)=(0, 0)+k\left(-\frac{3}{2}, \frac{3}{2}\right), k\in\mathbb{R}$, ou seja, $(x, y)=k\left(-\frac{3}{2}, \frac{3}{2}\right), k\in\mathbb{R}$

4.1. Por exemplo, $\overrightarrow{AB}=(3, -2, -7)$

4.2. Por exemplo: $P=A+k\overrightarrow{AB}$, $k\in\mathbb{R}$, ou seja, $(x, y, z)=(-1, 2, 3)+k(3, -2, -7), k\in\mathbb{R}$

5.1. Por exemplo: $(x, y, z)=(1, -2, -1)+k(3, -1, 1), k\in\mathbb{R}$

5.2. As coordenadas de qualquer ponto do plano xOz são do tipo $(x, 0, z)$.

$(x, 0, z)=(2, 1, 0)+k(3, -1, 1) \Leftrightarrow (x, 0, z)=(2+3k, 1-k, 0+k) \Leftrightarrow$

$\Leftrightarrow x=2+3k \wedge 0=1-k \wedge z=k \Leftrightarrow x=5 \wedge k=1 \wedge z=1$, logo as coordenadas do ponto são $(5, 0, 1)$.

6. $M\left(\frac{0+2}{2}, \frac{-3+1}{2}, \frac{4+4}{2}\right)=(1, -1, 4)$, logo $\overrightarrow{BM}=M-B=(-3, 2, 4)$.

Uma equação vetorial da reta BM é: $(x, y, z)=(4, -3, 0)+k(-3, 2, 4)$, $k\in\mathbb{R}$

As coordenadas do ponto T são do tipo $(x, y, 3)$.

$(x, y, 3)=(4, -3, 0)+k(-3, 2, 4) \Leftrightarrow$

$\Leftrightarrow 4-3k=x \wedge -3+2k=y \wedge 0+4k=3 \Leftrightarrow$

$\Leftrightarrow k=\frac{3}{4} \wedge x=\frac{7}{4} \wedge y=-\frac{3}{2}$

$T\left(\frac{7}{4}, -\frac{3}{2}, 3\right)$

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1.1. Declive: $\frac{-3}{1}=-3$; ponto de coordenadas $(3\,,\,-4)$

1.2. Declive: $\frac{6}{3}=2$; ponto de coordenadas $(0\,,\,-5)$

1.3. Declive: $\frac{1}{3}:(-2)=-\frac{1}{6}$; ponto de coordenadas $\left(-1\,,\,\frac{1}{2}\right)$

2.1. Por exemplo:

Ponto: $(0\,,\,-3)$; vetor diretor da reta: $(1\,,\,2)$

2.2. Por exemplo:

Ponto: $(0\,,\,7)$; vetor diretor da reta: $(1\,,\,-5)$

2.3. Por exemplo:

Ponto: $\left(0\,,\,\frac{1}{3}\right)$; vetor diretor da reta: $\left(1\,,\,-\frac{3}{2}\right)$

3.1. $(x\,,\,y)=(0\,,\,2)+k(1\,,\,-4)\,,\ k\in\mathbb{R}$

3.2. $m=\frac{4}{-2}=-2$, logo a equação é do tipo $y=-2x+b$.

O ponto de coordenadas $(1\,,\,3)$ pertence à reta s .

Então, as coordenadas $(1\,,\,3)$ satisfazem a equação $y=-2x+b$, pelo que: $3=-2\times1+b \Leftrightarrow b=5$

Assim, $y=-2x+5$.

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4. Opção **(B)**

5.1. Uma equação da reta que passa na origem é do tipo $y=mx$.

Como retas paralelas têm igual declive, a resposta é $y=x$.

5.2. O ponto C é a interseção das retas AC e BC .

O declive da reta BC é $-\frac{1}{2}$ e um ponto da reta BC é, por exemplo, o ponto de coordenadas $(1\,,\,3)$. Como a equação da reta BC é do tipo $y=-\frac{1}{2}x+b$, então: $3=-\frac{1}{2}+b \Leftrightarrow b=\frac{7}{2}$. Logo, a equação da reta BC é: $y=-\frac{1}{2}x+\frac{7}{2}$

$$\begin{cases} y=x+5 \\ y=-\frac{1}{2}x+\frac{7}{2} \end{cases} \Leftrightarrow \begin{cases} y=x+5 \\ 2y=-x+7 \end{cases} \Leftrightarrow \begin{cases} y=x+5 \\ 2x+10=-x+7 \end{cases} \Leftrightarrow \begin{cases} y=x+5 \\ x=-1 \end{cases} \Leftrightarrow \begin{cases} y=4 \\ x=-1 \end{cases}$$

O vértice C tem coordenadas $(-1\,,\,4)$.

6.1. O ponto D tem coordenadas $(0\,,\,2)$ e a ordenada do ponto A é 0 . $0=-\frac{2}{3}x+2 \Leftrightarrow 0=-2x+6 \Leftrightarrow x=3$, logo o ponto A tem coordenadas $(3\,,\,0)$.

$C=B+\overrightarrow{AD}$, logo $C=(8\,,\,3)+(-3\,,\,2)=(5\,,\,5)$.

$A(3\,,\,0)$, $C(5\,,\,5)$ e $D(0\,,\,2)$

6.2. $\overrightarrow{AC}=C-A=(2\,,\,5)$

A equação vetorial da reta AC é da forma $P=A+k\overrightarrow{AC}$, $k\in\mathbb{R}$, ou seja: $(x\,,\,y)=(3\,,\,0)+k(2\,,\,5)\,,\ k\in\mathbb{R}$

7. $y=mx+\frac{1}{m}$. O ponto de interseção com Ox tem ordenada 0 , logo: $0=mx+\frac{1}{m} \Leftrightarrow mx=-\frac{1}{m} \Leftrightarrow x=-\frac{1}{m^2}$