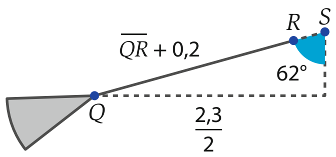
Caderno de atividades

1. Trigonometria

PÁG. 3

1.

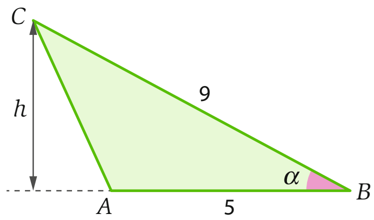


$$\frac{\frac{2{,}3}{2}}{\overline{QR}+0{,}2}=\operatorname{sen}62° \Leftrightarrow \overline{QR}=\frac{1{,}15}{\operatorname{sen}62°}-0{,}2 \Leftrightarrow \overline{QR}\approx 1{,}10\text{ m}$$

2. $\frac{\overline{RS}}{9}=\operatorname{sen}75° \Leftrightarrow \overline{RS}=9\times\operatorname{sen}75°$

$$A=\frac{6\times 9\times\operatorname{sen}75°}{2}=27\times\operatorname{sen}75°\approx 26\text{ u.a.}$$

3.



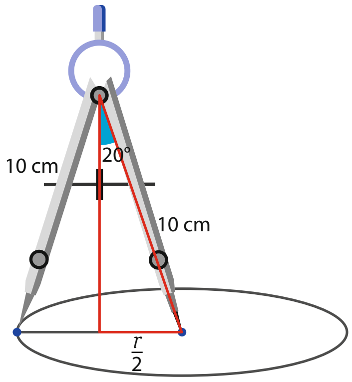
$$A=\frac{15}{2}\sqrt{2} \Leftrightarrow \frac{5\times h}{2}=\frac{15}{2}\sqrt{2} \Leftrightarrow h=\frac{15}{2}\sqrt{2}\times\frac{2}{5} \Leftrightarrow h=3\sqrt{2}$$

$$\frac{h}{9}=\operatorname{sen}\alpha \Leftrightarrow \frac{3\sqrt{2}}{9}=\operatorname{sen}\alpha \Leftrightarrow \alpha=\operatorname{sen}^{-1}\left(\frac{\sqrt{2}}{3}\right) \Leftrightarrow \alpha\approx 28{,}1°$$

PÁG. 4

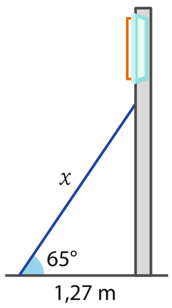
4. $\frac{\frac{r}{2}}{10}=\operatorname{sen}20° \Leftrightarrow r=20\times\operatorname{sen}20°$

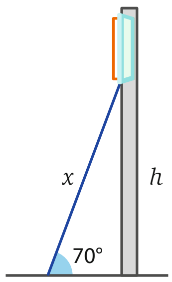
$$A=\pi\times(20\times\operatorname{sen}20°)^2\approx 147\text{ cm}^2$$



5. Designemos por x o comprimento da escada e por h a altura a que se encontra o parapeito da janela.

$$\frac{1{,}27}{x}=\cos 65° \Leftrightarrow x=\frac{1{,}27}{\cos 65°}$$

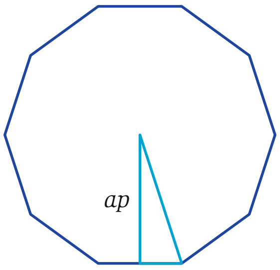


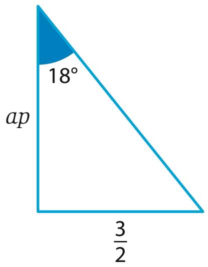


$$\frac{h}{x}=\operatorname{sen}70°$$

$$\frac{h}{x}=\operatorname{sen}70° \Leftrightarrow h=x\operatorname{sen}70° \Leftrightarrow h=\frac{1{,}27}{\cos 65°}\times\operatorname{sen}70° \Leftrightarrow h\approx 2{,}8\text{ m}$$

6.

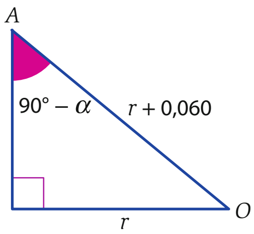




$$\frac{\frac{3}{2}}{\text{ap}} = \text{tg}\,18° \Leftrightarrow \text{ap} = \frac{\frac{3}{2}}{\text{tg}\,18°} \quad \text{e} \quad \text{A} = \frac{\text{P}}{2} \times \text{ap} = \frac{30}{2} \times \frac{\frac{3}{2}}{\text{tg}\,18°} \approx 69\ \text{cm}^2$$

PÁG. 5

7.



$$\frac{r}{r+0{,}060} = \text{sen}\,(90° - \alpha) \Leftrightarrow \frac{r}{r+0{,}060} = \text{sen}\,(89{,}7517°) \Leftrightarrow$$

$$\Leftrightarrow r = (r + 0{,}060)\ \text{sen}\,(89{,}7517°) \Leftrightarrow r = r\ \text{sen}\,(89{,}7517°) + 0{,}060\ \text{sen}\,(89{,}7517°) \Leftrightarrow$$

$$\Leftrightarrow r - r\ \text{sen}\,(89{,}7517°) = 0{,}060\ \text{sen}\,(89{,}7517°) \Leftrightarrow$$

$$\Leftrightarrow r\,(1 - \text{sen}\,(89{,}7517°)) = 0{,}060\ \text{sen}\,(89{,}7517°) \Leftrightarrow$$

$$\Leftrightarrow r = \frac{0{,}060\ \text{sen}\,(89{,}7517°)}{1 - \text{sen}\,(89{,}7517°)} \Leftrightarrow r \approx 6390\ \text{km}$$

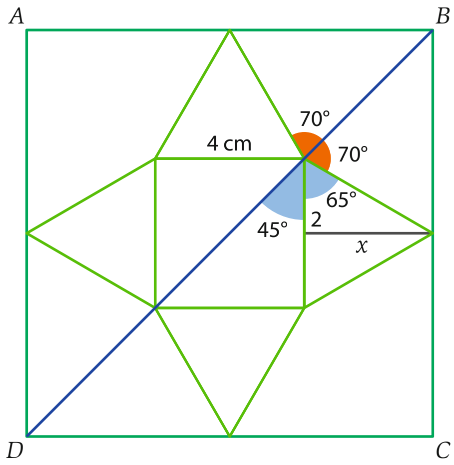
8. $\pi r^2 = 16\pi \underset{r>0}{\Leftrightarrow} r = 4$

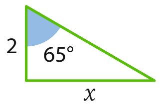
O triângulo é retângulo, pois está inscrito numa semicircunferência, e a sua hipotenusa mede 8 cm. Designando, respetivamente, por b e h a base (lado oposto ao ângulo de amplitude α) e a altura (lado adjacente ao ângulo de amplitude α) do triângulo, tem-se $\frac{b}{8} = \text{sen}\,\alpha \Leftrightarrow b = 8\ \text{sen}\,\alpha$ e $\frac{h}{8} = \cos\alpha \Leftrightarrow h = 8\cos\alpha$.

Assim, $\text{A} = \frac{b \times h}{2} = \frac{8\ \text{sen}\,\alpha \times 8\cos\alpha}{2} = 32\ \text{sen}\,\alpha\cos\alpha$.

Opção correta: **(C)**

9.





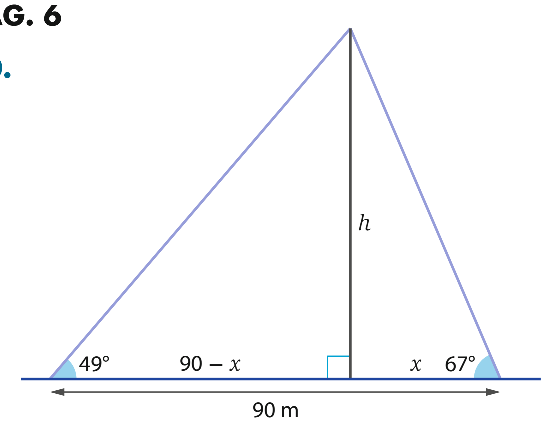
$\frac{x}{2} = \text{tg}\,65° \iff x = 2\,\text{tg}\,65°$

$\overline{AB} = 2x + 4 = 2 \times 2\ \text{tg}\,65° + 4$

Área $= (4\,\text{tg}\,65° + 4)^2 \approx 158{,}2\ \text{cm}^2$

PÁG. 6

10.



$$\begin{cases} \frac{h}{x} = \text{tg}\,67° \\ \frac{h}{90-x} = \text{tg}\,49° \end{cases} \iff \begin{cases} h = x\ \text{tg}\,67° \\ h = (90-x)\text{tg}\,49° \end{cases} \iff \begin{cases} h = x\ \text{tg}\,67° \\ x\ \text{tg}\,67° = (90-x)\text{tg}\,49° \end{cases} \iff$$

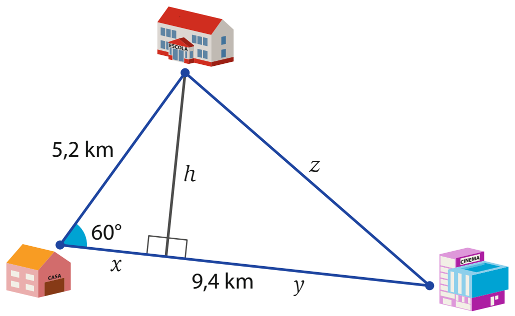
$$\iff \begin{cases} h = x\ \text{tg}\,67° \\ x\ \text{tg}\,67° = 90\ \text{tg}\,49° - x\ \text{tg}\,49° \end{cases} \iff \begin{cases} h = x\ \text{tg}\,67° \\ x\ \text{tg}\,67° + x\ \text{tg}\,49° = 90\ \text{tg}\,49° \end{cases}$$

$$\iff \begin{cases} h = x\ \text{tg}\,67° \\ x\,(\text{tg}\,67° + \text{tg}\,49°) = 90\ \text{tg}\,49° \end{cases} \iff \begin{cases} h = x\ \text{tg}\,67° \\ x = \frac{90\ \text{tg}\,49°}{\text{tg}\,67° + \text{tg}\,49°} \end{cases}$$

$$\iff \begin{cases} h = \frac{90\ \text{tg}\,49°}{\text{tg}\,67° + \text{tg}\,49°} \times \text{tg}\,67° \\ x = \frac{90\ \text{tg}\,49°}{\text{tg}\,67° + \text{tg}\,49°} \end{cases}$$

Logo, a altura do poste é $h = \frac{90\ \text{tg}\,49°}{\text{tg}\,67° + \text{tg}\,49°} \times \text{tg}\,67° \approx 69{,}6\ \text{m}$.

11.



$$\frac{h}{5,2} = \operatorname{sen} 60° \Leftrightarrow \frac{h}{5,2} = \frac{\sqrt{3}}{2} \Leftrightarrow h = 5,2 \times \frac{\sqrt{3}}{2} \Leftrightarrow h = 2,6\sqrt{3}$$

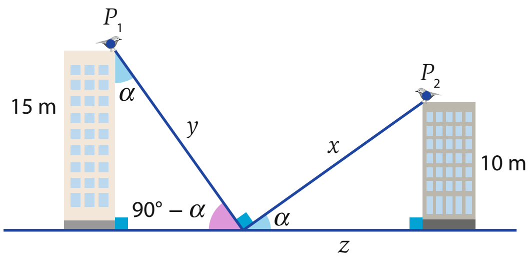
$$\frac{x}{5,2} = \cos 60° \Leftrightarrow \frac{x}{5,2} = \frac{1}{2} \Leftrightarrow x = \frac{5,2}{2} \Leftrightarrow x = 2,6$$, pelo que $y = 9,4 - 2,6 = 6,8$ km

Portanto, pelo teorema de Pitágoras:

$$z^2 = h^2 + y^2 \Leftrightarrow z^2 = \left(2,6\sqrt{3}\right)^2 + 6,8^2 \underset{z>0}{\Leftrightarrow} z = \sqrt{\left(2,6\sqrt{3}\right)^2 + 6,8^2}$$

Logo, a distância entre a escola e o cinema é $z = \sqrt{\left(2,6\sqrt{3}\right)^2 + 6,8^2} \approx 8,2$ km .

12.



$$\frac{z}{x} = \cos\alpha \Leftrightarrow \frac{z}{x} = \frac{4}{5} \Leftrightarrow x = \frac{5z}{4}$$

$$x^2 = z^2 + 10^2 \Leftrightarrow \left(\frac{5z}{4}\right)^2 = z^2 + 100 \Leftrightarrow \frac{25z^2}{16} - z^2 = 100 \Leftrightarrow$$

$$\Leftrightarrow \frac{9z^2}{16} = 100 \Leftrightarrow z^2 = \frac{100 \times 16}{9} \underset{z>0}{\Leftrightarrow} z = \frac{10 \times 4}{3} \Leftrightarrow z = \frac{40}{3}$$

$$x = \frac{5}{4} \times \frac{40}{3} \Leftrightarrow x = \frac{50}{3}$$

$$\frac{15}{y} = \cos\alpha \Leftrightarrow \frac{15}{y} = \frac{4}{5} \Leftrightarrow y = \frac{15 \times 5}{4} \Leftrightarrow y = \frac{75}{4}$$

O pássaro P_2 chega primeiro ao pedaço de pão, pois dista $\frac{50}{3} \approx 16,7$ m deste, enquanto o pássaro P_1 dista $\frac{75}{5} = 18,75$ m .

13. $\frac{\overline{AB}}{\overline{AC}} = \cos\alpha \Leftrightarrow \frac{\overline{AB}}{\overline{AC}} = \frac{1}{3} \Leftrightarrow \overline{AC} = 3\overline{AB}$

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \Leftrightarrow \left(3\overline{AB}\right)^2 = \overline{AB}^2 + 4^2 \Leftrightarrow 9\overline{AB}^2 - \overline{AB}^2 = 16 \Leftrightarrow$$

$$\Leftrightarrow 8\overline{AB}^2 = 16 \Leftrightarrow \overline{AB}^2 = 2 \underset{\overline{AB}>0}{\Leftrightarrow} \overline{AB} = \sqrt{2}$$

$$\text{Área} = \frac{\overline{AB} \times \overline{BC}}{2} = \frac{\sqrt{2} \times 4}{2} = 2\sqrt{2}$$

PÁG. 7

14. $\frac{\overline{BP}}{\overline{AB}} = \text{tg}\,\alpha$

$\frac{\overline{BC}}{\overline{AB}} = \text{tg}\,\beta \underset{\overline{BC} = 2\overline{BP}}{\Leftrightarrow} \frac{2\overline{BP}}{\overline{AB}} = \text{tg}\,\beta \Leftrightarrow 2\text{tg}\,\alpha = \text{tg}\,\beta$

Opção correta: **(A)**

15.1 $1500° = 60° + 4 \times 360°$

15.2 $-2300° = -140° - 6 \times 360°$

15.3 $-4600° = -280° - 12 \times 360°$

15.4 $12\,000° = 120° + 33 \times 360°$

PÁG. 8

16. $360° : 12 = 30°$

16.1

a. $1 \times 60° = 60°$

b. $-4 \times 60° = -240°$

16.2

a. $120° = 4 \times 30°$; ponto B .

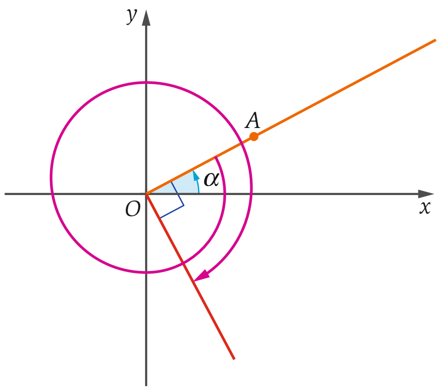
b. $-300° = -10 \times 30°$; ponto L .

c. $780° = 60 + 2 \times 360°$; $60° = 2 \times 30°$; ponto L .

d. $-570° = -210° - 1 \times 360°$; $-210° = -7 \times 30°$; ponto C .

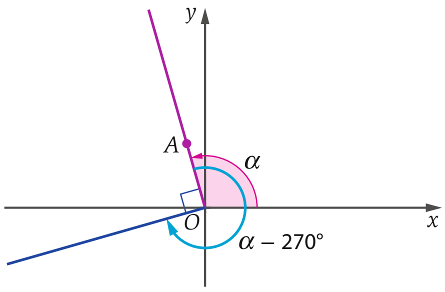
16.3 $-9 \times 30° - 3 \times 360° = -1350°$

17. $-450° = -90° - 1 \times 360°$



Opção correta: **(D)**

18.



Opção correta: **(A)**

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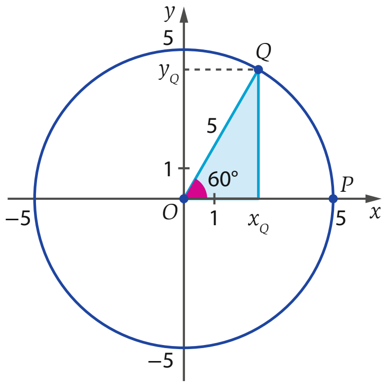
19. O ponto Q também é o transformado do ponto P por uma rotação de centro O e amplitude $60°$, pois $-660° + 2 \times 360° = 60°$.



$Q(x_Q, y_Q)$, $\frac{x_Q}{5} = \cos 60° \Leftrightarrow x_Q = 5 \times \frac{1}{2} \Leftrightarrow x_Q = \frac{5}{2}$ e $\frac{y_Q}{5} = \operatorname{sen} 60° \Leftrightarrow y_Q = 5 \times \frac{\sqrt{3}}{2} \Leftrightarrow y_Q = \frac{5\sqrt{3}}{2}$

$Q\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

20. $4 \text{ rad} \approx 4 \times 57{,}3° \approx 229{,}2°$

Opção correta: **(A)**

21. $\frac{\frac{5}{6}\pi \times 180°}{\pi} = 150°$

Opção correta: **(A)**

22. $\frac{\pi \times 820°}{180°} = \frac{41}{9}\pi$

Opção correta: **(D)**

23. $-3\pi = -2\pi - \pi$, passou $1 \text{ h } 30 \text{ min}$, pelo que, o relógio marcava $10\text{h}45$.

PÁG. 10

24. $360° : 10 = 36° = \frac{\pi}{5}$

a. $\frac{2\pi}{10} = \frac{\pi}{5}$; ponto V.

b. $-\frac{14\pi}{10} = -\frac{7\pi}{5} = -7 \times \frac{\pi}{5}$; ponto X.

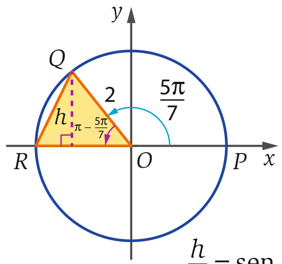
c. $\frac{6\pi}{5} = 6 \times \frac{\pi}{5}$; ponto Q.

d. $-\frac{21\pi}{5} + 2 \times 2\pi = -\frac{\pi}{5}$; ponto T.

25. $A = \frac{\frac{2\pi}{9} \times 4^2}{2} = \frac{16\pi}{9} \approx 5{,}6$ u.a.

26. $\dfrac{2\times 2\,\text{sen}\left(\frac{2\pi}{7}\right)}{2}=2\,\text{sen}\left(\frac{2\pi}{7}\right)\approx 1{,}56$

Opção correta: **(A)**



$\dfrac{h}{2}=\text{sen}\left(\pi-\dfrac{5\pi}{7}\right) \Leftrightarrow h=2\,\text{sen}\left(\dfrac{2\pi}{7}\right)$

27. Como $A\hat{D}E=D\hat{B}G=\frac{\pi}{6}$, a altura do triângulo $[ADE]$, em relação a $[DE]$, é $\frac{x}{2}=\text{tg}\,\frac{\pi}{6} \Leftrightarrow x=2\frac{\sqrt{3}}{3}$.

A altura do triângulo $[ABC]$, em relação a $[BC]$, é $2+\frac{2\sqrt{3}}{3}$.

PÁG. 12

28. $-\frac{7\pi}{2}=-7\times\frac{\pi}{2}$

Opção correta: **(C)**

PÁG. 13

29.1 $1230°=150°+3\times 360°$

Como, $90°<150°<180°$, o ângulo de amplitude $1230°$ pertence ao 2.° quadrante.

29.2 $-\frac{23\pi}{11}+2\pi=-\frac{\pi}{11}$

Como $-\frac{\pi}{2}<-\frac{\pi}{11}<0$, o ângulo de amplitude $-\frac{23\pi}{11}$ pertence ao 4.° quadrante.

29.3 $\frac{22\pi}{3}-3\times 2\pi=\frac{4\pi}{3}$

Como $\pi<\frac{4\pi}{3}<\frac{3\pi}{2}$, o ângulo de amplitude $\frac{4\pi}{3}$ pertence ao 3.° quadrante.

29.4 $-\frac{31\pi}{6}+3\times 2\pi=\frac{5\pi}{6}$

Como $\frac{\pi}{2}<\frac{5\pi}{6}<\pi$, o ângulo de amplitude $\frac{5\pi}{6}$ pertence ao 2.° quadrante.

30. Como $\alpha\in\left]\frac{\pi}{2},\pi\right[$, conclui-se que $\text{sen}\,\alpha>0$, $\cos\alpha<0$ e $\text{tg}\,\alpha<0$.

(A) $\underbrace{\text{tg}\,\alpha}_{<0}+\underbrace{\cos\alpha}_{<0}<0$

(B) $\underbrace{\text{sen}\,\alpha}_{>0}-\underbrace{\text{tg}\,\alpha}_{<0}>0$

(C) $\underbrace{\cos\alpha}_{<0}\times\underbrace{\text{tg}\,\alpha}_{<0}>0$

(D) $\underbrace{\text{sen}\,\alpha}_{>0}-\underbrace{\cos\alpha}_{<0}>0$

Opção correta: **(A)**

31. Como $\alpha \in \left]-\pi, -\frac{\pi}{2}\right[$, conclui-se que $\text{sen}\,\alpha < 0$, $\cos\alpha < 0$ e $\text{tg}\,\alpha > 0$.

(A) $\underbrace{\text{sen}\,\alpha}_{<0} + \underbrace{\cos\alpha}_{<0} < 0$

(B) $\underbrace{\cos\alpha}_{<0} - \underbrace{\text{tg}\,\alpha}_{>0} < 0$

(C) $\underbrace{\text{sen}\,\alpha}_{<0} \times \underbrace{\text{tg}\,\alpha}_{>0} < 0$

(D) $\underbrace{\text{tg}\,\alpha}_{>0} - \underbrace{\text{sen}\,\alpha}_{<0} > 0$

Opção correta: **(D)**

32. Como $\alpha \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$, conclui-se que $\alpha \in 2.^\circ$ Q ou $\alpha \in 3.^\circ$ Q.

Como $\cos\alpha \times \text{sen}\,\alpha > 0$, $\cos\alpha$ e $\text{sen}\,\alpha$ têm o mesmo sinal, pelo que $\alpha \in 3.^\circ$ Q e, portanto, $\text{sen}\,\alpha < 0$, $\cos\alpha < 0$ e $\text{tg}\,\alpha > 0$.

(A) $\underbrace{\cos\alpha}_{<0} + \underbrace{\text{sen}\,\alpha}_{<0} < 0$

(B) $\underbrace{\text{sen}\,\alpha}_{<0} \times \underbrace{\text{tg}\,\alpha}_{>0} < 0$

(C) $\text{tg}\,\alpha > 0$

(D) $\cos\alpha < 0$

Opção correta: **(C)**

33. O ponto A tem coordenadas $\left(\cos\frac{\pi}{5}, \text{sen}\frac{\pi}{5}\right)$.

O ponto D é a imagem do ponto A pela rotação de centro na origem e amplitude $-\frac{\pi}{2}$.

Logo, as coordenadas do ponto D são $\left(\cos\left(\frac{\pi}{5}-\frac{\pi}{2}\right), \text{sen}\left(\frac{\pi}{5}-\frac{\pi}{2}\right)\right)$, ou seja,

$\left(\cos\left(-\frac{3\pi}{10}\right), \text{sen}\left(-\frac{3\pi}{10}\right)\right)$.

Arredondando às milésimas, obtém-se $(0{,}588\,; -0{,}809)$.

34.1 $-1 \leqslant \text{sen}\,x \leqslant 1 \Leftrightarrow -1 \leqslant k^2 - 1 \leqslant 1 \Leftrightarrow 0 \leqslant k^2 \leqslant 2 \Leftrightarrow k^2 \leqslant 2 \Leftrightarrow k^2 - 2 \leqslant 0$

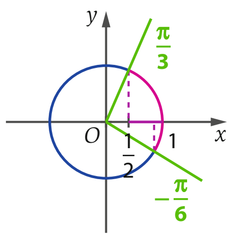
Zeros de $k^2 - 2$: $k^2 - 2 = 0 \Leftrightarrow k^2 = 2 \Leftrightarrow k = -\sqrt{2} \vee k = \sqrt{2}$

$k^2 - 2 \leqslant 0 \Leftrightarrow k \in \left[-\sqrt{2}, \sqrt{2}\right]$

34.2 Como $-\frac{\pi}{2} \leqslant x < \frac{\pi}{2}$, conclui-se que:

$0 \leqslant \cos x \leqslant 1 \Leftrightarrow 0 \leqslant k - 3 \leqslant 1 \Leftrightarrow 3 \leqslant k \leqslant 4 \Leftrightarrow k \in [3, 4]$

34.3 Para $x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right[$, tem-se:

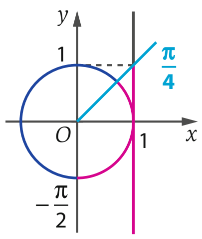


$\frac{1}{2} < \cos x \leqslant 1 \Leftrightarrow \frac{1}{2} < 1 - 3k \leqslant 1 \Leftrightarrow$

$\Leftrightarrow 1 - 3k > \frac{1}{2} \wedge 1 - 3k \leqslant 1 \Leftrightarrow -3k > \frac{1}{2} - 1 \wedge -3k \leqslant 1 - 1 \Leftrightarrow$

$\Leftrightarrow -3k > -\frac{1}{2} \wedge -3k \leqslant 0 \Leftrightarrow 3k < \frac{1}{2} \wedge 3k \geqslant 0 \Leftrightarrow k < \frac{1}{6} \wedge k \geqslant 0 \Leftrightarrow k \in \left[0, \frac{1}{6}\right[$

34.4 Para $x \in \left]-\frac{\pi}{2}, \frac{\pi}{4}\right]$, tem-se $\text{tg}\,x \leqslant 1 \Leftrightarrow k^2 + 1 \leqslant 1 \Leftrightarrow k^2 \leqslant 0 \Leftrightarrow k = 0$.



PÁG. 14

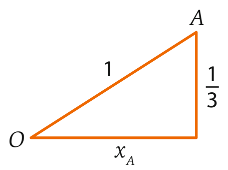
35.1 $A(\cos\alpha\ ,\ \operatorname{sen}\alpha)$

Como $\alpha \in 1.° \text{Q}$, $\cos\alpha > 0$ e $\operatorname{sen}\alpha > 0$; logo:

$$\text{Área} = \frac{\overline{AB} \times y_A}{2} = \frac{2\cos\alpha \times \operatorname{sen}\alpha}{2} = \cos\alpha \times \operatorname{sen}\alpha$$

35.2 $y_A = \frac{1}{3}$

$\text{P} = \overline{OA} + \overline{AB} + \overline{OB} = 2 + \overline{AB}$



$$1^2 = \left(\frac{1}{3}\right)^2 + x_A{}^2 \Leftrightarrow x_A{}^2 = 1 - \frac{1}{9} \underset{x_A > 0}{\Leftrightarrow} x_A = \sqrt{\frac{8}{9}} \Leftrightarrow x_A = \frac{\sqrt{8}}{3}$$

$$\text{P} = 2 + 2x_A = 2 + 2\frac{\sqrt{8}}{3} = \frac{2\sqrt{8}}{3} + 2$$

36. $S\,(\cos\alpha\ ,\ \operatorname{sen}\alpha)$; $U\,(0\ ,\ \operatorname{sen}\alpha)$; $T\,(1\ ,\ \operatorname{tg}\alpha)$

Como $\alpha \in 1.° \text{Q}$, $\cos\alpha > 0$ e $\operatorname{sen}\alpha > 0$; logo:

$$\text{Área} = \frac{\overline{US} \times (y_T - y_S)}{2} = \frac{\cos\alpha \times (\operatorname{tg}\alpha - \operatorname{sen}\alpha)}{2} =$$

$$= \frac{\cos\alpha \times \left(\frac{\operatorname{sen}\alpha}{\cos\alpha} - \operatorname{sen}\alpha\right)}{2} = \frac{\operatorname{sen}\alpha - \cos\alpha \times \operatorname{sen}\alpha}{2} = \frac{1}{2}\operatorname{sen}\alpha - \frac{1}{2}\cos\alpha \times \operatorname{sen}\alpha$$

37. $A\,(\cos\alpha\ ,\ \operatorname{sen}\alpha)$; $B\,(\cos\alpha\ ,\ -\operatorname{sen}\alpha)$; $C\,(0\ ,\ -\operatorname{sen}\alpha)$

A região sombreada é composta por um setor circular e por um quadrilátero.

Como $\alpha \in 2.° \text{Q}$, $\cos\alpha < 0$ e $\operatorname{sen}\alpha > 0$. Logo, $\overline{BC} = -\cos\alpha$ e $\overline{OC} = \operatorname{sen}\alpha$.

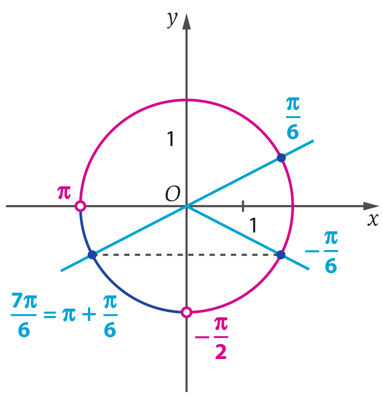
$$\text{Área} = \frac{1}{2}\pi \times 1^2 - \frac{\alpha \times 1^2}{2} + \overline{BC} \times \overline{OC} = \frac{\pi}{2} - \frac{\alpha}{2} - \cos\alpha \times \operatorname{sen}\alpha$$

PÁG. 15

38. $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$

Como $\operatorname{sen}\left(-\frac{\pi}{6}\right) = \operatorname{sen}\frac{7\pi}{6}$ e não há mais nenhuma amplitude pertencente a $\left]-\frac{\pi}{2}\ ,\ \pi\right[$ cujo seno seja igual a $\operatorname{sen}\frac{7\pi}{6}$, conclui-se que há apenas uma.

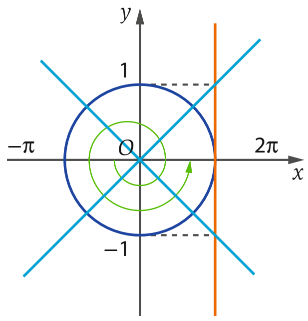
Opção correta: **(B)**



39. $\text{tg}^2\alpha = 1 \iff \text{tg}\,\alpha = -1 \lor \text{tg}\,\alpha = 1$

Em $]-\pi\,,\,0[$ há dois valores de amplitude, e em $[0\,,\,2\pi[$, há quatro valores.

Opção correta: **(D)**



PÁG. 16

40.1 $\text{tg}\,(\pi+\alpha)\times \text{sen}\left(-\frac{\pi}{2}+\alpha\right)-\cos\left(\frac{5\pi}{2}-\alpha\right)=\text{tg}\,\alpha\times(-\cos\alpha)-\cos\left(\left(\frac{\pi}{2}+2\pi\right)-\alpha\right)=$

$=-\text{tg}\,\alpha\times\cos\alpha=-\frac{\text{sen}\,\alpha}{\cos\alpha}\times\cos\alpha-\text{sen}\,\alpha=-\text{sen}\,\alpha-\text{sen}\,\alpha=-2\ \text{sen}\,\alpha$

40.2 $\frac{\text{sen}\,(3\pi+\alpha)}{\cos\,(-8\pi-\alpha)}-\text{tg}(\alpha-10\pi)+\text{sen}\left(-\frac{3\pi}{2}+\alpha\right)=$

$=\frac{\text{sen}\,((\pi+2\pi)+\alpha)}{\cos\,((-4\times 2\pi)-\alpha)}-\text{tg}\,\alpha+\text{sen}\left(-\frac{3\pi}{2}+2\pi+\alpha\right)=$

$=\frac{\text{sen}\,(\pi+\alpha)}{\cos\,(-\alpha)}-\text{tg}\,\alpha+\text{sen}\left(\frac{\pi}{2}+\alpha\right)=\frac{-\text{sen}\,\alpha}{\cos\alpha}-\text{tg}\,\alpha+\cos\alpha=$

$=-\text{tg}\,\alpha-\text{tg}\,\alpha+\cos\alpha=-2\text{tg}\,\alpha+\cos\alpha$

40.3 $\text{tg}\,(-\alpha)+\text{sen}\left(\frac{5\pi}{2}+\alpha\right)+\cos\,(-3\pi-\alpha)=$

$=-\text{tg}\,\alpha+\text{sen}\left(\left(\frac{\pi}{2}+2\pi\right)+\alpha\right)+\cos\,(-3\pi+2\times2\pi-\alpha)=$

$=-\text{tg}\,\alpha+\text{sen}\left(\frac{\pi}{2}+\alpha\right)+\cos\,(\pi-\alpha)=-\text{tg}\,\alpha+\cos\alpha+(-\cos\alpha)=$

$=-\text{tg}\,\alpha+\cos\alpha-\cos\alpha=-\text{tg}\,\alpha$

41.1 $\text{sen}\left(-\frac{\pi}{4}\right)+\cos\left(\frac{5\pi}{6}\right)\times\text{sen}\left(-\frac{\pi}{2}\right)=-\text{sen}\left(\frac{\pi}{4}\right)+\cos\left(\pi-\frac{\pi}{6}\right)\times(-1)=$

$=-\frac{\sqrt{2}}{2}+\left(-\cos\left(\frac{\pi}{6}\right)\right)\times(-1)=-\frac{\sqrt{2}}{2}+\left(-\frac{\sqrt{3}}{2}\right)\times(-1)=-\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}$

41.2 $\cos\,(150°)+\text{sen}\,(300°)-\cos\,(-120°)=$

$=\cos\,(180°-30°)+\text{sen}\,(360°-60°)-\cos\,(-180°+60°)=$

$=-\cos\,(30°)+\text{sen}\,(-60°)+\cos\,(60°)=$

$=-\frac{\sqrt{3}}{2}-\text{sen}\,(60°)+\frac{1}{2}=$

$=-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}+\frac{1}{2}=-\sqrt{3}+\frac{1}{2}$

41.3 $\frac{\text{tg}\left(-\frac{5\pi}{4}\right)}{\text{sen}\left(\frac{7\pi}{6}\right)}+\cos\left(-\frac{8\pi}{3}\right)+\text{sen}(-8\pi)=\frac{-\text{tg}\left(\frac{5\pi}{4}\right)}{\text{sen}\left(\pi+\frac{\pi}{6}\right)}+\cos\left(\frac{8\pi}{3}\right)+\text{sen}(-4\times 2\pi)=$

$=\frac{-\text{tg}\left(\pi+\frac{\pi}{4}\right)}{-\text{sen}\left(\frac{\pi}{6}\right)}+\cos\left(\frac{2\pi}{3}+2\pi\right)+\text{sen}(0)=\frac{-\text{tg}\left(\frac{\pi}{4}\right)}{-\frac{1}{2}}+\cos\left(\frac{2\pi}{3}\right)+0=$

$=\frac{-1}{-\frac{1}{2}}+\cos\left(\pi-\frac{\pi}{3}\right)=2+\left(-\cos\left(\frac{\pi}{3}\right)\right)=2-\frac{1}{2}=\frac{3}{2}$

41.4 $\text{sen}\left(-\frac{5\pi}{4}\right)+\cos\left(-\frac{9\pi}{6}\right)-\cos\left(\frac{7\pi}{3}\right)=\text{sen}\left(-\pi-\frac{\pi}{4}\right)+\cos\left(-\frac{3\pi}{2}\right)-\cos\left(\frac{\pi}{3}+2\pi\right)=$

$=\text{sen}\left(\frac{\pi}{4}\right)+0-\cos\left(\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}-\frac{1}{2}$

42. Como $\alpha\in\left]-\frac{\pi}{2},0\right[$, conclui-se que $\text{sen}\,\alpha<0$ e $\cos\alpha>0$.

(A) $\cos(\pi-\alpha)=-\underbrace{\cos\alpha}_{>0}<0$

(B) $\text{sen}\left(-\frac{\pi}{2}+\alpha\right)=-\underbrace{\cos\alpha}_{>0}<0$

(C) $\cos\left(-\frac{\pi}{2}+\alpha\right)=\text{sen}\,\alpha<0$

(D) $\text{sen}(10\pi-\alpha)=\text{sen}(5\times 2\pi-\alpha)=\text{sen}(-\alpha)=-\underbrace{\text{sen}\,\alpha}_{<0}>0$

Opção correta: **(D)**

43.1 $A(\alpha)=\text{sen}(-11\pi+\alpha)+\text{tg}(-13\pi+\alpha)\times\frac{2}{\cos\left(-\frac{3\pi}{2}-\alpha\right)}=$

$=\text{sen}(-11\pi+12\pi+\alpha)+\text{tg}\,\alpha\times\frac{2}{\cos\left(-\frac{3\pi}{2}+2\pi-\alpha\right)}=$

$=\text{sen}(\pi+\alpha)+\text{tg}\,\alpha\times\frac{2}{\cos\left(\frac{\pi}{2}-\alpha\right)}=-\text{sen}\,\alpha+\text{tg}\,\alpha\times\frac{2}{\text{sen}\,\alpha}=-\text{sen}\,\alpha+\frac{\text{sen}\,\alpha}{\cos\alpha}\times\frac{2}{\text{sen}\,\alpha}=$

$=\frac{2}{\cos\alpha}-\text{sen}\,\alpha$

43.2 $A\left(\frac{4\pi}{3}\right)=\frac{2}{\cos\left(\frac{4\pi}{3}\right)}-\text{sen}\left(\frac{4\pi}{3}\right)=\frac{2}{\cos\left(\pi+\frac{\pi}{3}\right)}-\text{sen}\left(\pi+\frac{\pi}{3}\right)=$

$=\frac{2}{-\cos\left(\frac{\pi}{3}\right)}-\left(-\text{sen}\left(\frac{\pi}{3}\right)\right)=\frac{2}{-\frac{1}{2}}+\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}-4$

44. $\alpha \in 3.^\circ$ Q ou $\alpha \in 4.^\circ$ Q .

$\text{tg}(-\alpha) \times \cos\left(\alpha - \frac{\pi}{2}\right) = -\text{tg}\,\alpha \times \text{sen}\,\alpha = -\frac{\text{sen}\,\alpha}{\cos\alpha} \times \text{sen}\,\alpha = -\frac{\text{sen}^2\alpha}{\cos\alpha}$

$\text{tg}(-\alpha) \times \cos\left(\alpha - \frac{\pi}{2}\right) > 0 \Leftrightarrow -\frac{\text{sen}^2\alpha}{\cos\alpha} > 0 \Leftrightarrow \frac{\text{sen}^2\alpha}{\cos\alpha} < 0 \Leftrightarrow \cos\alpha < 0$

Conclui-se que $\alpha \in 3.^\circ$ Q , pelo que $\text{sen}\,\alpha < 0$, $\cos\alpha < 0$ e $\text{tg}\,\alpha > 0$.

(A) $\underbrace{\cos\alpha}_{<0} - \underbrace{\text{tg}\,\alpha}_{>0} < 0$

(B) $\dfrac{\overbrace{\text{tg}\,\alpha}^{>0}}{\underbrace{\text{sen}\,\alpha}_{<0}} < 0$

(C) $\underbrace{\cos\alpha}_{<0} \times \underbrace{\text{sen}\,\alpha}_{<0} > 0$

(D) $\underbrace{\text{sen}\,\alpha}_{<0} + \underbrace{\cos\alpha}_{<0} < 0$

Opção correta: **(C)**

PÁG. 17

45. $\frac{\pi}{2} + \frac{\pi}{2} + \alpha + \beta = 2\pi \Leftrightarrow \alpha + \beta = \pi \Leftrightarrow \alpha = \pi - \beta$

$\text{sen}\,\alpha = \text{sen}\,(\pi - \beta) = \text{sen}\,\beta$

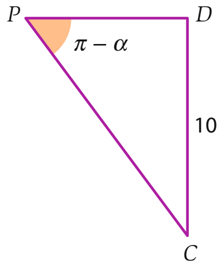
Opção correta: **(C)**

46.1 Se o ponto P coincidisse com o vértice D , teríamos $\alpha = \frac{\pi}{2}$, e se coincidisse com o vértice A , teríamos $\alpha = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$.

Assim, α toma todos os valores de amplitude entre esses valores, ou seja, $\alpha \in \left]\frac{\pi}{2}, \frac{3\pi}{4}\right[$.

46.2 Área $= \dfrac{\overline{AP} + \overline{BC}}{2} \times \overline{AB}$

$C\hat{P}D = \pi - P\hat{D}C - D\hat{C}P = \pi - \frac{\pi}{2} - \left(\alpha - \frac{\pi}{2}\right) = \pi - \alpha$



$\frac{\overline{CD}}{\overline{DP}} = \text{tg}(\pi - \alpha) \Leftrightarrow \frac{10}{\overline{DP}} = -\text{tg}\alpha \Leftrightarrow \overline{DP} = -\frac{10}{\text{tg}\alpha}$

$\overline{AP} = 10 - \overline{DP} = 10 - \left(-\frac{10}{\text{tg}\alpha}\right) = 10 + \frac{10}{\text{tg}\alpha}$

Área $= \dfrac{10 + \frac{10}{\text{tg}\alpha} + 10}{2} \times 10 = \left(20 + \frac{10}{\text{tg}\alpha}\right) \times 5 = 100 + \frac{50}{\text{tg}\alpha} = 100 + 50\frac{\cos\alpha}{\text{sen}\,\alpha}$

46.3 $100+50\dfrac{\cos\left(\frac{2\pi}{3}\right)}{\text{sen}\left(\frac{2\pi}{3}\right)}=100+50\dfrac{\cos\left(\pi-\frac{\pi}{3}\right)}{\text{sen}\left(\pi-\frac{\pi}{3}\right)}=100+50\dfrac{-\cos\left(\frac{\pi}{3}\right)}{\text{sen}\left(\frac{\pi}{3}\right)}=$

$$=100+50\times\dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=100-50\times\dfrac{1}{\sqrt{3}}=100-\dfrac{50\sqrt{3}}{3}$$

47. Como $\alpha\in\left]\pi,\dfrac{3\pi}{2}\right[$, conclui-se que $\text{sen}\,\alpha<0$ e $\cos\alpha<0$.

$1+3^2=\dfrac{1}{\cos^2\alpha}\Leftrightarrow\cos^2\alpha=\dfrac{1}{10}\underset{\cos\alpha<0}{\Leftrightarrow}\cos\alpha=-\dfrac{1}{\sqrt{10}}$

$\text{tg}\,\alpha=\dfrac{\text{sen}\,\alpha}{\cos\alpha}\Leftrightarrow\text{sen}\,\alpha=\cos\alpha\times\text{tg}\,\alpha$

$\text{sen}\,\alpha=\cos\alpha\times\text{tg}\,\alpha=-\dfrac{1}{\sqrt{10}}\times3=-\dfrac{3}{\sqrt{10}}$

48. Como $\alpha\in\left]0,\pi\right[$, conclui-se que $\text{sen}\,\alpha>0$.

$\left(-\dfrac{2}{5}\right)^2+\text{sen}^2\alpha=1\Leftrightarrow\text{sen}^2\alpha=1-\dfrac{4}{25}\Leftrightarrow\text{sen}^2\alpha=\dfrac{21}{25}\underset{\text{sen}\,\alpha>0}{\Leftrightarrow}\text{sen}\,\alpha=\dfrac{\sqrt{21}}{5}$

$\text{tg}\,\alpha=\dfrac{\text{sen}\,\alpha}{\cos\alpha}=\dfrac{\frac{\sqrt{21}}{5}}{-\frac{2}{5}}=-\dfrac{\sqrt{21}}{2}$

$\text{sen}\,\alpha\times\text{tg}\,\alpha=\dfrac{\sqrt{21}}{5}\times\left(-\dfrac{\sqrt{21}}{2}\right)=-\dfrac{21}{10}$

PÁG. 18

49.1 $A(x)=(\text{sen}\,x-\cos x)^2+\text{sen}\,(5\pi+x)\times\cos(-x)=$

$=\text{sen}^2\,x-2\,\text{sen}\,x\cos x+\cos^2x+\text{sen}\,(\pi+x)\times\cos x=$

$=\text{sen}^2\,x+\cos^2x-2\,\text{sen}\,x\cos x+(-\text{sen}\,x)\times\cos x=$

$=1-2\,\text{sen}\,x\cos x-\text{sen}\,x\times\cos x=1-3\,\text{sen}\,x\cos x$

49.2 $A\left(\dfrac{13\pi}{4}\right)=1-3\,\text{sen}\left(\dfrac{13\pi}{4}\right)\cos\left(\dfrac{13\pi}{4}\right)=1-3\,\text{sen}\left(\dfrac{5\pi}{4}+2\pi\right)\cos\left(\dfrac{5\pi}{4}+2\pi\right)=$

$=1-3\,\text{sen}\left(\dfrac{5\pi}{4}\right)\cos\left(\dfrac{5\pi}{4}\right)=1-3\,\text{sen}\left(\pi+\dfrac{\pi}{4}\right)\cos\left(\pi+\dfrac{\pi}{4}\right)=$

$=1-3\left(-\text{sen}\left(\dfrac{\pi}{4}\right)\right)\left(-\cos\left(\dfrac{\pi}{4}\right)\right)=1-3\times\dfrac{\sqrt{2}}{2}\times\dfrac{\sqrt{2}}{2}=1-\dfrac{3}{2}=-\dfrac{1}{2}$

49.3 Como $\alpha\in\left]-\dfrac{\pi}{2},\dfrac{\pi}{2}\right[$, conclui-se que $\cos\alpha>0$.

$\cos^2\alpha+\left(-\dfrac{3}{4}\right)^2=1\Leftrightarrow\cos^2\alpha=1-\dfrac{9}{16}\Leftrightarrow\cos^2\alpha=\dfrac{7}{16}\underset{\cos\alpha>0}{\Leftrightarrow}\cos\alpha=\dfrac{\sqrt{7}}{4}$

$A(\alpha)=1-3\,\text{sen}\,\alpha\cos\alpha=1-3\times\left(-\dfrac{3}{4}\right)\times\dfrac{\sqrt{7}}{4}=1+\dfrac{9\sqrt{7}}{16}$

50. Comecemos por simplificar a expressão cujo valor é pedido.

$$\operatorname{tg}(-\alpha)+\operatorname{sen}\left(-\frac{3\pi}{2}+\alpha\right)=-\operatorname{tg}\alpha+\operatorname{sen}\left(-\frac{3\pi}{2}+2\pi+\alpha\right)=$$
$$=-\operatorname{tg}\alpha+\operatorname{sen}\left(\frac{\pi}{2}+\alpha\right)=-\operatorname{tg}\alpha+\cos\alpha$$

A partir de $\operatorname{tg}\alpha=-\sqrt{8}$, determinemos $\cos\alpha$.

$$1+\left(-\sqrt{8}\right)^2=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{1}{9} \underset{\substack{\alpha\in 4.^\circ Q\\ \cos\alpha>0}}{\Leftrightarrow} \cos\alpha=\frac{1}{3}$$

Concluindo:

$$\operatorname{tg}(-\alpha)+\operatorname{sen}\left(-\frac{3\pi}{2}+\alpha\right)=-\operatorname{tg}\alpha+\cos\alpha=-\left(-\sqrt{8}\right)+\frac{1}{3}=\sqrt{8}+\frac{1}{3}$$

51. $\operatorname{sen}\left(-\frac{\pi}{2}+\alpha\right)=\operatorname{sen}\left(\alpha-\frac{\pi}{2}\right)=-\cos\alpha$

$$\operatorname{sen}\left(-\frac{\pi}{2}+\alpha\right)>0 \Leftrightarrow -\cos\alpha>0 \Leftrightarrow \cos\alpha<0$$

$$\operatorname{tg}(\pi+\alpha)<0 \Leftrightarrow \operatorname{tg}\alpha<0$$

Assim, como $\cos\alpha<0$ e $\operatorname{tg}\alpha<0$, conclui-se que $\alpha\in 2.^\circ Q$, pelo que $\operatorname{sen}\alpha>0$.

$$\cos^2\alpha+\operatorname{sen}^2\alpha=1 \Leftrightarrow \operatorname{sen}^2\alpha=1-\cos^2\alpha \Leftrightarrow \operatorname{sen}\alpha=\pm\sqrt{1-\cos^2\alpha}$$

Como $\operatorname{sen}\alpha>0$, conclui-se que $\operatorname{sen}\alpha=\sqrt{1-\cos^2\alpha}$.

Opção correta: **(A)**

52.1 $\text{Área}_{[ABC]}=\frac{\overline{AB}\times\overline{AC}}{2}$

$$\frac{\overline{AB}}{\overline{BC}}=\cos\beta \Leftrightarrow \frac{\overline{AB}}{10}=\cos\beta \Leftrightarrow \overline{AB}=10\cos\beta$$

$$\frac{\overline{AC}}{\overline{BC}}=\operatorname{sen}\beta \Leftrightarrow \frac{\overline{AC}}{10}=\operatorname{sen}\beta \Leftrightarrow \overline{AC}=10\operatorname{sen}\beta$$

$$\text{Área}_{[ABC]}=\frac{10\cos\beta\times 10\operatorname{sen}\beta}{2}=50\cos\beta\times\operatorname{sen}\beta$$

52.2 $(\operatorname{sen}\beta-\cos\beta)^2=\frac{1}{4} \Leftrightarrow \operatorname{sen}^2\beta-2\operatorname{sen}\beta\cos\beta+\cos^2\beta=\frac{1}{4} \Leftrightarrow$

$$\Leftrightarrow \operatorname{sen}^2\beta+\cos^2\beta-2\operatorname{sen}\beta\cos\beta=\frac{1}{4} \Leftrightarrow 1-2\operatorname{sen}\beta\cos\beta=\frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow -2\operatorname{sen}\beta\cos\beta=\frac{1}{4}-1 \Leftrightarrow \operatorname{sen}\beta\cos\beta=\frac{-\frac{3}{4}}{-2} \Leftrightarrow \operatorname{sen}\beta\cos\beta=\frac{3}{8}$$

Logo, o valor exato da área do triângulo $[ABC]$, para esse valor de β, é $50\times\frac{3}{8}=\frac{75}{4}$.

PÁG. 19

53.1 Como $[AB]$ é um diâmetro da circunferência, o triângulo $[ABC]$ é retângulo em C .

$$\text{Área}_{[ABC]} = \text{Área}_{[AOC]} + \text{Área}_{[OBC]} = \frac{\overline{OC} \times y_A}{2} + \frac{\overline{OC} \times |y_B|}{2}$$

$\overline{OC} = 1$, os pontos A e B têm coordenadas $(\cos\alpha , \operatorname{sen}\alpha)$ e $(-\cos\alpha , -\operatorname{sen}\alpha)$, respetivamente.

$$\text{Área}_{[ABC]} = \frac{1 \times \operatorname{sen}\alpha}{2} + \frac{1 \times |-\operatorname{sen}\alpha|}{2} = 2 \times \frac{\operatorname{sen}\alpha}{2} = \operatorname{sen}\alpha$$

Alternativamente:

$$\text{Área}_{[ABC]} = \frac{\overline{AC} \times \overline{BC}}{2}$$

$$\overline{AC}^2 = (1 - \cos\alpha)^2 + \operatorname{sen}^2\alpha = 1 - 2\cos\alpha + \cos^2\alpha + \operatorname{sen}^2\alpha = 1 - 2\cos\alpha + 1 = 2 - 2\cos\alpha$$

Como $\overline{AC} > 0$, $\overline{AC} = \sqrt{2 - 2\cos\alpha}$

$$\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 \Leftrightarrow 2^2 = 2 - 2\cos\alpha + \overline{BC}^2 \Leftrightarrow$$

$$\Leftrightarrow \overline{BC}^2 = 4 - 2 + 2\cos\alpha \underset{\overline{BC} > 0}{\Leftrightarrow} \overline{BC} = \sqrt{2 + 2\cos\alpha}$$

$$\text{Área}_{[ABC]} = \frac{\sqrt{2 - 2\cos\alpha} \times \sqrt{2 + 2\cos\alpha}}{2} =$$

$$= \frac{\sqrt{2^2 - (2\cos\alpha)^2}}{2} = \frac{\sqrt{4 - 4\cos^2\alpha}}{2} = \frac{\sqrt{4(1 - \cos^2\alpha)}}{2} = \frac{2\sqrt{\operatorname{sen}^2\alpha}}{2} \underset{\substack{\alpha \in 1.^\circ Q \\ \operatorname{sen}\alpha > 0}}{=} \operatorname{sen}\alpha$$

53.2 $$\left(\operatorname{sen}\left(\frac{\pi}{2} + \alpha\right) - 1\right)^2 + \operatorname{sen}^2\alpha = (\cos\alpha - 1)^2 + \operatorname{sen}^2\alpha =$$

$$= \cos^2\alpha - 2\cos\alpha + 1 + \operatorname{sen}^2\alpha = 1 - 2\cos\alpha + 1 = 2 - 2\cos\alpha$$

$$\left(\operatorname{sen}\left(\frac{\pi}{2} + \alpha\right) - 1\right)^2 + \operatorname{sen}^2\alpha = \frac{3}{2} \Leftrightarrow 2 - 2\cos\alpha = \frac{3}{2} \Leftrightarrow -2\cos\alpha = \frac{3}{2} - 2 \Leftrightarrow$$

$$\Leftrightarrow -2\cos\alpha = -\frac{1}{2} \Leftrightarrow \cos\alpha = \frac{1}{4}$$

$$\left(\frac{1}{4}\right)^2 + \operatorname{sen}^2\alpha = 1 \Leftrightarrow \operatorname{sen}^2\alpha = 1 - \frac{1}{16} \Leftrightarrow \operatorname{sen}^2\alpha = \frac{15}{16} \underset{\operatorname{sen}\alpha > 0}{\Leftrightarrow} \operatorname{sen}\alpha = \frac{\sqrt{15}}{4}$$

Logo, o valor exato da área do triângulo $[ABC]$, para esse valor de α , é $\frac{\sqrt{15}}{4}$.

54.1 Para que se tenha $\alpha = \frac{\pi}{2}$, o ponto P coincide com o ponto T e o ponto Q coincide com a origem do referencial.

Assim, as suas coordenadas, para $\alpha = \frac{\pi}{2}$, são: $P(0 , 1)$ e $Q(0 , 0)$.

Alternativamente:

Os pontos P e Q têm coordenadas $(\cos\alpha , \operatorname{sen}\alpha)$ e $(\cos\alpha , 0)$, respetivamente.

Assim, para $\alpha = \frac{\pi}{2}$, obtém-se: $P(0 , 1)$ e $Q(0 , 0)$.

54.2 $P(\cos\alpha\ ,\ \operatorname{sen}\alpha)$ e $R(4\ ,\ 0)$.

$$\overline{PR}=\sqrt{(4-\cos\alpha)^2+(0-\operatorname{sen}\alpha)^2}=$$

$$=\sqrt{16-8\cos\alpha+\cos^2\alpha+\operatorname{sen}^2\alpha}=\sqrt{16-8\cos\alpha+1}=\sqrt{17-8\cos\alpha}$$

54.3 $\cos\left(\frac{\pi}{2}-\alpha\right)=\frac{3}{5} \Leftrightarrow \operatorname{sen}\alpha=\frac{3}{5}$

$$\cos^2\alpha+\left(\frac{3}{5}\right)^2=1 \Leftrightarrow \cos^2\alpha=1-\frac{9}{25} \Leftrightarrow \cos^2\alpha=\frac{16}{25} \underset{\substack{\alpha\in 2.^\circ Q\\ \cos\alpha<0}}{\Leftrightarrow} \cos\alpha=-\frac{4}{5}$$

Logo, $\overline{PR}=\sqrt{17-8\left(-\frac{4}{5}\right)}=\sqrt{17+\frac{32}{5}}=\sqrt{\frac{117}{5}}$.

54.4 $P(\cos\alpha\ ,\ \operatorname{sen}\alpha)$, com $\cos\alpha<0$ e $\operatorname{sen}\alpha>0$, pelo que $\overline{OQ}=-\cos\alpha$ e $\overline{PQ}=\operatorname{sen}\alpha$.

$$\text{Área}_{[PQR]}=\frac{\overline{PQ}\times\overline{QR}}{2}=\frac{\overline{PQ}\times\left(\overline{OR}+\overline{OQ}\right)}{2}=\frac{\operatorname{sen}\alpha\,(4-\cos\alpha)}{2}=$$

$$=\frac{4\operatorname{sen}\alpha-\operatorname{sen}\alpha\times\cos\alpha}{2}=\frac{4\operatorname{sen}\alpha}{2}-\frac{\operatorname{sen}\alpha\times\cos\alpha}{2}=2\operatorname{sen}\alpha-\frac{\operatorname{sen}\alpha\times\cos\alpha}{2}$$

54.5 $2\operatorname{sen}\left(\frac{2\pi}{3}\right)-\frac{\cos\left(\frac{2\pi}{3}\right)\times\operatorname{sen}\left(\frac{2\pi}{3}\right)}{2}=2\operatorname{sen}\left(\pi-\frac{\pi}{3}\right)-\frac{\cos\left(\pi-\frac{\pi}{3}\right)\times\operatorname{sen}\left(\pi-\frac{\pi}{3}\right)}{2}=$

$$=2\operatorname{sen}\left(\frac{\pi}{3}\right)-\frac{\left(-\cos\left(\frac{\pi}{3}\right)\right)\times\operatorname{sen}\left(\frac{\pi}{3}\right)}{2}=$$

$$=2\times\frac{\sqrt{3}}{2}-\frac{\left(-\frac{1}{2}\right)\times\frac{\sqrt{3}}{2}}{2}=$$

$$=\sqrt{3}+\frac{\frac{\sqrt{3}}{4}}{2}=\sqrt{3}+\frac{\sqrt{3}}{8}=\frac{9\sqrt{3}}{8}$$