Trigonometria

PÁG. 10

Diagnóstico

1.1 $x^2 = 3^2 + 3^2 \Leftrightarrow x^2 = 18 \underset{x>0}{\Leftrightarrow} x = \sqrt{18}$

1.2 $8^2 = 4^2 + x^2 \Leftrightarrow x^2 = 64 - 16 \Leftrightarrow x^2 = 48 \underset{x>0}{\Leftrightarrow} x = \sqrt{48}$

1.3 $4^2 = 3^2 + x^2 \Leftrightarrow x^2 = 16 - 9 \Leftrightarrow x^2 = 7 \underset{x>0}{\Leftrightarrow} x = \sqrt{7}$

2.1 $\operatorname{sen}\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$, $\operatorname{tg}\alpha = \frac{3}{4}$, $\alpha = \operatorname{tg}^{-1}\left(\frac{3}{4}\right) \approx 37°$

$\operatorname{sen}\beta = \frac{4}{5}$, $\cos\beta = \frac{3}{5}$, $\operatorname{tg}\beta = \frac{4}{3}$, $\beta = \operatorname{tg}^{-1}\left(\frac{4}{3}\right) \approx 53°$

2.2 $x^2 = 8^2 + 15^2 \Leftrightarrow x^2 = 289 \underset{x>0}{\Leftrightarrow} x = \sqrt{289} \Leftrightarrow x = 17$

$\operatorname{sen}\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, $\operatorname{tg}\alpha = \frac{8}{15}$, $\alpha = \operatorname{tg}^{-1}\left(\frac{8}{15}\right) \approx 28°$

$\operatorname{sen}\beta = \frac{15}{17}$, $\cos\beta = \frac{8}{17}$, $\operatorname{tg}\beta = \frac{15}{8}$, $\beta = \operatorname{tg}^{-1}\left(\frac{15}{8}\right) \approx 62°$

3.1 $\cos 50° = \frac{\overline{AB}}{8} \Leftrightarrow \overline{AB} = 8\cos 50° \Leftrightarrow \overline{AB} \approx 5{,}1$

$\operatorname{sen} 50° = \frac{\overline{BC}}{8} \Leftrightarrow \overline{BC} = 8\operatorname{sen} 50° \Leftrightarrow \overline{BC} \approx 6{,}1$

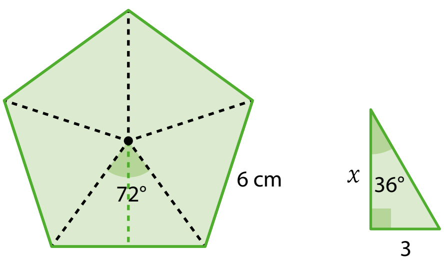
$A\hat{C}B = 90° - 50° = 40°$

3.2 $10^2 = 5^2 + \overline{AC}^2 \Leftrightarrow \overline{AC}^2 = 100 - 25 \Leftrightarrow \overline{AC}^2 = 75 \underset{x>0}{\Leftrightarrow} x = \sqrt{75}$; logo, $x \approx 8{,}7$.

$\cos\left(A\hat{B}C\right) = \frac{5}{10} \Leftrightarrow A\hat{B}C = 60°$

$C\hat{A}B = 90° - 60° = 30°$

4.



$\operatorname{tg} 36° = \frac{3}{x} \Leftrightarrow x = \frac{3}{\operatorname{tg} 36°}$; logo, $x \approx 4{,}129$.

$A = 5 \times \frac{6x}{2} = 15x \approx 15 \times 4{,}129 = 61{,}935 \approx 62 \text{ cm}^2$

PÁG. 11
Diagnóstico

5. Seja x a medida da altura do trapézio.

Tem-se $\frac{x}{5} = \text{sen}\,45° \Leftrightarrow x = 5 \times \text{sen}\,45°$; logo, $x \approx 3{,}536$.

Assim, $\text{A} \approx \frac{3+12}{2} \times 3{,}536 = 26{,}52 \approx 26{,}5\ \text{cm}^2$.

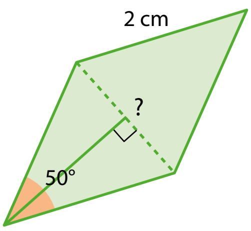
6. $\text{sen}\,43° = \frac{\overline{AC}}{10} \Leftrightarrow \overline{AC} = 10 \times \text{sen}\,43°$; logo, $\overline{AC} \approx 6{,}8998$.

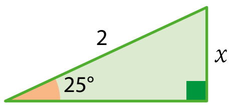
$6{,}81998 + 1{,}54 \approx 8{,}36$ m

7. $\text{tg}\,40° = \frac{x}{2{,}4} \Leftrightarrow x = 2{,}4 \times \text{tg}\,40°$; logo, $x \approx 2{,}014$.

$h = x + 1{,}62 \approx 2{,}014 + 1{,}62 = 3{,}634 \approx 3{,}63$ m

PÁG. 15
Tarefa 1

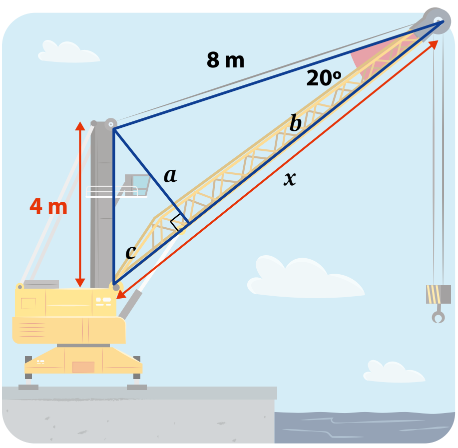


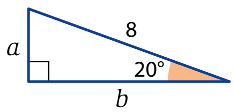


$\frac{x}{2} = \text{sen}\,25° \Leftrightarrow x = 2 \times \text{sen}\,25°$; logo, $x \approx 0{,}845$.

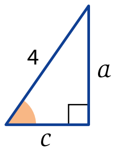
$2x \approx 2 \times 0{,}845 = 1{,}69$ cm

PÁG. 15
Tarefa 2





$\frac{a}{8} = \text{sen}\,20° \Leftrightarrow a = 8 \times \text{sen}\,20° \Leftrightarrow a \approx 2{,}736$ e $\frac{b}{8} = \cos 20° \Leftrightarrow b = 8 \times \cos 20°$; logo, $b \approx 7{,}518$.



$4^2 \approx 2{,}736^2 + c^2 \Leftrightarrow c^2 \approx 16 - 7{,}486 \Leftrightarrow c^2 = 8{,}514 \underset{c>0}{\Leftrightarrow} c = \sqrt{8{,}514}$; logo, $c \approx 2{,}918$.

$x = b + c \approx 7{,}518 + 2{,}918 = 10{,}436 \approx 10{,}4$ m

PÁG. 17

Tarefa 3

1.

a. Como o triângulo é equilátero, ao traçar a altura relativa ao lado $[AB]$, esse lado é intersetado no seu ponto médio e, como o lado do triângulo é unitário, tem-se $\overline{AD} = \frac{1}{2}$.

$$\overline{AC}^2 = \overline{AD}^2 + \overline{CD}^2 \Leftrightarrow \overline{CD}^2 = 1^2 - \left(\frac{1}{2}\right)^2 \Leftrightarrow \overline{CD}^2 = \frac{3}{4} \underset{\overline{CD}>0}{\Leftrightarrow} \overline{CD} = \sqrt{\frac{3}{4}} \Leftrightarrow \overline{CD} = \frac{\sqrt{3}}{2}$$

b. Sabe-se que:

- o triângulo é equilátero;
- num triângulo, a lados iguais opõem-se ângulos iguais;
- a soma das amplitudes dos ângulos internos de um triângulo é igual a $180°$;

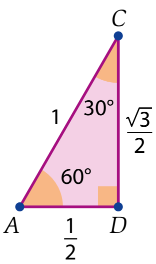
  portanto, conclui-se que as amplitudes dos ângulos internos do triângulo são iguais e $D\hat{A}C = \frac{180°}{3} = 60°$.

Sabe-se que:

- o triângulo é equilátero;
- $B\hat{C}A = 60°$;
- $[CD]$ é a altura relativa ao lado $[AB]$;

  portanto, conclui-se que $D\hat{C}A = \frac{60°}{2} = 30°$.

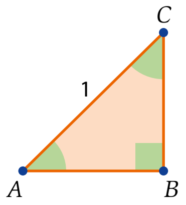
c.



$$\operatorname{sen} 30° = \frac{\frac{1}{2}}{1} = \frac{1}{2} \text{ ; } \cos 30° = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2} \text{ ; } \operatorname{tg} 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} .$$

$$\operatorname{sen} 60° = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2} \text{ ; } \cos 60° = \frac{\frac{1}{2}}{1} = \frac{1}{2} \text{ ; } \operatorname{tg} 60° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} .$$

2.



a. $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \underset{\overline{AB}=\overline{BC}}{\Leftrightarrow} 1^2 = 2\overline{AB}^2 \Leftrightarrow \overline{AB}^2 = \frac{1}{2} \underset{\overline{AB}>0}{\Leftrightarrow} \overline{AB} = \sqrt{\frac{1}{2}} \Leftrightarrow \overline{AB} = \frac{\sqrt{2}}{2}$

b. Sabe-se que:

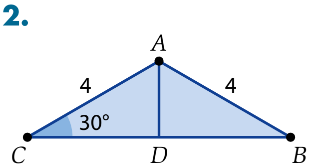
- num triângulo, a lados iguais opõem-se ângulos iguais;
- a soma das amplitudes dos ângulos internos de um triângulo é igual a $180°$;
  portanto, conclui-se que as amplitudes dos ângulos internos agudos do triângulo são iguais e $C\hat{A}B = B\hat{C}A = \frac{180° - 90°}{2} = 45°$.

c. $\operatorname{sen} 45° = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$; $\cos 45° = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$; $\operatorname{tg} 45° = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$.

PÁG. 18

Aplicar

2.



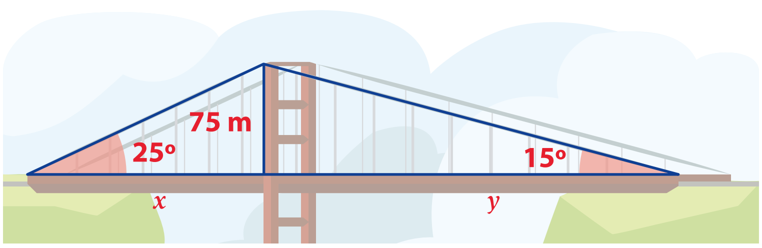
$\cos 30° = \frac{\overline{CD}}{4} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{\overline{CD}}{4} \Leftrightarrow \overline{CD} = \frac{4\sqrt{3}}{2} \Leftrightarrow \overline{CD} = 2\sqrt{3}$

$\overline{BC} = 2\overline{CD} = 2 \times 2\sqrt{3} = 4\sqrt{3}$

$\operatorname{sen} 30° = \frac{\overline{AD}}{4} \Leftrightarrow \frac{1}{2} = \frac{\overline{AD}}{4} \Leftrightarrow \overline{AD} = \frac{4}{2} \Leftrightarrow \overline{AD} = 2$

$A_{[ABC]} = \frac{\overline{BC} \times \overline{AD}}{2} = \frac{4\sqrt{3} \times 2}{2} = 4\sqrt{3} \approx 6{,}93 \text{ cm}^2$

3.

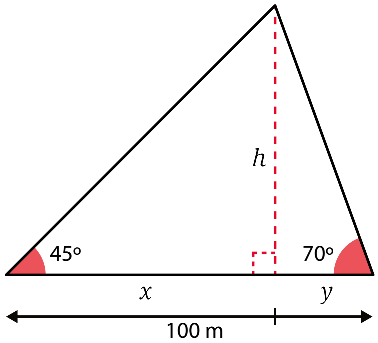


$\operatorname{tg} 25° = \frac{75}{x} \Leftrightarrow x = \frac{75}{\operatorname{tg} 25°} \Leftrightarrow x \approx 160{,}838$ e $\operatorname{tg} 15° = \frac{75}{y} \Leftrightarrow y = \frac{75}{\operatorname{tg} 15°}$; logo, $y \approx 279{,}904$.

$x + y \approx 160{,}838 + 279{,}904 = 440{,}742 \approx 441$ m

PÁG. 19
Aplicar

5.



$\text{tg}\,45° = \frac{h}{x} \Leftrightarrow h = x \times \text{tg}\,45° \Leftrightarrow h = x$ e $\text{tg}\,70° = \frac{h}{y} \Leftrightarrow h = y \times \text{tg}\,70°$

$x + y = 100 \Leftrightarrow y = 100 - x \underset{h=x}{\Leftrightarrow} y = 100 - h$

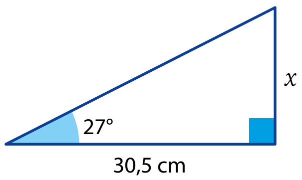
$h = (100 - h)\text{tg}\,70° \Leftrightarrow h = 100 \times \text{tg}\,70° - h \times \text{tg}\,70° \Leftrightarrow h + h \times \text{tg}\,70° = 100\text{tg}\,70° \Leftrightarrow$

$\Leftrightarrow h(1 + \text{tg}\,70°) = 100 \times \text{tg}\,70° \Leftrightarrow h = \frac{100 \times \text{tg}\,70°}{1 + \text{tg}\,70°}$; logo, $h \approx 73{,}315$.

$73{,}315 + 1{,}70 \approx 75$ m

PÁG. 20
Aplicar

6.

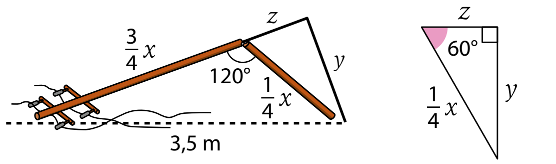


$\text{tg}\,27° = \frac{x}{30{,}5} \Leftrightarrow x = 30{,}5 \times \text{tg}\,27°$; logo, $x \approx 15{,}54$ cm .

A profundidade de cada degrau é de 30,5 cm (valor superior a 28 cm , como é exigido no Decreto-Lei).
A altura de cada degrau é, aproximadamente, 15,5 cm (valor inferior a 18 cm = 0,18m).

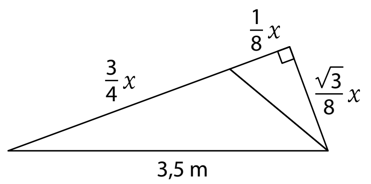
Como os 26 degraus são todos iguais, as suas dimensões são constantes, pelo que as escadarias da Casa da Música satisfazem as normas indicadas pelo citado Decreto-Lei.

7. Seja x o comprimento, em metros, do poste.



$$\operatorname{sen} 60° = \frac{y}{\frac{1}{4}x} \Leftrightarrow y = \frac{\sqrt{3}}{2} \times \frac{1}{4}x \Leftrightarrow y = \frac{\sqrt{3}}{8}x$$

$$\cos 60° = \frac{z}{\frac{1}{4}x} \Leftrightarrow z = \frac{1}{2} \times \frac{1}{4}x \Leftrightarrow z = \frac{1}{8}x$$



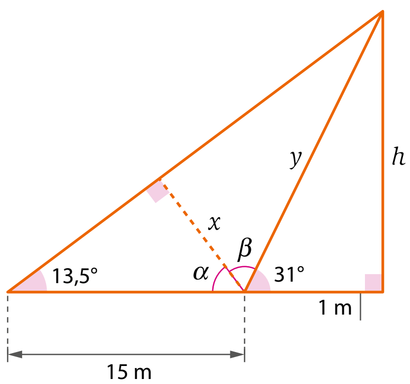
$$\left(\frac{3}{4}x + \frac{1}{8}x\right)^2 + \left(\frac{\sqrt{3}}{8}x\right)^2 = 3{,}5^2 \Leftrightarrow \left(\frac{7}{8}x\right)^2 + \frac{3}{64}x^2 = \left(\frac{7}{2}\right)^2 \Leftrightarrow \frac{49}{64}x^2 + \frac{3}{64}x^2 = \frac{49}{4} \Leftrightarrow$$

$$\Leftrightarrow \frac{52}{64}x^2 = \frac{49}{4} \Leftrightarrow x^2 = \frac{49 \times 64}{4 \times 52} \underset{x=0}{\Leftrightarrow} x = \sqrt{\frac{49 \times 64}{4 \times 52}} \text{ ; logo, } x \approx 3{,}9 \text{ m .}$$

PÁG. 21

Aplicar

8.



$\alpha = 180° - 90° - 13{,}5° = 76{,}5°$

$\beta = 180° - 76{,}5° - 31° = 72{,}5°$

$\operatorname{sen}(13{,}5°) = \frac{x}{15} \Leftrightarrow x = 15 \times \operatorname{sen}(13{,}5°)$; logo, $x \approx 3{,}5$ m .

$\cos(72{,}5°) = \frac{x}{y}$; logo, $y \approx \frac{3{,}5}{\cos(72{,}5°)} \approx 11{,}64$ m .

$\operatorname{sen}(31°) = \frac{h}{y}$; logo, $h \approx 11{,}64 \times \operatorname{sen}(31°) \approx 5{,}995$ m .

A altura pedida é, aproximadamente, $5{,}995 + 1 \approx 7$ metros.

Por outro processo:

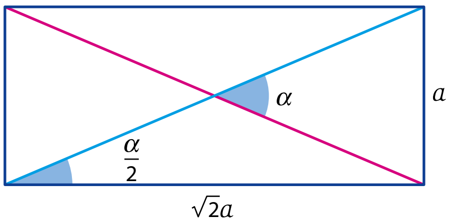
$$\begin{cases} \frac{h}{x} = \text{tg}\,31° \\ \frac{h}{x+15} = \text{tg}\,13{,}5° \end{cases} \Leftrightarrow \begin{cases} h = x \times \text{tg}\,31° \\ h = (x+15)\text{tg}\,13{,}5° \end{cases} \Leftrightarrow \begin{cases} x \times \text{tg}\,31° = (x+15)\text{tg}\,13{,}5° \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x \times \text{tg}\,31° = x \times \text{tg}\,13{,}5° + 15 \times \text{tg}\,13{,}5° \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} x \times \text{tg}\,31° - x \times \text{tg}\,13{,}5° = 15 \times \text{tg}\,13{,}5° \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x(\text{tg}\,31° - \text{tg}\,13{,}5°) = 15 \times \text{tg}\,13{,}5° \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} x = \frac{15\text{tg}\,13{,}5°}{\text{tg}\,31° - \text{tg}\,13{,}5°} \\ \_\_\_\_\_\_ \end{cases} \text{; logo, } \begin{cases} x \approx 9{,}98 \\ h \approx 5{,}99755 \end{cases}.$$

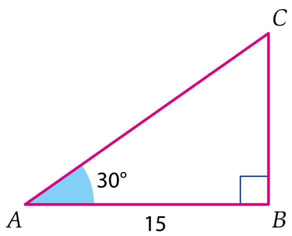
A altura pedida é, aproximadamente, $5{,}99755 + 1 = 6{,}99755 \approx 7$ m .

9. Seja a a medida da aresta do cubo. A medida da diagonal facial do cubo é $\sqrt{2}\,a$.



$$\text{tg}\left(\frac{\alpha}{2}\right) = \frac{a}{\sqrt{2}\,a} \Leftrightarrow \frac{\alpha}{2} = \text{tg}^{-1}\left(\frac{1}{\sqrt{2}}\right) \text{; logo, } \frac{\alpha}{2} \approx 35{,}2644° \text{ , pelo que, } \alpha \approx 70{,}53° \text{ .}$$

10.1



$$\cos 30° = \frac{15}{\overline{AC}} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{15}{\overline{AC}} \Leftrightarrow \overline{AC} = \frac{30}{\sqrt{3}} \text{; logo, } \overline{AC} \approx 17{,}3 \text{ cm .}$$

10.2 $\text{tg}\,30° = \frac{\overline{BC}}{\overline{AB}} \Leftrightarrow \overline{BC} = 15 \times \frac{\sqrt{3}}{3} \Leftrightarrow \overline{BC} = 5\sqrt{3}$

$$\overline{CE}^2 = \overline{BC}^2 + \overline{EB}^2 \Leftrightarrow \overline{CE}^2 = (5\sqrt{3})^2 + \left(\frac{15}{2}\right)^2 \Leftrightarrow \overline{CE}^2 = \frac{525}{4} \underset{\overline{CE}>0}{\Leftrightarrow} \overline{CE} = \sqrt{\frac{525}{4}} \text{; logo, } \overline{CE} \approx 11{,}5 \text{ cm .}$$

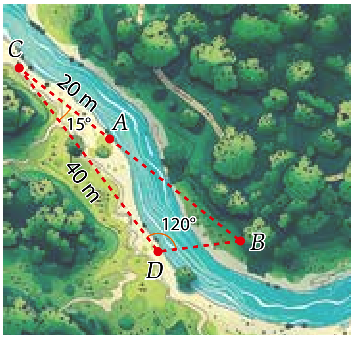
10.3 $\text{tg}\left(B\hat{E}C\right) = \frac{\overline{BC}}{\overline{EB}} \Leftrightarrow \text{tg}\left(B\hat{E}C\right) = \frac{5\sqrt{3}}{\frac{15}{2}} \Leftrightarrow B\hat{E}C = \text{tg}^{-1}\left(\frac{2\sqrt{3}}{3}\right)$; logo, $B\hat{E}C \approx 49{,}107°$.

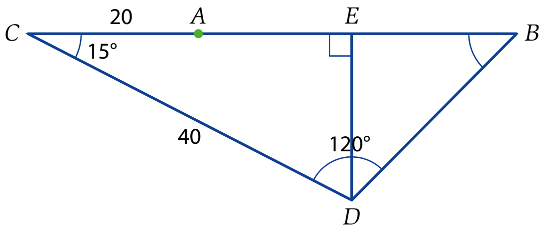
$A\hat{E}C = 180° - B\hat{E}C \approx 130{,}9°$

PÁG. 22

Aplicar

11.





$D\hat{B}C = 180° - 120° - 15° = 45°$

$B\hat{D}E = 90° - 45° = 45°$

$\frac{\overline{CE}}{40} = \cos 15° \Leftrightarrow \overline{CE} = 40 \times \cos 15°$; logo, $\overline{CE} \approx 38{,}64$ m .

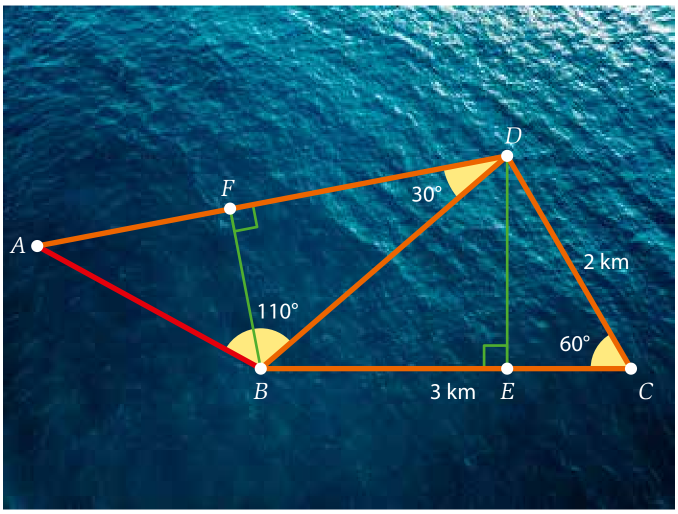
$\frac{\overline{DE}}{40} = \text{sen}\, 15° \Leftrightarrow \overline{DE} = 40 \times \text{sen}\, 15°$; logo, $\overline{DE} \approx 10{,}35$ m .

$\frac{\overline{DE}}{\overline{BE}} = \text{tg}\, 45° \Leftrightarrow \frac{\overline{DE}}{\overline{BE}} = 1 \Leftrightarrow \overline{DE} = \overline{BE}$

$\overline{CB} = \overline{CE} + \overline{EB} \approx 38{,}64 + 10{,}35 = 48{,}99$

$\overline{AB} = \overline{CB} - 20 \approx 48{,}99 - 20 = 28{,}99 \approx 29$ m

12.



$\text{sen}\, 60° = \frac{\overline{DE}}{2} \Leftrightarrow \overline{DE} = 2 \times \frac{\sqrt{3}}{2} \Leftrightarrow \overline{DE} = \sqrt{3}$

$\cos 60° = \frac{\overline{EC}}{2} \Leftrightarrow \overline{EC} = 2 \times \frac{1}{2} \Leftrightarrow \overline{EC} = 1$

$\overline{BE} = 3 - \overline{EC} = 3 - 1 = 2$

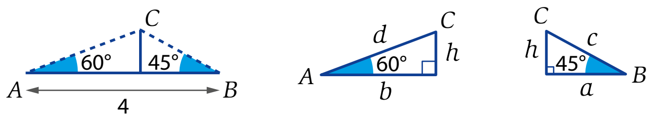
$\overline{BD}^2 = \overline{BE}^2 + \overline{ED}^2 \Leftrightarrow \overline{BD}^2 = 2^2 + \left(\sqrt{3}\right)^2 \Leftrightarrow \overline{BD} = \sqrt{7}$

$B\hat{A}D = 180° - 110° - 30° = 40°$

$\text{sen}\,30° = \frac{\overline{BF}}{\sqrt{7}} \Leftrightarrow \frac{1}{2} = \frac{\overline{BF}}{\sqrt{7}} \Leftrightarrow \overline{BF} = \frac{\sqrt{7}}{2}$

$\text{sen}\,40° = \frac{\overline{BF}}{\overline{AB}} \Leftrightarrow \overline{AB} = \frac{\frac{\sqrt{7}}{2}}{\text{sen}\,40°}$; logo, $\overline{AB} \approx 2{,}058\text{ km} = 2058\text{ m}$.

13.1



$\frac{h}{a} = \text{tg}\,45° \Leftrightarrow h = a \times 1 \Leftrightarrow h = a$ e $\frac{h}{b} = \text{tg}\,60° \Leftrightarrow h = b\sqrt{3}$

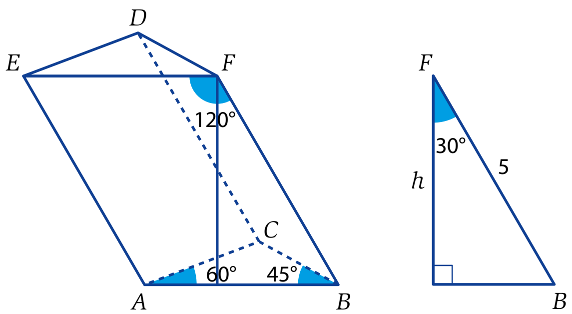
Destas igualdades, conclui-se que $a = b\sqrt{3}$.

Por outro lado, $a + b = 4 \Leftrightarrow b\sqrt{3} + b = 4 \Leftrightarrow b(\sqrt{3}+1) = 4 \Leftrightarrow b = \frac{4}{\sqrt{3}+1}$.

Assim, $h = \frac{4}{\sqrt{3}+1}\sqrt{3} = \frac{4\sqrt{3}}{\sqrt{3}+1}$.

Logo, $A_{[ABC]} = \frac{\overline{AB} \times h}{2} = \frac{4 \times \frac{4\sqrt{3}}{\sqrt{3}+1}}{2} = \frac{8\sqrt{3}}{\sqrt{3}+1}$.

13.2



$\cos 30° = \frac{h}{5} \Leftrightarrow h = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$

$V = \frac{8\sqrt{3}}{\sqrt{3}+1} \times \frac{5\sqrt{3}}{2} = \frac{60}{\sqrt{3}+1}$

PÁG. 26

Aplicar

14.1 A., C. e D.

14.2 B., E. e F.

14.3 A. e F.

14.4 C., E. e F.

PÁG. 27

Aplicar

15.1 $\frac{360°}{8}=45°$; $\alpha=45°$, $\beta=-3\times45°=-135°$, $\gamma=-45°$.

15.2

a. G $(-90°=-2\times45°)$

b. F $(225°=5\times45°)$

c. D $(-225°=-5\times45°)$

d. A $(720°=16\times45°$ ou $720°=2\times360°)$

e. B

f. B

g. F $(-1215°=-27\times45°$ ou $-1215°=-3\times45°-3\times360°)$

h. I $(-405°=-9\times45°$ ou $-405°=-45°-360°)$

17.1 $35°$

17.2 $-35°$

17.3 $-90°$

17.4 $90°$, porque $450°=360°+90°$.

17.5 $-1°$, porque $-361°=-360°-1°$.

17.6 $-65°$, porque $-785°=-2\times360°-65°$.

17.7 $100°$, porque $820°=2\times360°+100°$.

17.8 $-150°$, porque $-1950°=-5\times360°-150°$.

17.9 $-65°$, porque $-425°=-360°-65°$.

17.10 $-300°$, porque $60°-4\times360°=-1380°$ e $-1380°=-3\times360°-300°$.

PÁG. 28

Aplicar

18.1 $\frac{360°}{12}=30°$; $4\times30°=120°$.

18.2 $120°$ e $-120°$.

18.3 Sim, porque $3000° = 8 \times 360° + 120°$.

18.4 $2880° : 360° = 8$

A roda deu 8 voltas completas.

19.1 $\frac{360°}{8} = 45°$

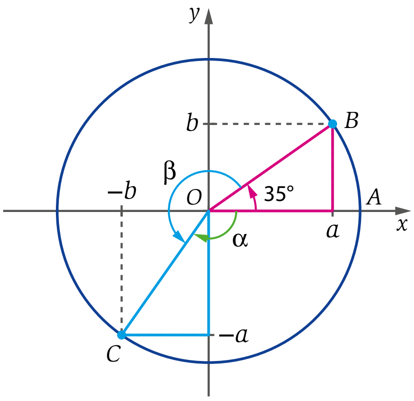
19.2

a. $5 \times 45° = 225°$

b. $-45°$

19.3 $-2 \times 360° - 4 \times 45° = -900°$

20.



Os triângulos representados a vermelho e a azul são geometricamente iguais (são retângulos e têm os catetos iguais, de um para o outro), pelo que têm ângulos internos iguais. Assim, $\alpha = -90° - 35° = -125°$ e

$\beta = 360° - 35° - 125° = 200°$.

20.1 $\alpha + 2000° = 1875°$

$1875° = 5 \times 360° + 75°$

A amplitude do ângulo orientado com os mesmos lados origem e extremidade do ângulo de amplitude $\alpha + 2000°$ é $75°$.

20.2 $\beta - 1575° = -1375°$

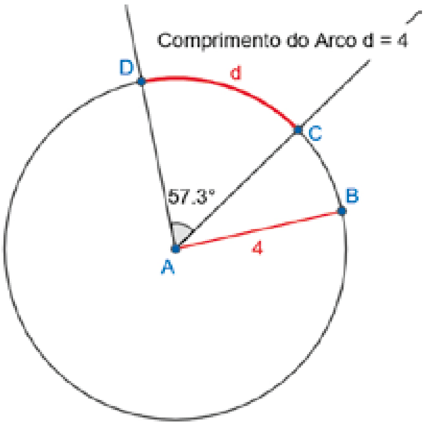
$-1375° = -3 \times 360° - 295°$

A amplitude do ângulo orientado com os mesmos lados origem e extremidade do ângulo de amplitude $\beta - 1575°$ é $-295°$.

PÁG. 30

Tarefa 4

Por exemplo:



De acordo com a construção 1 radiano corresponde aproximadamente a $57{,}3°$, pois o comprimento do arco d é igual ao raio da circunferência.

PÁG. 31

Tarefa 5

$90° = \frac{1}{4} \times 360°$; logo, a $90°$ correspondem $\frac{1}{4} \times 2\pi \text{ rad} = \frac{\pi}{2} \text{rad}$.

$180° = \frac{1}{2} \times 360°$; logo, a $180°$ correspondem $\frac{1}{2} \times 2\pi \text{ rad} = \pi \text{ rad}$.

$270° = \frac{3}{4} \times 360°$; logo, a $270°$ correspondem $\frac{3}{4} \times 2\pi \text{ rad} = \frac{3}{2}\pi \text{ rad}$.

$30° = \frac{1}{12} \times 360°$; logo, a $30°$ correspondem $\frac{1}{12} \times 2\pi \text{ rad} = \frac{\pi}{6} \text{rad}$.

$45° = \frac{1}{8} \times 360°$; logo, a $45°$ correspondem $\frac{1}{8} \times 2\pi \text{ rad} = \frac{\pi}{4} \text{rad}$.

$60° = \frac{1}{6} \times 360°$; logo, a $60°$ correspondem $\frac{1}{6} \times 2\pi \text{ rad} = \frac{\pi}{3} \text{rad}$.

PÁG. 32

Aplicar

23. $\frac{r \text{ cm}}{1 \text{ rad}} = \frac{8 \text{ cm}}{2 \text{ rad}} \Leftrightarrow r = 4 \text{ cm}$.

Por outro processo:

O comprimento de um arco de amplitude α e raio r é dado por $\alpha \times \pi$, pelo que $2 \times r = 8 \Leftrightarrow r = 4 \text{ cm}$.

24.1 $x = \frac{20°}{180°}\pi = \frac{\pi}{9}$

24.2 $x = \frac{86°}{180°}\pi = \frac{43\pi}{90}$

24.3 $x = \frac{135°}{180°}\pi = \frac{3\pi}{4}$

24.4 $x=\frac{190°}{180°}\pi=\frac{19\pi}{18}$

24.5 $x=\frac{150°}{180°}\pi=\frac{5\pi}{6}$

24.6 $x=\frac{350°}{180°}\pi=\frac{35\pi}{18}$

25.1 $x=\frac{\frac{\pi}{5}\times 180°}{\pi}=36°$

25.2 $x=\frac{\frac{\pi}{10}\times 180°}{\pi}=18°$

25.3 $x=\frac{\frac{\pi}{12}\times 180°}{\pi}=15°$

25.4 $x=\frac{\frac{5\pi}{12}\times 180°}{\pi}=75°$

25.5 $x=\frac{0{,}8\times 180°}{\pi}\approx 45{,}8°$

25.6 $x=\frac{1\times 180°}{\pi}\approx 57{,}3°$

25.7 $x=\frac{4\times 180°}{\pi}\approx 229{,}2°$

25.8 $x=\frac{6{,}3\times 180°}{\pi}\approx 361{,}0°$

PÁG. 34

Tarefa 6

a. $-35°\in]-90°\ ,\ 0°[$

O ângulo de amplitude $-35°$ pertence ao 4.° quadrante.

b. $451°-360°=91°\in]90°\ ,\ 180°[$

O ângulo de amplitude $451°$ pertence 2.° quadrante.

c. $-820°+2\times 360°=-100°\in]-180°\ ,\ -90°[$

O ângulo de amplitude $-820°$ pertence 3.° quadrante.

d. $\frac{\pi}{4}\text{rad}\in\left]0\ ,\ \frac{\pi}{2}\text{rad}\right[$

O ângulo de amplitude $\frac{\pi}{4}\text{rad}$ pertence 1.° quadrante.

e. $-\frac{5\pi}{6}\text{rad}\in\left]-\pi\text{ rad},-\frac{\pi}{2}\text{rad}\right[$

O ângulo de amplitude $-\frac{5\pi}{6}\text{rad}$ pertence 3.° quadrante.

PÁG. 35

Aplicar

26.1 $35° \in]0°, 90°[$

1.° quadrante

26.2 $720° = 2 \times 360°$

Semieixo positivo Ox

26.3 $-\frac{7\pi}{36} \in \left]-\frac{\pi}{2}, 0\right[$

4.° quadrante .

26.4 $-\frac{\pi}{2} - 70\pi + 35 \times 2\pi = -\frac{\pi}{2}$

Semieixo negativo Oy .

26.5 $450° - 360° = 90°$

Semieixo positivo Oy

26.6 $-5\pi + 3 \times 2\pi = \pi$

Semieixo negativo Ox

26.7 $-361° + 360° = -1° \in]-90°, 0°[$

4.° quadrante

26.8 $-885° + 2 \times 360° = -165° \in]-180°, -90°[$

3.° quadrante .

26.9 $\frac{41\pi}{9} - 2 \times 2\pi = \frac{5\pi}{9} \in \left]\frac{\pi}{2}, \pi\right[$

2.° quadrante

26.10 $630° - 360° = 270°$

Semieixo negativo Oy

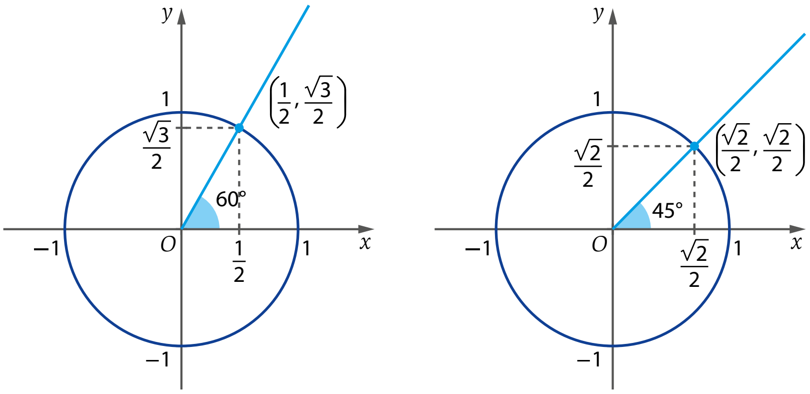
26.11 $-\frac{65\pi}{6} + 6 \times 2\pi = \frac{7\pi}{6} \in \left]\pi, \frac{3\pi}{2}\right[$

3.° quadrante

26.12 $-605° + 360° = -245° \in]-270°, -180°[$

2.° quadrante

PÁG. 38
Tarefa 8



PÁG. 38
Tarefa 9

1. Entre -1 e 1 .

2. O seno é positivo nos 1.° e 2.° quadrantes e negativo nos 3.° e 4.° quadrantes.
O cosseno é positivo nos 1.° e 4.° quadrantes e negativo nos 2.° e 3.° quadrantes.

3. Considerando que os valores do cosseno e do seno são, respetivamente, a abcissa e a ordenada do ponto de interseção do lado extremidade de cada ângulo com a circunferência trigonométrica, tem-se:

	Cosseno	Seno
$0°$ (0)	1	0
$90°$ $\left(\frac{\pi}{2}\right)$	0	1
$180°$ (π)	-1	0
$270°$ $\left(\frac{3\pi}{2}\right)$	0	-1

PÁG. 41
Tarefa 10

$$P_1 : (2\cos 30°\ ,\ 2\ \text{sen}\, 30°) = \left(2\times\frac{\sqrt{3}}{2}\ ,\ 2\times\frac{1}{2}\right) = \left(\sqrt{3}\ ,\ 1\right)$$

$$Q_1 : (2{,}2\ \text{tg}\, 30°) = \left(2{,}2\times\frac{\sqrt{3}}{3}\right) = \left(2\ ,\ \frac{2\sqrt{3}}{3}\right)$$

$$P_2 : (-2\cos 30°\ ,\ 2\ \text{sen}\, 30°) = \left(-2\times\frac{\sqrt{3}}{2}\ ,\ 2\times\frac{1}{2}\right) = \left(-\sqrt{3}\ ,\ 1\right)$$

$$P_3 : (-2\cos 30°\ ,\ -2\ \text{sen}\, 30°) = \left(-2\times\frac{\sqrt{3}}{2}\ ,\ -2\times\frac{1}{2}\right) = \left(-\sqrt{3}\ ,\ -1\right)$$

$$P_4 : (2\cos 30°\ ,\ -2\ \text{sen}\, 30°) = \left(2\times\frac{\sqrt{3}}{2}\ ,\ -2\times\frac{1}{2}\right) = \left(\sqrt{3}\ ,\ -1\right)$$

$$Q_2 : (2\ ,\ -2\ \text{tg}\, 30°) = \left(2\ ,\ -2\times\frac{\sqrt{3}}{3}\right) = \left(2\ ,\ -\frac{2\sqrt{3}}{3}\right)$$

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PÁG. 42

Tarefa 11

$$\operatorname{tg} 30° = \frac{\operatorname{sen} 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\operatorname{tg} 45° = \frac{\operatorname{sen} 45°}{\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\operatorname{tg} 60° = \frac{\operatorname{sen} 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

PÁG. 43

Aplicar

29. Como $0° < \alpha < 180°$ e $\cos\alpha < 0$, conclui-se que $\alpha \in 2.°$ Q.

Assim, $\operatorname{sen}\alpha > 0$ e $\operatorname{tg}\alpha < 0$.

Logo, $\operatorname{sen}\alpha - \operatorname{tg}\alpha > 0$.

30. Como $\alpha \in 2.°$ Q., $\operatorname{sen}\alpha > 0$, $\cos\alpha < 0$ e $\operatorname{tg}\alpha < 0$.

Como $\cos\beta \times \operatorname{tg}\alpha > 0$ e $\operatorname{tg}\alpha < 0$, conclui-se que $\cos\beta < 0$.

Como $\beta \in]-\pi, 0[$, conclui-se que $\beta \in 3.°$ Q., $\operatorname{sen}\beta < 0$ e $\operatorname{tg}\beta > 0$.

(A) $\operatorname{sen}\alpha - \operatorname{sen}\beta > 0$

(B) $\cos\alpha \times \cos\beta > 0$

(C) $\cos\alpha + \operatorname{sen}\beta < 0$

(D) $\operatorname{tg}\beta - \operatorname{tg}\alpha > 0$

Opção correta: **(C)**

31.1 $\operatorname{sen}(45°) - 2\cos(30°) - \operatorname{tg}(180°) = \frac{\sqrt{2}}{2} - 2 \times \frac{\sqrt{3}}{2} - 0 = \frac{\sqrt{2}}{2} - \sqrt{3}$

31.2 $\operatorname{sen}(-630°) \times \cos(-330°) - \operatorname{tg}(225°) = \operatorname{sen}(90°) \times \cos(30°) - \operatorname{tg}(45°) = 1 \times \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3}}{2} - 1$

31.3 $\frac{(\operatorname{sen}(765°) + \cos(45°)) \times \operatorname{tg}(60°)}{\operatorname{tg}(390°)} = \frac{(\operatorname{sen}(45°) + \cos(45°)) \times \operatorname{tg}(60°)}{\operatorname{tg}(30°)} =$

$$= \frac{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \times \sqrt{3}}{\frac{\sqrt{3}}{3}} = \frac{\sqrt{2} \times \sqrt{3}}{\frac{\sqrt{3}}{3}} = 3\sqrt{2}$$

31.4 $\dfrac{2\text{tg}(45°)-\cos(-1020°)-\cos(90°)}{\text{sen}(270°)+4\text{sen}(30°)}=\dfrac{2\times 1-\cos(60°)-0}{-1+4\times\frac{1}{2}}=\dfrac{2-\frac{1}{2}}{1}=\dfrac{3}{2}$

32.1 $\text{sen}\left(\frac{3\pi}{2}\right)\times\text{sen}\left(\frac{\pi}{6}\right)-\text{tg}\left(\frac{\pi}{4}\right)+\cos(\pi)=-1\times\frac{1}{2}-1+(-1)=-\frac{1}{2}-2=-\frac{5}{2}$

32.2 $\text{sen}\left(\frac{\pi}{3}\right)+\cos\left(\frac{\pi}{4}\right)-\text{tg}\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}-\sqrt{3}=\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}=\frac{\sqrt{2}-\sqrt{3}}{2}$

32.3 $\text{sen}(\pi)+\text{sen}\left(\frac{\pi}{2}\right)\times\text{tg}\left(\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{2}\right)=0+1\times\frac{\sqrt{3}}{3}-0=\frac{\sqrt{3}}{3}$

32.4 $\text{tg}(2\pi)-\cos\left(\frac{\pi}{3}\right)\times\text{sen}\left(\frac{\pi}{4}\right)+\text{tg}\left(\frac{\pi}{6}\right)=0-\frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{3}=-\frac{\sqrt{2}}{4}+\frac{\sqrt{3}}{3}=\frac{4\sqrt{3}-3\sqrt{2}}{12}$

32.5 $\cos\left(-\frac{3\pi}{4}\right)-\text{tg}\left(\frac{11\pi}{3}\right)-2\text{sen}\left(\frac{7\pi}{4}\right)=\cos\left(\frac{5\pi}{4}\right)-\text{tg}\left(-\frac{\pi}{3}\right)-2\text{sen}\left(-\frac{\pi}{4}\right)=$

$=\cos\left(\pi+\frac{\pi}{4}\right)+\text{tg}\left(\frac{\pi}{3}\right)+2\text{sen}\left(\frac{\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)+\sqrt{3}+2\frac{\sqrt{2}}{2}=-\frac{\sqrt{2}}{2}+\sqrt{3}+2\frac{\sqrt{2}}{2}=\sqrt{3}+\frac{\sqrt{2}}{2}$

Cálculos auxiliares:

$-\frac{3\pi}{4}+2\pi=\frac{5\pi}{4}=\pi+\frac{\pi}{4}$

$\frac{11\pi}{3}-2\times 2\pi=-\frac{\pi}{3}$

32.6 $\cos\left(-\frac{25\pi}{4}\right)+\text{sen}\left(-\frac{13\pi}{4}\right)-\text{tg}\left(\frac{14\pi}{3}\right)=\cos\left(-\frac{\pi}{4}\right)+\text{sen}\left(\frac{3\pi}{4}\right)-\text{tg}\left(\frac{2\pi}{3}\right)=$

$=\cos\left(\frac{\pi}{4}\right)+\text{sen}\left(\pi-\frac{\pi}{4}\right)-\text{tg}\left(\pi-\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}+\text{sen}\left(\frac{\pi}{4}\right)+\text{tg}\left(\frac{\pi}{3}\right)=$

$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+\sqrt{3}=\sqrt{2}+\sqrt{3}$

Cálculos auxiliares:

$-\frac{25\pi}{4}+3\times 2\pi=-\frac{\pi}{4}$

$-\frac{13\pi}{4}+2\times 2\pi=\frac{3\pi}{4}=\pi-\frac{\pi}{4}$

$\frac{14\pi}{3}-2\times 2\pi=\frac{2\pi}{3}=\pi-\frac{\pi}{3}$

32.7 $\cos\left(\frac{23\pi}{6}\right)\times\text{tg}\left(\frac{7\pi}{3}\right)=\cos\left(-\frac{\pi}{6}\right)\times\text{tg}\left(\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{6}\right)\times\sqrt{3}=\frac{\sqrt{3}}{2}\times\sqrt{3}=\frac{3}{2}$

Cálculos auxiliares:

$\frac{23\pi}{6}-2\times 2\pi=-\frac{\pi}{6}$

$\frac{7\pi}{3}-2\pi=\frac{\pi}{3}$

32.8 $\dfrac{\cos\left(\frac{11\pi}{2}\right)-\text{sen}\left(-\frac{10\pi}{3}\right)}{\text{tg}\left(\frac{8\pi}{3}\right)-\text{tg}\left(\frac{11\pi}{6}\right)}=\dfrac{\cos\left(\frac{3\pi}{2}\right)-\text{sen}\left(\frac{2\pi}{3}\right)}{\text{tg}\left(\frac{2\pi}{3}\right)-\text{tg}\left(-\frac{\pi}{6}\right)}=\dfrac{0-\text{sen}\left(\pi-\frac{\pi}{3}\right)}{\text{tg}\left(\pi-\frac{\pi}{3}\right)+\text{tg}\left(\frac{\pi}{6}\right)}=$

$=\dfrac{-\text{sen}\left(\frac{\pi}{3}\right)}{-\text{tg}\left(\frac{\pi}{3}\right)+\text{tg}\left(\frac{\pi}{6}\right)}=\dfrac{-\frac{\sqrt{3}}{2}}{-\sqrt{3}+\frac{\sqrt{3}}{3}}=\dfrac{-\frac{\sqrt{3}}{2}}{-\frac{2\sqrt{3}}{3}}=\dfrac{3}{4}$

Cálculos auxiliares:

$\frac{11\pi}{2}-2\times 2\pi=\frac{3\pi}{2}$

$-\frac{10\pi}{3}+2\times 2\pi=\frac{2\pi}{3}=\pi-\frac{\pi}{3}$

$\frac{8\pi}{3}-2\pi=\frac{2\pi}{3}=\pi-\frac{\pi}{3}$

$\frac{11\pi}{6}-2\pi=-\frac{\pi}{6}$

33.1 $\text{sen}\,x=\frac{2-3k}{4}\wedge x\in\mathbb{R} \Leftrightarrow -1\leqslant\frac{2-3k}{4}\leqslant 1 \Leftrightarrow -4\leqslant 2-3k\leqslant 4 \Leftrightarrow -4-2\leqslant -3k\leqslant 4-2 \Leftrightarrow$

$\Leftrightarrow -6\leqslant -3k\leqslant 2 \Leftrightarrow 6\geqslant 3k\geqslant -2 \Leftrightarrow 2\geqslant k\geqslant -\frac{2}{3} \Leftrightarrow k\in\left[-\frac{2}{3}, 2\right]$

33.2 $\cos x=k^2\wedge x\in\left]0, \frac{\pi}{2}\right] \Leftrightarrow 0\leqslant k^2<1 \Leftrightarrow -1<k<1 \Leftrightarrow k\in]-1,1[$

33.3 $\cos x=\frac{5k+8}{3}\wedge 60°<x\leqslant 180° \Leftrightarrow -1\leqslant\frac{5k+8}{3}<\frac{1}{2} \Leftrightarrow -3\leqslant 5k+8<\frac{3}{2} \Leftrightarrow$

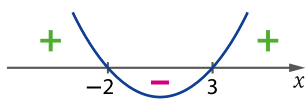
$\Leftrightarrow -11\leqslant 5k<-\frac{13}{2} \Leftrightarrow -\frac{11}{5}\leqslant k<-\frac{13}{10} \Leftrightarrow k\in\left[-\frac{11}{5}, -\frac{13}{10}\right[$

33.4 $6\text{tg}\,x=k^2-k\wedge x\in\left]\frac{\pi}{4}, \frac{\pi}{2}\right[\Leftrightarrow \text{tg}\,x=\frac{k^2-k}{6}\wedge x\in\left]\frac{\pi}{4}, \frac{\pi}{2}\right[\Leftrightarrow \frac{k^2-k}{6}\geqslant 1 \Leftrightarrow k^2-k-6\geqslant 0$

Cálculo auxiliar:

$k^2-k-6=0 \Leftrightarrow k=\frac{-(-1)\pm\sqrt{(-1)^2-4\times 1(-6)}}{2\times 1} \Leftrightarrow k=\frac{1\pm 5}{2} \Leftrightarrow$

$\Leftrightarrow k=\frac{1-5}{2}\vee k=\frac{1+5}{2} \Leftrightarrow k=-2\vee k=3$

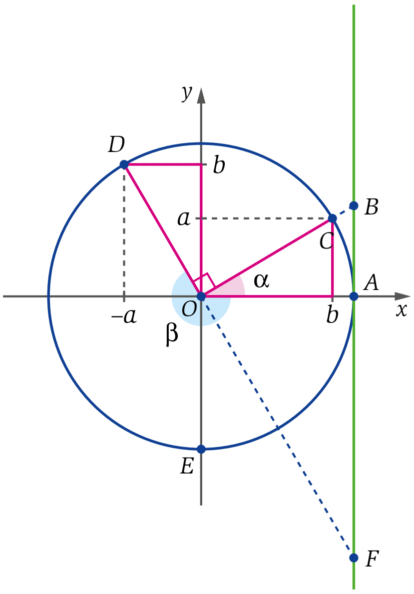


$k^2-k-6\geqslant 0 \Leftrightarrow k\in]-\infty, -2[\cup]3, +\infty[$

PÁG. 45

Aplicar

35.



Os triângulos representados a vermelho são geometricamente iguais, pois têm, de um para o outro, os ângulos internos agudos iguais, com amplitudes α e $90° - \alpha$, e hipotenusas iguais (raios da circunferência trigonométrica).

35.1 $\operatorname{sen}\alpha = a$, $\cos\alpha = b$, $\operatorname{sen}\beta = b$, $\cos\beta = -a$

$\cos^2\alpha + \cos^2\beta = b^2 + (-a)^2 = a^2 + b^2 = 1$

35.2 $\operatorname{tg}\alpha - \operatorname{tg}\beta = \frac{\operatorname{sen}\alpha}{\cos\alpha} - \frac{\operatorname{sen}\beta}{\cos\beta} = \frac{a}{b} - \frac{b}{-a} = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ba} = \frac{1}{ab}$

36. A área do paralelogramo é dada por $\overline{AB} \times \overline{AC}$.

$B(2\cos\theta\ ,\ 2\operatorname{sen}\theta)$, $A(0{,}2\operatorname{sen}\theta)$, $C(0\ ,\ -2\operatorname{sen}\theta)$

$\overline{AB} = -2\cos\theta$, $\overline{AC} = 2 \times 2\operatorname{sen}\theta = 4\operatorname{sen}\theta$

Área: $\overline{AB} \times \overline{AC} = -2\cos\theta \times 4\operatorname{sen}\theta = -8\operatorname{sen}\theta \times \cos\theta$

PÁG. 52

Tarefa 12

$\operatorname{tg}(0°) = \operatorname{tg}(360°) = 0$

$\operatorname{tg}(30°) = \frac{\sqrt{3}}{3}$

$\operatorname{tg}(60°) = \sqrt{3}$

Não existe $\operatorname{tg}(90°)$.

$\operatorname{tg}(120°) = -\sqrt{3}$

$\operatorname{tg}(135°) = -1$

$\operatorname{tg}(150°) = -\frac{\sqrt{3}}{3}$

$\operatorname{tg}(180°) = 0$

$\operatorname{tg}(210°) = \frac{\sqrt{3}}{3}$

$\operatorname{tg}(225°) = 1$

$\operatorname{tg}(240°) = \sqrt{3}$

Não existe $\operatorname{tg}(270°)$.

$\operatorname{tg}(300°) = -\sqrt{3}$

$\operatorname{tg}(315°) = -1$

$\operatorname{tg}(330°) = -\frac{\sqrt{3}}{3}$

PÁG. 53

Aplicar

38.1 $\cos\left(\frac{\pi}{2}+\alpha\right)+2\operatorname{sen}(\alpha+\pi)=-\operatorname{sen}\alpha+2(-\operatorname{sen}\alpha)=-3\operatorname{sen}\alpha$

38.2 $\operatorname{sen}\left(\frac{3\pi}{2}-\alpha\right)\times\operatorname{tg}(\alpha+\pi)+\operatorname{sen}(\pi-\alpha)=-\cos\alpha\times\operatorname{tg}\alpha+\operatorname{sen}\alpha=$

$$=-\cos\alpha\times\frac{\operatorname{sen}\alpha}{\cos\alpha}+\operatorname{sen}\alpha=-\operatorname{sen}\alpha+\operatorname{sen}\alpha=0$$

38.3 $\operatorname{tg}\left(\frac{\pi}{2}+\alpha\right)-\cos(3\pi+\alpha)=\dfrac{\operatorname{sen}\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{\pi}{2}+\alpha\right)}-\cos(\pi+\alpha)=\dfrac{\cos\alpha}{-\operatorname{sen}\alpha}-(-\cos\alpha)=$

$$=-\frac{1}{\operatorname{tg}\alpha}+\cos\alpha=\cos\alpha-\frac{1}{\operatorname{tg}\alpha}$$

38.4 $\dfrac{\operatorname{tg}(-\alpha-5\pi)}{\cos\left(\alpha-\frac{3\pi}{2}\right)}+\operatorname{sen}(-\alpha)=\dfrac{\operatorname{tg}(-\alpha)}{\cos\left(\frac{\pi}{2}+\alpha\right)}-\operatorname{sen}\alpha=\dfrac{-\operatorname{tg}\alpha}{-\operatorname{sen}\alpha}-\operatorname{sen}\alpha=\dfrac{1}{\cos\alpha}-\operatorname{sen}\alpha$

39. $\operatorname{sen}\left(\alpha-\frac{\pi}{2}\right)+2\cos(\alpha-3\pi)=-\cos\alpha+2\cos(\alpha-\pi)=-\cos\alpha+2(-\cos\alpha)=-3\cos\alpha$

$-3\cos\alpha<0 \Leftrightarrow \cos\alpha>0$

Como $\alpha\in]-\pi,0[$, conclui-se que $\alpha\in 4.^\circ$ Q.; portanto, tem-se $\operatorname{sen}\alpha<0$ e $\operatorname{tg}\alpha<0$.

(A) $\operatorname{sen}\alpha\cos\alpha<0$

(B) $\operatorname{tg}\alpha+\operatorname{sen}\alpha<0$

(C) $\cos\alpha-\operatorname{sen}\alpha>0$

(D) $\operatorname{tg}\alpha\cos\alpha<0$

Opção correta: **(C)**

40.1

$$A(x)=\frac{\cos(x+4\pi)\times\cos(\pi-x)}{\operatorname{tg}\left(\frac{\pi}{2}+x\right)}=\frac{\cos x\times(-\cos x)}{\dfrac{\operatorname{sen}\left(\frac{\pi}{2}+x\right)}{\cos\left(\frac{\pi}{2}+x\right)}}=\frac{-\cos^2 x}{\dfrac{\cos x}{-\operatorname{sen}x}}=\operatorname{sen}x\times\cos x$$

40.2 $(\operatorname{sen}\alpha+\cos\alpha)^2=\frac{2}{3}\Leftrightarrow \operatorname{sen}^2\alpha+2\operatorname{sen}\alpha\times\cos\alpha+\cos^2\alpha=\frac{2}{3}\Leftrightarrow 1+2\operatorname{sen}\alpha\times\cos\alpha=\frac{2}{3}\Leftrightarrow$

$$\Leftrightarrow \operatorname{sen}\alpha\times\cos\alpha=\frac{\frac{2}{3}-1}{2}\Leftrightarrow \operatorname{sen}\alpha\times\cos\alpha=-\frac{1}{6}$$

$A(\alpha)=\operatorname{sen}\alpha\cos\alpha=-\frac{1}{6}$

PÁG. 54

Aplicar

41. Como o ângulo de amplitude β pertence ao 3.º quadrante ou ao 4.º quadrante , conclui-se que $\operatorname{sen}\beta < 0$.

$\cos(-\alpha) \times \operatorname{sen}\beta < 0 \iff \cos\alpha \times \operatorname{sen}\beta < 0$

Como $\operatorname{sen}\beta < 0$, conclui-se que $\cos\alpha > 0$.

$\operatorname{tg}\alpha \times \operatorname{sen}(\pi - \beta) > 0 \iff \operatorname{tg}\alpha \times \operatorname{sen}\beta > 0$

Como $\operatorname{sen}\beta < 0$, conclui-se que $\operatorname{tg}\alpha < 0$.

De $\cos\alpha > 0$ e $\operatorname{tg}\alpha < 0$, conclui-se que o ângulo de amplitude α pertence ao 4.º quadrante.

Opção correta: **(D)**

42.1 $(-1\ ,\ -\operatorname{tg}\theta)$

42.2 Como o arco de circunferência CD está centrado em B , conclui-se que $\overline{BD} = \overline{BC}$. Por outro lado, $\overline{BD}$ é igual ao simétrico da ordenada do ponto C . Assim, a abcissa do ponto D obtém-se subtraindo a -1 , abcissa de B , $\overline{BD}$, ou seja, $-1 - \operatorname{tg}\theta$.

42.3 $3\operatorname{sen}\theta + 5\cos(\pi + \theta) = 0 \iff 3\operatorname{sen}\theta - 5\cos\theta = 0$

Dividindo por $\cos\theta$, obtém-se $3 \times \frac{\operatorname{sen}\theta}{\cos\theta} - 5 \times \frac{\cos\theta}{\cos\theta} = 0 \iff 3\operatorname{tg}\theta - 5 = 0 \iff \operatorname{tg}\theta = \frac{5}{3}$.

As coordenadas do ponto D são, neste caso, $\left(-1 - \frac{5}{3}\ ,\ 0\right)$, ou seja, $\left(-\frac{8}{3}\ ,\ 0\right)$.

43.1 $A(x) = 2\cos x + 2\operatorname{sen}\left(x - \frac{\pi}{2}\right) - \cos(3\pi - x) - \operatorname{tg}(\pi - x) =$

$= 2\cos x - 2\cos x - \cos(\pi - x) + \operatorname{tg}x = \cos x + \operatorname{tg}x$

43.2 $A\left(\frac{27\pi}{4}\right) - \operatorname{sen}\left(-\frac{13\pi}{6}\right) = \cos\left(\frac{27\pi}{4}\right) + \operatorname{tg}\left(\frac{27\pi}{4}\right) - \operatorname{sen}\left(-\frac{13\pi}{6}\right) =$

$= \cos\left(\pi - \frac{\pi}{4}\right) + \operatorname{tg}\left(\pi - \frac{\pi}{4}\right) - \operatorname{sen}\left(-\frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{4}\right) - \operatorname{tg}\left(\frac{\pi}{4}\right) + \operatorname{sen}\left(\frac{\pi}{6}\right) =$

$= -\frac{\sqrt{2}}{2} - 1 + \frac{1}{2} = -\frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{-\sqrt{2} - 1}{2} = -\frac{\sqrt{2} + 1}{2}$

Cálculos auxiliares:

$\frac{27\pi}{4} - 3 \times 2\pi = \frac{3\pi}{4} = \pi - \frac{\pi}{4}$

$-\frac{13\pi}{6} + 2\pi = -\frac{\pi}{6}$

43.3 Como $\theta \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$ e $\text{sen}\,\theta < 0$, conclui-se que $\theta \in 3.° \text{ Q}$. Assim, $\cos\theta < 0$ e $\text{tg}\,\theta > 0$.

(A) $A(\theta) = \cos\theta + \text{tg}\,\theta$ pode ser negativa ou positiva.

(B) $A(\pi + \theta) = \cos(\pi + \theta) + \text{tg}(\pi + \theta) = -\cos\theta + \text{tg}\,\theta > 0$

(C) $A(\theta - \pi) = \cos(\theta - \pi) + \text{tg}(\theta - \pi) = -\cos\theta + \text{tg}\,\theta > 0$

(D) $A(-\theta) = \cos(-\theta) + \text{tg}(-\theta) = \cos\theta - \text{tg}\,\theta < 0$

Opção correta: **(D)**

PÁG. 55

Tarefa 13

$\text{sen}(\alpha) = \frac{b}{c} \Leftrightarrow b = c\,\text{sen}(\alpha)$; $\cos(\alpha) = \frac{a}{c} \Leftrightarrow a = c\cos(\alpha)$.

Pelo Teorema de Pitágoras, $c^2 = b^2 + a^2$.

Substituindo, $c^2 = (c\,\text{sen}(\alpha))^2 + (c\cos(\alpha))^2 \Leftrightarrow c^2 = c^2\text{sen}^2(\alpha) + c^2\cos^2(\alpha) \Leftrightarrow c^2 = c^2(\text{sen}^2(\alpha) + \cos^2(\alpha))$

Dividindo ambos os membros da equação por c^2, tem-se $1 = \text{sen}^2(\alpha) + \cos^2(\alpha)$.

PÁG. 55

Tarefa 14

Seja $P(x, y)$ um ponto da circunferência centrada na origem e de raio 1 . A equação reduzida dessa circunferência é $x^2 + y^2 = 1$.

Por outro lado, $P(\cos\alpha, \text{sen}\,\alpha)$.

Substituindo na equação, tem-se $\text{sen}^2(\alpha) + \cos^2(\alpha) = 1$.

PÁG. 55

Tarefa 15

Fórmula fundamental da trigonometria: $\text{sen}^2\alpha + \cos^2\alpha = 1$.

Dividindo ambos os membros da equação por $\cos^2\alpha$,

obtém-se: $\frac{\text{sen}^2\alpha}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} \Leftrightarrow \left(\frac{\text{sen}\,\alpha}{\cos\alpha}\right)^2 + 1 = \frac{1}{\cos^2\alpha}$

Como $\text{tg}\,\alpha = \frac{\text{sen}\,\alpha}{\cos\alpha}$, conclui-se que $\text{tg}^2\alpha + 1 = \frac{1}{\cos^2\alpha}$.

PÁG. 57

Aplicar

46. $\text{tg}(\alpha-\pi)+\cos\left(\frac{\pi}{2}+\alpha\right)=\text{tg}\,\alpha+(-\text{sen}\,\alpha)=\text{tg}\,\alpha-\text{sen}\,\alpha$

$\text{sen}^2\alpha+\left(\frac{\sqrt{3}}{3}\right)^2=1 \Leftrightarrow \text{sen}^2\alpha=1-\frac{1}{3} \Leftrightarrow \text{sen}^2\alpha=\frac{2}{3}$

Como $\alpha\in]-\pi\,,\,0[$ e $\cos\alpha>0$, α pertence ao 4.° quadrante, pelo que $\text{sen}\,\alpha<0$ e, portanto,

$\text{sen}\,\alpha=-\sqrt{\frac{2}{3}}=-\frac{\sqrt{6}}{3}$

$\text{tg}\,\alpha=\frac{-\frac{\sqrt{6}}{3}}{\frac{\sqrt{3}}{3}}=-\frac{\sqrt{6}}{\sqrt{3}}=-\sqrt{2}$

$\text{tg}\,\alpha-\text{sen}\,\alpha=-\sqrt{2}+\frac{\sqrt{6}}{3}=\frac{\sqrt{6}}{3}-\sqrt{2}$

47.1 $\text{sen}^2x+\cos^2x=1 \Leftrightarrow (k-1)^2+\left(\frac{k-2}{3}\right)^2=1 \Leftrightarrow k^2-2k+1+\frac{k^2-4k+4}{9}=1 \Leftrightarrow$

$\Leftrightarrow 9k^2-18k+9+k^2-4k+4=9 \Leftrightarrow 10k^2-22k+4=0$

$\Leftrightarrow 5k^2-11k+2=0 \Leftrightarrow$

$\Leftrightarrow k=\frac{11\pm\sqrt{(-11)^2-4\times5\times2}}{2\times5} \Leftrightarrow k=\frac{11\pm9}{10} \Leftrightarrow k=\frac{11-9}{10}\vee k=\frac{11+9}{10}$

$\Leftrightarrow k=\frac{1}{5}\vee k=2$

$k\in\left\{\frac{1}{5}\,,\,2\right\}$

47.2 $\text{tg}^2x+1=\frac{1}{\cos^2x} \Leftrightarrow k^2+1=\frac{1}{\left(\frac{1}{\sqrt{2k+4}}\right)^2} \Leftrightarrow k^2+1=2k+4 \Leftrightarrow k^2-2k-3=0 \Leftrightarrow$

$\Leftrightarrow k=\frac{2\pm\sqrt{(-2)^2-4(-3)}}{2} \Leftrightarrow k=\frac{2\pm4}{2} \Leftrightarrow k=\frac{2\pm4}{2}\vee k=\frac{2\pm4}{2} \Leftrightarrow k=-1\vee k=3$

$k\in\{-1\,,\,3\}$

48. $\text{tg}\,x=\frac{\text{sen}\,x}{\cos x} \Leftrightarrow 3a=\frac{a}{\cos x} \Leftrightarrow \cos x=\frac{a}{3a} \Leftrightarrow \cos x=\frac{1}{3}$

$\text{sen}^2x+\cos^2x=1 \Leftrightarrow \text{sen}^2x=1-\left(\frac{1}{3}\right)^2 \Leftrightarrow \text{sen}^2x=\frac{8}{9}$

$\text{sen}^2x+\cos x=\frac{8}{9}+\frac{1}{3}=\frac{11}{9}$

Opção correta: **(C)**

PÁG. 58

Aplicar

49.

$A_{[ABCD]} = \frac{\overline{AB} + \overline{CD}}{2} \times \overline{BC}$

$A(\cos\alpha\ ,\ \operatorname{sen}\alpha)\ ;\ B(\cos\alpha\ ,\ 0)\ ;\ C(1{,}0)\ ;\ D(1\ ,\ \tan\alpha)$

$\overline{AB} = \operatorname{sen}\alpha\ ;\ \overline{CD} = \tan\alpha\ ;\ \overline{BC} = 1 - \cos\alpha$

$A_{[ABCD]} = \frac{\operatorname{sen}\alpha + \tan\alpha}{2} \times (1 - \cos\alpha) = \frac{\operatorname{sen}\alpha - \operatorname{sen}\alpha\cos\alpha + \tan\alpha - \tan\alpha\cos\alpha}{2} =$

$= \frac{\operatorname{sen}\alpha - \operatorname{sen}\alpha\cos\alpha + \tan\alpha - \operatorname{sen}\alpha}{2} = \frac{-\operatorname{sen}\alpha\cos\alpha + \tan\alpha}{2} = \frac{\tan\alpha - \operatorname{sen}\alpha\cos\alpha}{2}$

Opção correta: **(C)**

50.1 $A(\cos\alpha\ ,\ \operatorname{sen}\alpha)$, com $\cos\alpha < 0$

$P_{[OABC]} = \overline{OA} + \overline{AB} + \overline{BC} + \overline{CD} + \overline{OD} = \overline{OA} + 3\overline{AB} + \overline{OD} = 1 + 3\operatorname{sen}\alpha - \cos\alpha$

50.2 $P\left(\frac{\pi}{2}\right) = 3\operatorname{sen}\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right) + 1 = 3 \times 1 - 0 + 1 = 4$

Quando $\alpha = \frac{\pi}{4}$, o trapézio é um quadrado de lado 1 , pelo que o seu perímetro é 4 .

50.3 $\operatorname{tg}(-\alpha - 3\pi) = \frac{12}{5} \Leftrightarrow \operatorname{tg}(-(\alpha + 3\pi)) = \frac{12}{5} \Leftrightarrow -\operatorname{tg}(\alpha + 3\pi) = \frac{12}{5} \Leftrightarrow \operatorname{tg}\alpha = -\frac{12}{5}$

$\left(-\frac{12}{5}\right)^2 + 1 = \frac{1}{\cos^2\alpha} \Leftrightarrow \frac{144}{25} + 1 = \frac{1}{\cos^2\alpha} \Leftrightarrow \frac{169}{25} = \frac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = \frac{25}{169} \Leftrightarrow \cos\alpha = \pm\sqrt{\frac{25}{169}} \underset{\alpha \in \left]\frac{\pi}{2}, \pi\right[}{\Leftrightarrow} \cos\alpha = -\frac{5}{13}$

$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \Leftrightarrow \operatorname{sen}\alpha = \operatorname{tg}\alpha \times \cos\alpha$

$\operatorname{sen}\alpha = \left(-\frac{12}{5}\right) \times \left(-\frac{5}{13}\right) = \frac{12}{13}$

$P(\alpha) = 3 \times \frac{12}{13} - \left(-\frac{5}{13}\right) + 1 = \frac{36}{13} + \frac{5}{13} + 1 = \frac{54}{13}$

PÁG. 59

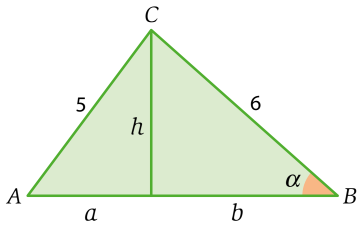
Aplicar +

1. $\frac{\overline{AB} \times \overline{BC}}{2} = 9 \Leftrightarrow \overline{AB} \times \overline{BC} = 18$

$\frac{\overline{AB}}{\overline{AC}} = \cos\alpha \Leftrightarrow \frac{\overline{AB}}{3\sqrt{5}} = \cos\alpha$ e $\frac{\overline{BC}}{\overline{AC}} = \operatorname{sen}\alpha \Leftrightarrow \frac{\overline{BC}}{3\sqrt{5}} = \operatorname{sen}\alpha$

$\operatorname{sen}\alpha \times \cos\alpha = \frac{\overline{BC}}{3\sqrt{5}} \times \frac{\overline{AB}}{3\sqrt{5}} = \frac{\overline{AB} \times \overline{BC}}{9 \times 5} = \frac{18}{9 \times 5} = \frac{2}{5}$

2.



$$\text{sen}^2\alpha = 1 - \left(\frac{\sqrt{5}}{3}\right)^2 \Leftrightarrow \text{sen}^2\alpha = \frac{4}{9} \Leftrightarrow \text{sen}\,\alpha = \frac{2}{3}$$

$$\text{sen}\,\alpha = \frac{h}{6} \Leftrightarrow h = 6\,\text{sen}\,\alpha \Leftrightarrow h = 6 \times \frac{2}{3} \Leftrightarrow h = 4$$

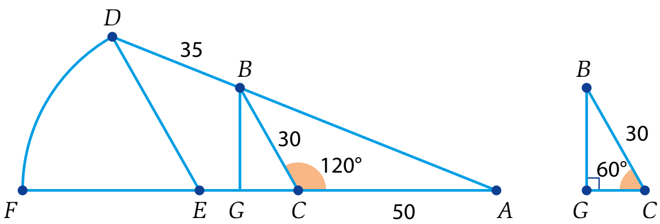
$$6^2 = h^2 + b^2 \Leftrightarrow b^2 = 6^2 - 4^2 \Leftrightarrow b = \sqrt{20} \Leftrightarrow b = 2\sqrt{5}$$

$$5^2 = h^2 + a^2 \Leftrightarrow a^2 = 5^2 - 4^2 \Leftrightarrow a = \sqrt{9} \Leftrightarrow a = 3$$

$$\overline{AB} = a + b = 3 + 2\sqrt{5}$$

Opção correta: **(C)**

3.1



$$\text{sen}\,60° = \frac{\overline{BG}}{30} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{\overline{BG}}{30} \Leftrightarrow \overline{BG} = 15\sqrt{3}$$

$$\cos 60° = \frac{\overline{CG}}{30} \Leftrightarrow \frac{1}{2} = \frac{\overline{CG}}{30} \Leftrightarrow \overline{CG} = 15$$

$$\overline{AB}^2 = \overline{AG}^2 + \overline{BG}^2 \Leftrightarrow \overline{AB}^2 = (50+15)^2 + \left(15\sqrt{3}\right)^2 \Leftrightarrow \overline{AB}^2 = 4900 \Leftrightarrow \overline{AB} = 70$$

$$\frac{\overline{AC}}{\overline{AE}} = \frac{\overline{AB}}{\overline{AD}} \Leftrightarrow \frac{50}{\overline{AE}} = \frac{70}{70+35} \Leftrightarrow \overline{AE} = 75$$

$$\overline{CE} = \overline{AE} - \overline{AC} = 75 - 50 = 25$$

$$\frac{\overline{BC}}{\overline{AC}} = \frac{\overline{DE}}{\overline{AE}} \Leftrightarrow \frac{30}{50} = \frac{\overline{DE}}{75} \Leftrightarrow \overline{DE} = 45$$

3.2 $2\pi \times 45 \times \frac{60}{360} = 15\pi$

4. $\begin{cases} A\hat{C}D = A\hat{C}B + 30° \\ A\hat{C}D + A\hat{C}B = 180° \end{cases} \Leftrightarrow \begin{cases} \overline{A\hat{C}B + 30° + A\hat{C}B = 180°} \end{cases} \Leftrightarrow \begin{cases} A\hat{C}D = 105° \\ A\hat{C}B = 75° \end{cases}$

$C\hat{D}A = 180° - 25° - 105° = 50°$, $B\hat{A}C = 90° - 75° = 15°$, $B\hat{A}D = 25° - 15° = 40°$

$$\begin{cases} \frac{\overline{BC}}{\overline{AB}} = \text{tg}\,15° \\ \frac{\overline{BC}+45}{\overline{AB}} = \text{tg}\,40° \end{cases} \Leftrightarrow \begin{cases} \overline{BC} = \overline{AB}\ \text{tg}\,15° \\ \overline{BC} + 45 = \overline{AB}\ \text{tg}\,40° \end{cases} \Leftrightarrow \begin{cases} \overline{AB}\ \text{tg}\,15° + 45 = \overline{AB}\ \text{tg}\,40° \\ \overline{} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \overline{AB}\ \text{tg}\,15° - \overline{AB}\ \text{tg}\,40° = -45 \\ \overline{} \end{cases} \Leftrightarrow \begin{cases} \overline{AB}(\text{tg}\,40° - \text{tg}\,15°) = 45 \\ \overline{} \end{cases}$$

$$\Leftrightarrow \begin{cases} \overline{} \\ \overline{AB} = \dfrac{45}{\text{tg}\,40° - \text{tg}\,15°} \end{cases}$$

$$A_{[ACD]} = \frac{\overline{AB} \times 45}{2} = \frac{\frac{45}{\text{tg}\,40° - \text{tg}\,15°} \times 45}{2} \approx 1773\ \text{m}^2$$

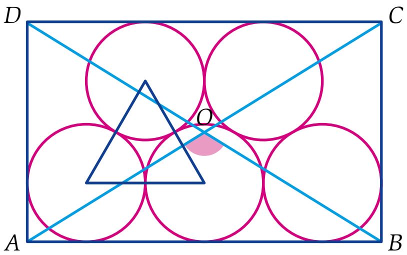
PÁG. 60

Aplicar +

5. Seja r a medida do raio de cada uma das circunferências.

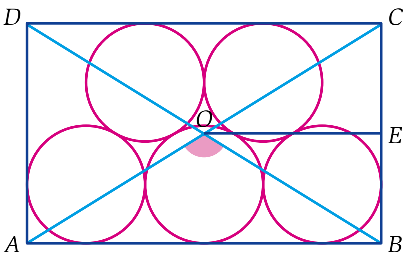
Então, o comprimento do retângulo $[ABCD]$ é $6r$.

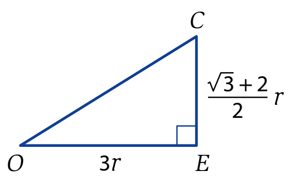
Para determinar a largura do retângulo, vamos considerar um triângulo equilátero de lado $2r$, como se ilustra na figura.



A altura do triângulo é $\sqrt{(2r)^2 - r^2} = \sqrt{3r^2} = \sqrt{3}\,r$.

Assim, a largura do retângulo é $\sqrt{3}\,r + 2r = (\sqrt{3}+2)r$.





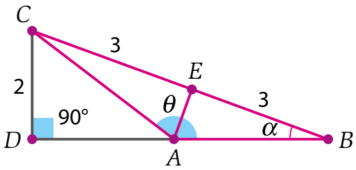
$$\frac{\frac{\sqrt{3}+2}{2}r}{3r} = \text{tg}\left(E\hat{O}C\right) \Leftrightarrow E\hat{O}C = \text{tg}^{-1}\left(\frac{\sqrt{3}+2}{6}\right)$$

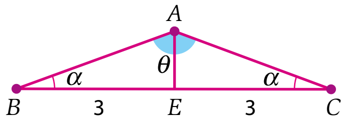
$$A\hat{O}B = 180° - B\hat{O}C = 180° - 2 \times E\hat{O}C = 180° - 2 \times \text{tg}^{-1}\left(\frac{\sqrt{3}+2}{6}\right) \approx 116°$$

6.1 $\text{sen}^2\alpha+\cos^2\alpha=1 \Leftrightarrow \text{sen}^2\alpha=1-\left(\frac{2\sqrt{2}}{3}\right)^2 \Leftrightarrow \text{sen}^2\alpha=\frac{1}{9} \underset{0°<\alpha<90°}{\Leftrightarrow} \text{sen}\,\alpha=\frac{1}{3}$

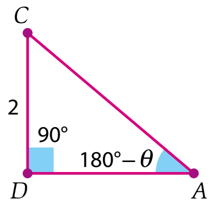
$\text{sen}\,\alpha=\frac{1}{3} \Leftrightarrow \frac{2}{\overline{BC}}=\frac{1}{3} \Leftrightarrow \overline{BC}=6$

Seja E o ponto médio de lado $\overline{BC}$; logo, o triângulo $[AEB]$ é retângulo em E e $\overline{CE}=3$.





$\cos\alpha=\frac{3}{\overline{AC}} \Leftrightarrow \frac{2\sqrt{2}}{3}=\frac{3}{\overline{AC}} \Leftrightarrow \overline{AC}=\frac{9}{2\sqrt{2}}$



$\text{sen}(180°-\theta)=\frac{2}{\overline{AC}}=\frac{2}{\frac{9}{2\sqrt{2}}}=\frac{4\sqrt{2}}{9}$

$\text{sen}\,\theta=\text{sen}(180°-\theta)=\frac{4\sqrt{2}}{9}$

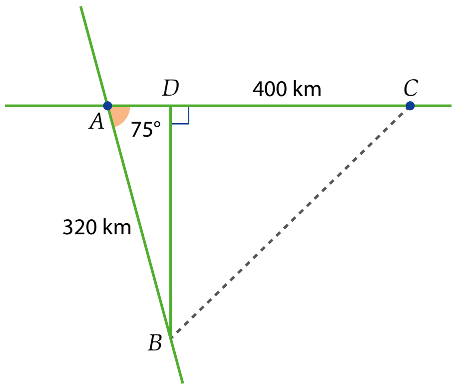
6.2 $P=\overline{AB}+\overline{AC}+\overline{BC}=2\times\frac{9}{2\sqrt{2}}+6=\frac{9\sqrt{2}}{2}+6$

7. $\begin{cases}\frac{\overline{BC}}{\overline{DC}}=\text{tg}\,50° \\ \frac{\overline{BC}}{\overline{DC}+4}=\text{tg}\,30°\end{cases} \Leftrightarrow \begin{cases}\overline{BC}=\overline{DC}\ \text{tg}\,50° \\ \overline{BC}=(\overline{DC}+4)\ \text{tg}\,30°\end{cases} \Leftrightarrow \begin{cases}\overline{DC}\ \text{tg}\,50°=(\overline{DC}+4)\ \text{tg}\,30° \\ \overline{}\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}\overline{DC}\ \text{tg}\,50°-\overline{DC}\ \text{tg}\,30°=4\ \text{tg}\,30° \\ \overline{}\end{cases} \Leftrightarrow \begin{cases}\overline{BC}=\frac{4\ \text{tg}\,30°}{\text{tg}\,50°-\text{tg}\,30°}\times\text{tg}\,50° \\ \overline{DC}=\frac{4\ \text{tg}\,30°}{\text{tg}\,50°-\text{tg}\,30°}\end{cases}$

Portanto, $\overline{BC}\approx 4{,}5$ cm e $\overline{DC}\approx 3{,}8$ cm .

8. $\overline{AB} = 80 \times 4 = 320$ km e $\overline{AC} = 100 \times 4 = 400$ km



$\frac{\overline{AD}}{320} = \cos 75° \Leftrightarrow \overline{AD} = 320 \times \cos 75°$

$\frac{\overline{BD}}{320} = \text{sen}\,75° \Leftrightarrow \overline{BD} = 320 \times \text{sen}\,75°$

$\overline{CD} = 400 - \overline{AD} = 400 - 320 \times \cos 75°$

$\overline{BC}^2 = \overline{BD}^2 + \overline{CD}^2 \Leftrightarrow \overline{BC}^2 = (320 \times \text{sen}\,75°)^2 + (400 - 320 \times \cos 75°)^2 \Leftrightarrow$

$\Leftrightarrow \overline{BC}^2 = 400^2 + 320^2 - 2 \times 400 \times 320 \times \cos 75° \Leftrightarrow$

$\Leftrightarrow \overline{BC} = \sqrt{400^2 + 320^2 - 2 \times 400 \times 320 \times \cos 75°}$; portanto, $\overline{BC} \approx 443$ km .

PÁG. 61

Aplicar +

9. $\frac{360°}{10} = 36°$

9.1 $2 \times 36° + n \times 360°$, $n \in \mathbb{N}$

9.2 $-6 \times 36° - n \times 360°$, $n \in \mathbb{N}$

9.3 $-2 \times 36° - n \times 360°$, $n \in \mathbb{N}$

10. $\frac{360°}{8} = 45°$

10.1 $-3 \times 45° - 2 \times 360° = -855°$

10.2 $3 \times 45° + n \times 360°$, $n \in \mathbb{N}$, isto é, $135° + n \times 360°$, $n \in \mathbb{N}$ ($\mathbb{N}$, tendo em conta que o ângulo deve ter amplitude positiva).

10.3 $6 \times 45° + n \times 360°$, $n \in \mathbb{N}$, isto é, $270° + n \times 360°$, $n \in \mathbb{N}$ ($\mathbb{N}$, tendo em conta que o ângulo deve ter amplitude positiva)..

11. $\frac{360°}{12} = 30°$; $8 \times 30° = 240°$.

12. $\frac{360°}{5}=72°$; $2\times72°+n\times360°$, $n\in\mathbb{N}$; $-3\times72°-n\times360°$, $n\in\mathbb{N}$.

13. $-1100°=-20°-3\times360°$

Opção correta: **(C)**

PÁG. 62

Aplicar +

14. $-780°=-60°-2\times360°$

Opção correta: **(B)**

15. Opção correta: **(D)**, pois $-720°=-2\times360°$ e nenhuma das restantes amplitudes é dada por $k\times360°$, com $k\in\mathbb{Z}$.

16. Opção correta: **(C)**, porque é a única opção em que os lados origem e extremidade coincidem.

17. Opção correta: **(B)**, porque $135°=-225°+360°$.

18.1 Não, porque os lados extremidade destes ângulos generalizados não coincidem; um está no primeiro quadrante e o outro está no quarto quadrante.

18.2 A amplitude do ângulo orientado correspondente ao ângulo generalizado de amplitude $-40°-2\times360°$ é $-40°$; a amplitude do ângulo orientado correspondente, com amplitude positiva, é dada por $-40°+360°$, ou seja, é $320°$.

18.3 A amplitude do ângulo orientado correspondente ao ângulo generalizado de amplitude $65°+3\times360°$ é $65°$; a amplitude do ângulo orientado correspondente, com amplitude negativa, é dada por $65°-360°$, ou seja, é $-295°$

PÁG. 63

Aplicar +

19. $\frac{360°}{12}=30°$

19.1 $-930°=-2\times360°-210°$, $\frac{210°}{30°}=7$

No final da rotação o ponteiro ficou a apontar para o número 6 .

19.2 $-460°=-360°-100°$ e $100°$ não é múltiplo de $30°$.

O participante não ganhou prémio, pois o ponteiro ficou entre os números 10 e 9 , dado que $-3\times30°=-90°$ e $-4\times30°=-120°$.

19.3 $-30°-n\times360°$, $n\in\mathbb{N}$.

19.4 $]-11\times30°-3\times360°\,,\,-10\times30°-3\times360°[\,=\,]-1410°\,,\,-1380°[$.

20. $\alpha = -\left(\frac{360°}{6} + \frac{360°}{5}\right) = -132°$; $\alpha + 1050° = -132° + 1050° = 918° = 2 \times 360° + 198°$.

Opção correta: **(C)**

21.1

a. $\dot{O}B$

b. $\dot{O}E$ $(-120° - 360° = -2 \times 60° - 360°)$

21.2

a. $300° + k \times 360°$, $k \in \mathbb{Z}$

b. Sim, porque $-780° = 300° - 3 \times 360°$.

21.3 Sim, porque $960° = 600° + 360°$.

PÁG. 64

Aplicar +

22. $\dfrac{\frac{2\pi}{3} \times 3^2}{2} = 3\pi \text{ cm}^2$

Opção correta: **(C)**

23.1 $\frac{2 \text{ cm}}{1 \text{ rad}} = \frac{2{,}5 \text{ cm}}{\alpha \text{ rad}} \Leftrightarrow \alpha = \frac{2{,}5}{2} = 1{,}25 \text{ rad}$

23.2 $\frac{5 \text{ cm}}{1 \text{ rad}} = \frac{7{,}5 \text{ cm}}{\alpha \text{ rad}} \Leftrightarrow \alpha = \frac{7{,}5}{5} = 1{,}5 \text{ rad}$

23.3 $\frac{\alpha \times 20^2}{2} = 160 \Leftrightarrow \alpha = 0{,}8 \text{ rad}$

24.1 1 radiano

24.2 A amplitude do ângulo giro é $\frac{2\pi r}{r} = 2\pi \text{ rad}$.

24.3 $\frac{3 \text{ cm}}{1 \text{ rad}} = \frac{3\pi \text{ cm}}{\alpha \text{ rad}} \Leftrightarrow \alpha = \frac{3\pi}{3} = \pi \text{ rad}$

24.4

a. $\frac{210°}{180°}\pi = \frac{7\pi}{6}$

b. $\dfrac{\frac{7\pi}{6} \times r^2}{2} = \frac{7\pi r^2}{12}$

25. $\theta r + 2r = 4r \Leftrightarrow \theta r = 2r \Leftrightarrow \theta = 2$ e $\operatorname{sen}(2) \approx 0{,}909$.

Opção correta: **(D)**

26. $P = 2 \times 5 + \left(\pi - \frac{\pi}{4}\right) \times 5 = 10 + \frac{15\pi}{4}\text{cm}$

$$A = \frac{\left(\pi - \frac{\pi}{4}\right) \times 5^2}{2} = \frac{75\pi}{8}\ \text{cm}^2$$

PÁG. 65

Aplicar +

27. $-\frac{15\pi}{4} + 2 \times 2\pi = \frac{\pi}{4} \in 1.^\circ\ \text{Q.}$

Opção correta: **(A)**

28. $-\frac{125\pi}{6} + 11 \times 2\pi = \frac{7\pi}{6}$

Opção correta: **(D)**

29. $-\frac{13\pi}{3} + 2 \times 2\pi = -\frac{\pi}{3}$; $\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$; $\operatorname{sen}\left(-\frac{\pi}{3}\right) = -\operatorname{sen}\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

Opção correta: **(C)**

30. $\operatorname{sen}\left(-\frac{11\pi}{3}\right) + 2\operatorname{tg}\left(\frac{9\pi}{4}\right) + \cos\left(-\frac{\pi}{6}\right) = \operatorname{sen}\left(\frac{\pi}{3}\right) + 2\operatorname{tg}\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + 2 \times 1 + \frac{\sqrt{3}}{2} = \sqrt{3} + 2$

Opção correta: **(B)**

Cálculos auxiliares:

$-\frac{11\pi}{3} + 2 \times 2\pi = \frac{\pi}{3}$; $\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$.

31. $\alpha \in \left]\frac{\pi}{2}, \pi\right[\wedge \cos\alpha = -\frac{1}{2} \Leftrightarrow \alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\alpha - \frac{5\pi}{6} = \frac{2\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{6}$

$\operatorname{sen}\alpha - \operatorname{tg}\left(\alpha - \frac{5\pi}{6}\right) = \operatorname{sen}\left(\frac{2\pi}{3}\right) - \operatorname{tg}\left(-\frac{\pi}{6}\right) = \operatorname{sen}\left(\frac{\pi}{3}\right) + \operatorname{tg}\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3} = \frac{5\sqrt{3}}{6}$

Opção correta: **(C)**

32. Para que a tangente seja negativa, o seno e o cosseno têm de ter sinais contrários.

Portanto, não existem ângulos nas circunstâncias do enunciado.

Opção correta: **(D)**

PÁG. 66

Aplicar +

33. Como $\alpha \in \left]\frac{\pi}{2}, \pi\right[$, tem-se $\cos\alpha < 0$, $\operatorname{sen}\alpha > 0$ e $\operatorname{tg}\alpha < 0$.

33.1 $\cos(\pi+\alpha) = -\cos\alpha > 0$

33.2 $-\operatorname{sen}(-\pi-\alpha) = -\operatorname{sen}(-(\pi+\alpha)) = \operatorname{sen}(\pi+\alpha) = -\operatorname{sen}\alpha < 0$

33.3 $\operatorname{sen}(\pi+\alpha)\times\cos(\pi-\alpha) = -\operatorname{sen}\alpha\times(-\cos\alpha) = \operatorname{sen}\alpha\times\cos\alpha < 0$

33.4 $\operatorname{sen}(-\alpha) - \cos(\pi-\alpha) = -\operatorname{sen}\alpha - (-\cos\alpha) = -\operatorname{sen}\alpha + \cos\alpha < 0$

33.5 $\operatorname{tg}(\pi+\alpha) - \operatorname{sen}\left(\frac{3\pi}{2}-\alpha\right) = \operatorname{tg}\alpha + \cos\alpha < 0$

33.6 $\cos\left(\frac{3\pi}{2}+\alpha\right)\times\operatorname{tg}(\pi-\alpha) = \operatorname{sen}\alpha\times(-\operatorname{tg}\alpha) = -\operatorname{sen}\alpha\times\operatorname{tg}\alpha > 0$

34.1 $\frac{\sqrt{2}}{2} < \operatorname{sen}\alpha < 1$, $-\frac{\sqrt{2}}{2} < \cos\alpha < 0$ e $\operatorname{tg}\alpha < -1$.

34.2 $\operatorname{sen}\alpha + 3\cos\left(\alpha - \frac{3\pi}{2}\right) = \operatorname{sen}\alpha + 3(-\operatorname{sen}\alpha) = -2\operatorname{sen}\alpha$

$-2\operatorname{sen}\alpha = 2\lambda + 3 \Leftrightarrow \operatorname{sen}\alpha = -\lambda - \frac{3}{2}$

$\frac{\sqrt{2}}{2} < \operatorname{sen}\alpha < 1 \Leftrightarrow \frac{\sqrt{2}}{2} < -\lambda - \frac{3}{2} < 1 \Leftrightarrow \frac{\sqrt{2}}{2} + \frac{3}{2} < -\lambda < 1 + \frac{3}{2} \Leftrightarrow$

$\Leftrightarrow -\left(1+\frac{3}{2}\right) < \lambda < -\left(\frac{\sqrt{2}}{2}+\frac{3}{2}\right) \Leftrightarrow -\frac{5}{2} < \lambda < -\frac{3+\sqrt{2}}{2}$

34.3 $\cos(\beta+\pi)\times\operatorname{sen}\left(-\frac{\pi}{2}-\beta\right) + \operatorname{tg}^2(\pi-\beta) = -\cos\beta\times(-\cos\beta) + (-\operatorname{tg}\beta)^2 = \cos^2\beta + \operatorname{tg}^2\beta$

$\operatorname{sen}^2\beta + \cos^2\beta = 1 \Leftrightarrow \cos^2\beta = 1 - \left(\frac{3}{\sqrt{10}}\right)^2 \Leftrightarrow \cos^2\beta = \frac{1}{10}$ e $\operatorname{tg}^2\beta = \frac{\left(\frac{3}{\sqrt{10}}\right)^2}{\frac{1}{10}} = 9$;

logo, $\cos^2\beta + \operatorname{tg}^2\beta = \frac{1}{10} + 9 = \frac{91}{10}$.

35. $\cos(3\pi-\alpha)\times\operatorname{tg}(\pi+\alpha) - \operatorname{sen}(\pi-\alpha) = \cos(\pi-\alpha)\times\operatorname{tg}\alpha - \operatorname{sen}\alpha = -\cos\alpha\times\operatorname{tg}\alpha - \operatorname{sen}\alpha = -2\operatorname{sen}\alpha$;

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \operatorname{sen}^2\alpha = 1 - \left(-\frac{3}{10}\right)^2 = \frac{91}{100} \underset{\alpha\in\left]-\pi,-\frac{\pi}{2}\right[}{\Leftrightarrow} \operatorname{sen}\alpha = -\frac{\sqrt{91}}{10}$;

$-2\operatorname{sen}\alpha = -2\left(-\frac{\sqrt{91}}{10}\right) = \frac{\sqrt{91}}{5}$.

36.1 De $C(x\,,\sqrt{5}x)$ e $C(\cos\alpha\,,\operatorname{sen}\alpha)$, resulta $x^2+(\sqrt{5}x)^2=1 \Leftrightarrow x^2+5x^2=1 \Leftrightarrow 6x^2=1 \Leftrightarrow$
$\Leftrightarrow x^2=\frac{1}{6}$;

$\cos^2\alpha=\frac{1}{6} \underset{\alpha\in 1.^{\circ}\text{Q.}}{\Leftrightarrow} \cos\alpha=\sqrt{\frac{1}{6}}$;

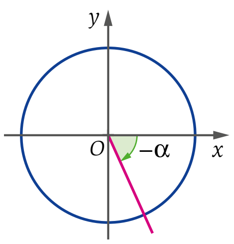
$\operatorname{sen}^2\alpha+\cos^2\alpha=1 \Leftrightarrow \operatorname{sen}^2\alpha=1-\frac{1}{6} \Leftrightarrow \operatorname{sen}^2\alpha=\frac{5}{6} \underset{\alpha\in 1.^{\circ}\text{Q.}}{\Leftrightarrow} \operatorname{sen}\alpha=\sqrt{\frac{5}{6}}$

$$\operatorname{tg}\alpha=\frac{\operatorname{sen}\alpha}{\cos\alpha}=\frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}}=\sqrt{5}$$

Por outro processo, pode-se começar por determinar a tangente , como se segue em baixo:

α é a inclinação da semirreta $\dot{O}C$. Como o seu declive é $\sqrt{5}$, então $\operatorname{tg}\alpha=\sqrt{5}$. Depois, obtém-se o seno a partir do cosseno e da tangente.

36.2



$\cos(-\alpha)=\cos\alpha=\frac{\sqrt{6}}{6}$, $\operatorname{sen}(-\alpha)=-\operatorname{sen}\alpha=-\frac{\sqrt{30}}{6}$ e $\operatorname{tg}(-\alpha)=-\operatorname{tg}\alpha=-\sqrt{5}$

37.1 $A(x)=\dfrac{\cos\left(-x-\frac{3\pi}{2}\right)}{\operatorname{tg}\left(\frac{\pi}{2}-x\right)}-\cos(x+\pi)-\operatorname{tg}(x-3\pi)=\dfrac{\operatorname{sen}x}{\frac{1}{\operatorname{tg}x}}+\cos x-\operatorname{tg}x=\operatorname{sen}x\times\operatorname{tg}x+\cos x-\operatorname{tg}x=$

$$=\operatorname{sen}x\times\frac{\operatorname{sen}x}{\cos x}+\cos x-\frac{\operatorname{sen}x}{\cos x}=\frac{\operatorname{sen}^2x+\cos^2x-\operatorname{sen}x}{\cos x}=\frac{1-\operatorname{sen}x}{\cos x}$$

37.2 $A\left(\frac{11\pi}{6}\right)-A\left(-\frac{4\pi}{3}\right)=\dfrac{1-\operatorname{sen}\left(\frac{11\pi}{6}\right)}{\cos\left(\frac{11\pi}{6}\right)}-\dfrac{1-\operatorname{sen}\left(-\frac{4\pi}{3}\right)}{\cos\left(-\frac{4\pi}{3}\right)}=$

$$=\frac{1-\left(-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}-\frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}=\sqrt{3}+2-\sqrt{3}=2$$

Cálculos auxiliares:

$\frac{11\pi}{6}-2\pi=-\frac{\pi}{6}$, $\operatorname{sen}\left(\frac{11\pi}{6}\right)=\operatorname{sen}\left(-\frac{\pi}{6}\right)=-\operatorname{sen}\left(\frac{\pi}{6}\right)=-\frac{1}{2}$ e $\cos\left(\frac{11\pi}{6}\right)=\cos\left(-\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

$-\frac{4\pi}{3}+2\pi=\frac{2\pi}{3}=\pi-\frac{\pi}{3}$, $\operatorname{sen}\left(-\frac{4\pi}{3}\right)=\operatorname{sen}\left(\pi-\frac{\pi}{3}\right)=\operatorname{sen}\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

e $\cos\left(-\frac{4\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\left(\frac{\pi}{3}\right)=-\frac{1}{2}$

37.3 $1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha} \Leftrightarrow 1+\left(-\sqrt{8}\right)^2=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{1}{9} \underset{\alpha\in]0,\pi[\wedge \mathrm{tg}\alpha<0}{\Leftrightarrow} \Leftrightarrow \cos\alpha=-\frac{1}{3}$

$\mathrm{tg}\alpha=\frac{\mathrm{sen}\alpha}{\cos\alpha} \Leftrightarrow -\sqrt{8}=\frac{\mathrm{sen}\alpha}{-\frac{1}{3}} \Leftrightarrow \mathrm{sen}\alpha=\frac{\sqrt{8}}{3}$

$A(\alpha)=\frac{1-\frac{\sqrt{8}}{3}}{-\frac{1}{3}}=\sqrt{8}-3=2\sqrt{2}-3$

PÁG. 67

Aplicar +

38.1 $\mathrm{tg}\left(\alpha-\frac{\pi}{2}\right)=k^2-2 \Leftrightarrow -\mathrm{tg}\left(\frac{\pi}{2}-\alpha\right)=k^2-2 \Leftrightarrow -\frac{1}{\mathrm{tg}\alpha}=k^2-2 \Leftrightarrow \mathrm{tg}\alpha=\frac{1}{2-k^2}$

$\mathrm{tg}(-\alpha)<0 \Leftrightarrow \mathrm{tg}\alpha>0\ ;\ \alpha\in\left]0\,,\frac{\pi}{2}\right[$.

$\mathrm{tg}\alpha>0 \Leftrightarrow \frac{1}{2-k^2}>0 \Leftrightarrow 2-k^2>0 \Leftrightarrow k\in\left]-\sqrt{2}\,,\sqrt{2}\right[$

38.2 $\mathrm{tg}(-\alpha)=-0{,}6 \Leftrightarrow -\mathrm{tg}\alpha=-0{,}6 \Leftrightarrow \mathrm{tg}\alpha=0{,}6$

$1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha} \Leftrightarrow 1+0{,}6^2=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{25}{34} \underset{\alpha\in 1.^{\circ}\mathrm{Q.}}{\Leftrightarrow} \cos\alpha=\frac{5\sqrt{34}}{34}$

$\mathrm{tg}\alpha=\frac{\mathrm{sen}\alpha}{\cos\alpha} \Leftrightarrow \frac{3}{5}=\frac{\mathrm{sen}\alpha}{\frac{5\sqrt{34}}{34}} \Leftrightarrow \mathrm{sen}\alpha=\frac{3\sqrt{34}}{34}$

$$\frac{\mathrm{tg}\alpha-\mathrm{sen}\alpha}{3\cos(\pi+\alpha)}=\frac{\mathrm{tg}\alpha-\mathrm{sen}\alpha}{-3\cos\alpha}=\frac{\frac{3}{5}-\frac{3\sqrt{34}}{34}}{-3\times\frac{5\sqrt{34}}{34}}=\frac{\frac{3\times 34-5\times 3\sqrt{34}}{5\times 34}}{-\frac{3\times 5\sqrt{34}}{34}}=$$

$$=-\frac{34-5\sqrt{34}}{5\times 5\sqrt{34}}=-\frac{\left(34-5\sqrt{34}\right)\sqrt{34}}{5\times 5\sqrt{34}\times\sqrt{34}}=-\frac{34\sqrt{34}-5\times 34}{5\times 5\times 34}=\frac{5-\sqrt{34}}{25}$$

39. $\alpha+\beta=\frac{\pi}{2}$

$\mathrm{sen}(\pi-\alpha)+\mathrm{sen}\left(\beta-\frac{\pi}{2}\right)+\mathrm{tg}\alpha+\mathrm{tg}(\pi+\beta)=\mathrm{sen}\alpha+\mathrm{sen}(-\alpha)+\mathrm{tg}\alpha+\mathrm{tg}\beta=$

$=\mathrm{sen}\alpha-\mathrm{sen}\alpha+\mathrm{tg}\alpha+\mathrm{tg}\left(\frac{\pi}{2}-\alpha\right)=\mathrm{tg}\alpha+\frac{1}{\mathrm{tg}\alpha}=\frac{b}{a}+\frac{a}{b}=\frac{b^2+a^2}{a\times b}=\frac{1}{ab}$

40. $\frac{\mathrm{sen}\theta+3\cos\theta}{\cos\theta}=1 \Leftrightarrow \frac{\mathrm{sen}\theta}{\cos\theta}+\frac{3\cos\theta}{\cos\theta}=1 \Leftrightarrow \mathrm{tg}\theta+3=1 \Leftrightarrow \mathrm{tg}\theta=-2$

$\mathrm{tg}\left(\frac{3\pi}{2}-\theta\right)+\mathrm{tg}(\pi+\theta)=\frac{1}{\mathrm{tg}\theta}+\mathrm{tg}\theta=\frac{1}{-2}+(-2)=-\frac{5}{2}$

41.1 O vértice A tem coordenadas $(5\cos\alpha\,,\,5\operatorname{sen}\alpha)$, com $\cos\alpha > 0$ e $\operatorname{sen}\alpha < 0$.

$\overline{AB} = 5 + (-5\operatorname{sen}\alpha) = 5 - 5\operatorname{sen}\alpha$

$\overline{BC} = 2 \times 5\cos\alpha = 10\cos\alpha$

$A(\alpha) = \overline{AB} \times \overline{BC} = (5 - 5\operatorname{sen}\alpha) \times 10\cos\alpha = 50\cos\alpha - 50\operatorname{sen}\alpha\cos\alpha = 50(\cos\alpha - \operatorname{sen}\alpha\cos\alpha)$

41.2 $A(0) = 50(\cos 0 - \operatorname{sen} 0 \times \cos 0) = 50(1 - 0 \times 1) = 50$

Quando $\alpha = 0$, $[ABCD]$ é um retângulo de dimensões 5 e 10, pelo que a sua área é 50.

41.3 $\operatorname{tg}(-\alpha - \pi) = 2\sqrt{6} \Leftrightarrow \operatorname{tg}(-(\alpha + \pi)) = 2\sqrt{6} \Leftrightarrow -\operatorname{tg}(\alpha + \pi) = 2\sqrt{6} \Leftrightarrow$

$$\Leftrightarrow -\operatorname{tg}\alpha = 2\sqrt{6} \Leftrightarrow \operatorname{tg}\alpha = -2\sqrt{6}$$

$$1 + \operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1 + \left(-2\sqrt{6}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{1}{25} \underset{\alpha \in 4.^\circ Q.}{\Leftrightarrow} \cos\alpha = \frac{1}{5}$$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \Leftrightarrow -2\sqrt{6} = \frac{\operatorname{sen}\alpha}{\frac{1}{5}} \Leftrightarrow \operatorname{sen}\alpha = -\frac{2\sqrt{6}}{5}$$

$$A(\alpha) = 50\left(\frac{1}{5} - \left(-\frac{2\sqrt{6}}{5}\right) \times \frac{1}{5}\right) = 50\left(\frac{5 + 2\sqrt{6}}{25}\right) = 10 + 4\sqrt{6}$$

PÁG. 68

Aplicar +

42.1 $\left(\operatorname{tg}^2 x + 1\right)\left(\operatorname{sen}^4 x - \cos^4 x\right) = \frac{1}{\cos^2 x}\left(\operatorname{sen}^4 x - \cos^4 x\right) = \frac{\operatorname{sen}^4 x}{\cos^2 x} - \frac{\cos^4 x}{\cos^2 x} = \operatorname{tg}^2 x \operatorname{sen}^2 x - \cos^2 x =$

$$= \operatorname{tg}^2 x \operatorname{sen}^2 x - \left(1 - \operatorname{sen}^2 x\right) = \operatorname{sen}^2 x\left(1 + \operatorname{tg}^2 x\right) - 1$$

$$= \operatorname{sen}^2 x \times \frac{1}{\cos^2 x} - 1$$

Por outro processo:

$$\left(\operatorname{tg}^2 x + 1\right)\left(\operatorname{sen}^4 x - \cos^4 x\right) = \frac{1}{\cos^2 x}\left[\left(\operatorname{sen}^2 x - \cos^2 x\right)\underbrace{\left(\operatorname{sen}^2 x + \cos^2 x\right)}_{1}\right]$$

$$= \frac{\operatorname{sen}^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} = \operatorname{tg}^2 x - 1 = (\operatorname{tg} x - 1)(\operatorname{tg} x + 1)$$

42.2 $\dfrac{\operatorname{sen}^3 x + \cos^2 x \operatorname{sen} x}{1 + \dfrac{1}{\operatorname{tg}^2 x}} = \dfrac{\operatorname{sen} x\left(\operatorname{sen}^2 x + \cos^2 x\right)}{1 + \dfrac{\cos^2 x}{\operatorname{sen}^2 x}} = \dfrac{\operatorname{sen} x}{\dfrac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^2 x}} = \operatorname{sen}^3 x$

43.1 $A(x) = 2\operatorname{sen}^2(\pi + x) + \operatorname{sen}^2\left(\frac{\pi}{2} + x\right) - 2\operatorname{tg} x \cos(2\pi - x) =$

$$= 2(-\operatorname{sen} x)^2 + (\cos x)^2 - 2\operatorname{tg} x \cos(-x) = 2\operatorname{sen}^2 x + \cos^2 x - 2\operatorname{tg} x \cos x =$$

$$= \operatorname{sen}^2 x + \operatorname{sen}^2 x + \cos^2 x - 2\operatorname{sen} x = \operatorname{sen}^2 x - 2\operatorname{sen} x + 1 = (\operatorname{sen} x - 1)^2$$

43.2 $\operatorname{tg}\left(\alpha+\frac{\pi}{2}\right)=\frac{4}{3} \Leftrightarrow -\frac{1}{\operatorname{tg}\alpha}=\frac{4}{3} \Leftrightarrow \operatorname{tg}\alpha=-\frac{3}{4}$

$1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha} \Leftrightarrow 1+\left(-\frac{3}{4}\right)^2=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{16}{25} \underset{\alpha\in\left]-\frac{\pi}{2},\frac{\pi}{2}\right[}{\Leftrightarrow} \cos\alpha=\frac{4}{5}$

$\operatorname{tg}\alpha=\frac{\operatorname{sen}\alpha}{\cos\alpha} \Leftrightarrow -\frac{3}{4}=\frac{\operatorname{sen}\alpha}{\frac{4}{5}} \Leftrightarrow \operatorname{sen}\alpha=-\frac{3}{5}$

$A(-\alpha-\pi)=(\operatorname{sen}(-\alpha-\pi)-1)^2=(\operatorname{sen}(-(\alpha+\pi))-1)^2=(\operatorname{sen}\alpha-1)^2=\left(-\frac{3}{5}-1\right)^2=\frac{64}{25}$

43.3 $\operatorname{sen}^2\theta+\cos^2\theta=1 \Leftrightarrow \operatorname{sen}^2\theta=1-\left(\frac{12}{13}\right)^2 \Leftrightarrow \operatorname{sen}^2\theta=\frac{25}{169} \underset{\theta\in 1.^{\circ}\text{Q.}}{\Leftrightarrow} \operatorname{sen}\theta=\frac{5}{13}$

$A(\theta)=1-5m^2 \Leftrightarrow (\operatorname{sen}\theta-1)^2=1-5m^2 \Leftrightarrow \left(\frac{5}{13}-1\right)^2=1-5m^2 \Leftrightarrow \left(-\frac{8}{13}\right)^2=1-5m^2 \Leftrightarrow$

$\Leftrightarrow 5m^2=1-\frac{64}{169} \Leftrightarrow m^2=\frac{105}{5\times 169} \Leftrightarrow m=\pm\sqrt{\frac{21}{169}} \Leftrightarrow m=-\frac{\sqrt{21}}{13} \vee m=\frac{\sqrt{21}}{13}$

44.1 $A(\alpha)=\frac{\overline{QP}+\overline{RS}}{2}\times(y_Q+|y_R|)$

O ponto Q tem coordenadas $(\cos\alpha\,,\,\operatorname{sen}\alpha)$.

O ponto R pertence à circunferência trigonométrica e tem ordenada $-\frac{1}{2}$:

$x^2+y^2=1 \Leftrightarrow x^2=1-\left(-\frac{1}{2}\right)^2 \Leftrightarrow x=-\frac{\sqrt{3}}{2}$;

logo, tem-se $R\left(-\frac{\sqrt{3}}{2}\,,\,-\frac{1}{2}\right)$ e $S\left(\frac{\sqrt{3}}{2}\,,\,-\frac{1}{2}\right)$;

$\overline{QP}=2\times(-\cos\alpha)=-2\cos\alpha$

$\overline{RS}=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$

$A(\alpha)=\frac{-2\cos\alpha+\sqrt{3}}{2}\times\left(\operatorname{sen}\alpha+\frac{1}{2}\right)=\left(\frac{\sqrt{3}}{2}-\cos\alpha\right)\left(\frac{1}{2}+\operatorname{sen}\alpha\right)$

44.2 $A\left(\frac{\pi}{2}\right)=\left(\frac{\sqrt{3}}{2}-\cos\left(\frac{\pi}{2}\right)\right)\left(\frac{1}{2}+\text{sen}\left(\frac{\pi}{2}\right)\right)=\left(\frac{\sqrt{3}}{2}-0\right)\left(\frac{1}{2}+1\right)=\frac{\sqrt{3}}{2}\times\frac{3}{2}=\frac{3\sqrt{3}}{4}$

Se $\alpha=\frac{\pi}{2}$, o polígono é um triângulo equilátero de lado $\sqrt{3}$, cuja área é $\frac{3\sqrt{3}}{4}$.

$$A\left(\frac{5\pi}{6}\right)=\left(\frac{\sqrt{3}}{2}-\cos\left(\frac{5\pi}{6}\right)\right)\left(\frac{1}{2}+\text{sen}\left(\frac{5\pi}{6}\right)\right)=\left(\frac{\sqrt{3}}{2}-\cos\left(\pi-\frac{\pi}{6}\right)\right)\left(\frac{1}{2}+\text{sen}\left(\pi-\frac{\pi}{6}\right)\right)=$$

$$=\left(\frac{\sqrt{3}}{2}+\cos\left(\frac{\pi}{6}\right)\right)\left(\frac{1}{2}+\text{sen}\left(\frac{\pi}{6}\right)\right)=\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{1}{2}\right)=\sqrt{3}$$

Se $\alpha=\frac{5\pi}{6}$, o polígono é um retângulo de dimensões $\sqrt{3}$ e 1, cuja área é $\sqrt{3}$.

44.3 $\text{sen}(\pi+\alpha)=-\frac{\sqrt{6}}{3} \Leftrightarrow -\text{sen}\,\alpha=-\frac{\sqrt{6}}{3} \Leftrightarrow \text{sen}\,\alpha=\frac{\sqrt{6}}{3}$

$$\text{sen}^2\alpha+\cos^2\alpha=1 \Leftrightarrow \cos^2\alpha=1-\left(\frac{\sqrt{6}}{3}\right)^2 \Leftrightarrow \cos^2\alpha=\frac{3}{9} \underset{\alpha\in\left]\frac{\pi}{2},\frac{7\pi}{6}\right[}{\Leftrightarrow} \cos\alpha=-\frac{\sqrt{3}}{3}$$

$$A(\alpha)=\left(\frac{\sqrt{3}}{2}-\cos\alpha\right)\left(\frac{1}{2}+\text{sen}\,\alpha\right)=\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{3}\right)\left(\frac{1}{2}+\frac{\sqrt{6}}{3}\right)=$$

$$=\frac{5\sqrt{3}}{6}\times\frac{3+2\sqrt{6}}{6}=\frac{5\sqrt{3}\times(3+2\sqrt{6})}{6\times6}=\frac{3\times5\sqrt{3}+2\times5\sqrt{3\times6}}{6\times6}=$$

$$=\frac{15\sqrt{3}+10\sqrt{18}}{36}=\frac{15\sqrt{3}+30\sqrt{2}}{36}=\frac{5\sqrt{3}+10\sqrt{2}}{12}$$

PÁG. 74

Autoavaliação

1. $A(x)=\text{sen}\left(\frac{\pi}{2}+x\right)+\cos(\pi+x)+\cos\left(\frac{\pi}{2}-x\right)=\cos x+(-\cos x)+\text{sen}\,x=\text{sen}\,x$

Opção correta: **(A)**

2. $\alpha+\beta+\gamma=180° \Leftrightarrow \alpha=180°-(\beta+\gamma)$

(B) $\text{sen}\,\alpha=\text{sen}\left(180°-(\beta+\gamma)\right)=\text{sen}(\beta+\gamma)$

Opção correta: **(B)**

3. $\frac{17\pi}{6}-2\pi=\frac{5\pi}{6}$ e $\frac{17\pi}{6}-\pi=\frac{11\pi}{6}$

Opção correta: **(C)**

4. Como $\frac{5\pi}{3} < \alpha < \theta < 2\pi$, $\cos\alpha > 0$, $\cos\theta > 0$, $\operatorname{sen}\alpha < 0$, $\operatorname{sen}\theta < 0$, $\cos\alpha < \cos\theta$, $\operatorname{sen}\alpha < \operatorname{sen}\theta$, $\operatorname{tg}\alpha < 0$ e $\operatorname{tg}\beta < 0$.

(A) $\cos\alpha - \cos\theta < 0$

(B) $\cos\theta \times \operatorname{tg}\alpha < 0$

(C) $\operatorname{sen}\theta - \operatorname{sen}\alpha > 0$

(D) $\operatorname{tg}\theta + \operatorname{tg}\alpha < 0$

Opção correta: **(C)**

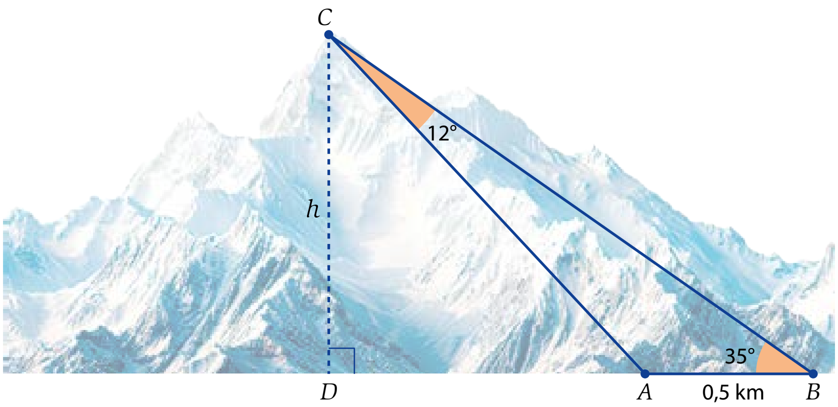
5. A área da região colorida da figura obtém-se subtraindo à área do triângulo a área do setor circular: $\frac{1 \times \operatorname{tg}\alpha}{2} - \frac{\alpha \times 1^2}{2} = \frac{1}{2}(\operatorname{tg}\alpha - \alpha)$.

Opção correta: **(D)**

PÁG. 75

Autoavalição

6.



$B\hat{A}C = 180° - 35° - 12° = 133°$ e $D\hat{A}C = 180° - 133° = 47°$

$$\begin{cases} \frac{h}{\overline{DA}} = \operatorname{tg}47° \\ \frac{h}{\overline{DA}+0,5} = \operatorname{tg}35° \end{cases} \Leftrightarrow \begin{cases} h = \overline{DA}\ \operatorname{tg}47° \\ h = (\overline{DA}+0,5)\operatorname{tg}35° \end{cases} \Leftrightarrow \begin{cases} \overline{DA}\ \operatorname{tg}47° = (\overline{DA}+0,5)\operatorname{tg}35° \\ \overline{\qquad} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \overline{DA}\ \operatorname{tg}47° = \overline{DA}\ \operatorname{tg}35° + 0,5\operatorname{tg}35° \\ \overline{\qquad} \end{cases} \Leftrightarrow \begin{cases} \overline{DA}\ \operatorname{tg}47° - \overline{DA}\ \operatorname{tg}35° = 0,5\operatorname{tg}35° \\ \overline{\qquad} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \overline{DA}\ (\operatorname{tg}47° - \operatorname{tg}35°) = 0,5\operatorname{tg}35° \\ \overline{\qquad} \end{cases} \Leftrightarrow \begin{cases} \overline{DA} = \frac{0,5\operatorname{tg}35°}{\operatorname{tg}47° - \operatorname{tg}35°} \\ \overline{\qquad} \end{cases}$$

$$\Leftrightarrow \begin{cases} h = \frac{0,5\operatorname{tg}35°}{\operatorname{tg}47° - \operatorname{tg}35°} \times \operatorname{tg}47° \\ \overline{\qquad} \end{cases}$$; logo, $h \approx 1009$ m .

7. Seja θ a amplitude do setor circular AOB .

$\theta r = 2\pi \Leftrightarrow r = \frac{2\pi}{\theta}$

$\frac{\theta r^2}{2} = 3\pi \Leftrightarrow \theta r^2 = 6\pi \Leftrightarrow \theta\left(\frac{2\pi}{\theta}\right)^2 = 6\pi \Leftrightarrow \frac{4\pi^2}{\theta} = 6\pi \Leftrightarrow \theta = \frac{4\pi^2}{6\pi} \Leftrightarrow \theta = \frac{2}{3}\pi$

O correspondente ângulo orientado tem amplitude $120° - 360° = -240°$. Assim, $\alpha = -240° - 2 \times 360° = -960°$.

8.1 $B(x) = \frac{\operatorname{sen}(-x-\pi) \times \operatorname{tg}(\pi + x) + \cos x}{\cos(\pi - x) \times \cos\left(-\frac{5\pi}{3}\right) + \operatorname{tg}\left(\frac{9\pi}{4}\right)} = \frac{\operatorname{sen} x \times \operatorname{tg} x + \cos x}{-\cos x \times \cos\left(\frac{\pi}{3}\right) + \operatorname{tg}\left(\frac{\pi}{4}\right)} =$

$= \frac{\frac{\operatorname{sen}^2 x + \cos^2 x}{\cos x}}{-\cos x \times \frac{1}{2} + 1} = \frac{\frac{1}{\cos x}}{\frac{-\cos x + 2}{2}} = \frac{2}{\cos x(-\cos x + 2)} = \frac{2}{2\cos x - \cos^2 x}$

8.2 $2\cos\left(\theta - \frac{\pi}{2}\right) + 1 = 0 \Leftrightarrow 2\operatorname{sen}\theta + 1 = 0 \Leftrightarrow \operatorname{sen}\theta = -\frac{1}{2}$

$\operatorname{sen}^2\theta + \cos^2\theta = 1 \Leftrightarrow \cos^2\theta = 1 - \left(-\frac{1}{2}\right)^2 \Leftrightarrow \cos^2\theta = \frac{3}{4} \underset{\theta \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[}{\Leftrightarrow} \cos\theta = -\frac{\sqrt{3}}{2}$

$B(\theta) = \frac{2}{2\cos\theta - \cos^2\theta} = \frac{2}{2\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)^2} = \frac{2}{-\sqrt{3} - \frac{3}{4}} = -\frac{8}{3 + 4\sqrt{3}}$

PÁG. 76

Autoavaliação

9.1 $-\frac{143\pi}{12} = \frac{\pi}{12} - 12\pi = \frac{\pi}{12} - 2 \times 6\pi$

Se $k = -6$, $-\frac{\pi}{4} - 2 \times 6\pi < \frac{\pi}{12} - 2 \times 6\pi < \frac{\pi}{4} - 2 \times 6\pi$, pelo que o lado extremidade do ângulo de amplitude $-\frac{143\pi}{12}$ interseta o setor circular AOB .

9.2 $3\operatorname{tg}\alpha = \frac{2 - 3\lambda}{5} \Leftrightarrow \operatorname{tg}\alpha = \frac{2 - 3\lambda}{15}$

$-1 < \operatorname{tg}\alpha < 1 \Leftrightarrow -1 < \frac{2 - 3\lambda}{15} < 1 \Leftrightarrow -15 < 2 - 3\lambda < 15 \Leftrightarrow -15 - 2 < -3\lambda < 15 - 2 \Leftrightarrow$

$\Leftrightarrow -13 < 3\lambda < 17 \Leftrightarrow -\frac{13}{3} < \lambda < \frac{17}{3} \Leftrightarrow \lambda \in \left]-\frac{13}{3}, \frac{17}{3}\right[$

9.3

$\operatorname{sen}^2\beta + \cos^2\beta = 1 \Leftrightarrow \cos^2\beta = 1 - \left(\frac{\sqrt{2}}{3}\right)^2 \Leftrightarrow \cos^2\beta = \frac{7}{9} \underset{\beta \in \left]0, \frac{\pi}{4}\right[}{\Leftrightarrow} \cos\beta = \frac{\sqrt{7}}{3}$

$\operatorname{tg}\left(\frac{3\pi}{2} - \beta\right) = \operatorname{tg}\left(\pi + \frac{\pi}{2} - \beta\right) = \operatorname{tg}\left(\frac{\pi}{2} - \beta\right) = \frac{\operatorname{sen}\left(\frac{\pi}{2} - \beta\right)}{\cos\left(\frac{\pi}{2} - \beta\right)} = \frac{\cos\beta}{\operatorname{sen}\beta} = \frac{\frac{\sqrt{7}}{3}}{\frac{\sqrt{2}}{3}} = \frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$

10.1 $\frac{\overline{AC}}{2}=\cos x \Leftrightarrow \overline{AC}=2\cos x$, $\frac{\overline{BC}}{2}=\operatorname{sen} x \Leftrightarrow \overline{BC}=2\operatorname{sen} x$

$A(\alpha)=\frac{\overline{AC}\times\overline{BC}}{2}=\frac{2\cos x 2\operatorname{sen} x}{2}=2\cos x\operatorname{sen} x$

10.2 $(\operatorname{sen} x-\cos x)^2=\operatorname{sen}^2 x-2\operatorname{sen} x\cos x+\cos^2 x=\operatorname{sen}^2 x+\cos^2 x-2\operatorname{sen} x\cos x=1-0{,}8=0{,}2$

10.3 $x=\frac{\pi}{3}$, pois o seno de um ângulo e o cosseno do seu complementar são iguais e vice-versa. Assim, tem-se $A\left(\frac{\pi}{3}\right)=2\cos\left(\frac{\pi}{3}\right)\operatorname{sen}\left(\frac{\pi}{3}\right)=2\operatorname{sen}\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)=A\left(\frac{\pi}{6}\right)$.

PÁG. 77

Começar a preparar o exame

1. $5-5\cos 45°\approx 1{,}46$ cm

Opção correta: **(C)**

2. $\alpha\times 4=2 \Leftrightarrow \alpha=\frac{2}{4}=\frac{1}{2}$rad

Opção correta: **(C)**

3.1 $1+\operatorname{tg}^2\beta=\frac{1}{\cos^2\beta} \Leftrightarrow 1+(-2)^2=\frac{1}{\cos^2\beta} \Leftrightarrow \cos^2\beta=\frac{1}{5} \underset{\beta\in 2.°Q.}{\Leftrightarrow} \cos\beta=-\frac{\sqrt{5}}{5}$

$\operatorname{sen}^2\beta+\cos^2\beta=1 \Leftrightarrow \operatorname{sen}^2\beta=1-\frac{1}{5} \Leftrightarrow \operatorname{sen}^2\beta=\frac{4}{5} \underset{\beta\in 2.°Q.}{\Leftrightarrow} \operatorname{sen}\beta=\frac{2\sqrt{5}}{5}$

$\cos^2\beta-\operatorname{tg}\beta-\operatorname{sen}(\pi-\beta)=\frac{1}{5}-(-2)-\frac{2\sqrt{5}}{5}=\frac{11-2\sqrt{5}}{5}$

3.2 $C\left(\cos\left(\beta+\frac{\pi}{2}\right),\operatorname{sen}\left(\beta+\frac{\pi}{2}\right)\right)=(-\operatorname{sen}\beta,\cos\beta)=\left(-\frac{2\sqrt{5}}{5},-\frac{\sqrt{5}}{5}\right)$

PÁG. 78

Começar a preparar o exame

4. $\alpha+\beta=\pi \Leftrightarrow \alpha=\pi-\beta$

$\operatorname{sen}(\pi+\alpha)+\operatorname{tg}(\pi-\alpha)=-\operatorname{sen}\alpha-\operatorname{tg}\alpha=-\operatorname{sen}(\pi-\beta)-\operatorname{tg}(\pi-\beta)=-\operatorname{sen}\beta+\operatorname{tg}\beta$

$\operatorname{sen}^2\beta+\cos^2\beta=1 \Leftrightarrow \operatorname{sen}^2\beta=1-(-0{,}8)^2 \Leftrightarrow \operatorname{sen}^2\beta=0{,}36 \underset{90°<\beta<180°}{\Leftrightarrow} \operatorname{sen}\beta=0{,}6$

$\operatorname{tg}\beta=\frac{\operatorname{sen}\beta}{\cos\beta}=\frac{0{,}6}{-0{,}8}=-0{,}75$

$-\operatorname{sen}\beta+\operatorname{tg}\beta=-0{,}6-0{,}75=-1{,}35$

5. A área do triângulo $[PQR]$ obtém-se, por exemplo, adicionando as áreas dos triângulos $[OPQ]$ e $[OPR]$, que têm a mesma medida da base, $\overline{OP}=2$, considerando o lado $[OP]$ para base.

Como os pontos Q e R são simétricos relativamente à origem do referencial, têm ordenadas simétricas e os dois triângulos têm a mesma altura, relativa à base $[OP]$, correspondente à ordenada do ponto Q.

Como $P\hat{O}Q=\frac{4}{3}\pi-\pi=\frac{\pi}{3}$, a ordenada do ponto Q é igual a $2\operatorname{sen}\left(\frac{\pi}{3}\right)=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$.

Assim, $A_{[PQR]}=2\times\frac{2\times\sqrt{3}}{2}=2\sqrt{3}$.

Opção correta: **(D)**

6. $\alpha+\beta=\pi \Leftrightarrow \beta=\pi-\alpha$

$\cos\beta=\cos(\pi-\alpha)=-\cos\alpha$

Opção correta: **(C)**

PÁG. 79

Começar a preparar o exame

7. Como $\theta\in 3.^{\text{o}}Q.$, conclui-se que $\cos\theta<0$, $\operatorname{sen}\theta<0$ e $\operatorname{tg}\theta>0$.

(A) $\operatorname{tg}\theta-\operatorname{sen}\theta>0$

(B) $\cos\theta+\operatorname{sen}\theta<0$

(C) $\cos\theta\times\operatorname{sen}\theta>0$

(D) $\operatorname{tg}\theta-\cos\theta>0$

Opção correta: **(B)**

8. Como $\frac{\pi}{2}<\alpha<\pi$, $\frac{\pi}{2}<\beta<\pi$ e $\alpha<\beta$, tem-se $\frac{\pi}{2}-\pi<\alpha-\beta<0$, pelo que $\alpha-\beta\in 4.^{\circ}$ Q., $\cos(\alpha-\beta)>0$ e $\operatorname{sen}(\alpha-\beta)<0$.

Por outro lado, $\pi<\alpha+\beta<2\pi$, pelo que $\alpha+\beta\in 3.^{\circ}$ Q., $\cos(\alpha+\beta)<0$ e $\operatorname{sen}(\alpha+\beta)<0$, ou $\alpha+\beta\in 4.^{\circ}$ Q., $\cos(\alpha+\beta)>0$ e $\operatorname{sen}(\alpha+\beta)<0$.

Opção correta: **(B)**

9.1 $A(\cos\alpha, \operatorname{sen}\alpha)$ e $C(\cos\alpha, -\operatorname{sen}\alpha)$

9.2 Seja D a projeção ortogonal do ponto A no semieixo positivo Ox.

$\frac{\overline{OD}}{1}=\cos\alpha \Leftrightarrow \overline{OD}=\cos\alpha$; $A=\frac{\overline{AC}\times\overline{BD}}{2}=\frac{2\operatorname{sen}\alpha(1+\cos\alpha)}{2}=\operatorname{sen}\alpha(1+\cos\alpha)$

9.3 O triângulo é isósceles, dado que A e C são simétricos em relação ao Ox. Assim, $\overline{BA}=\overline{BC}$.

Logo, o triângulo é equilátero se $A\hat{B}C=\frac{\pi}{3}$, ou seja, se $A\hat{O}C=\frac{2\pi}{3}$ (o ângulo ABC está inscrito no arco AC).

Portanto, o triângulo é equilátero para $\alpha=\frac{A\hat{O}C}{2}=\frac{\pi}{3}$.

O lado do triângulo é $\overline{AC}=2\operatorname{sen}\alpha=2\operatorname{sen}\left(\frac{\pi}{3}\right)=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$.

9.4 O triângulo é retângulo se $A\hat{B}C=\frac{\pi}{2}$, ou seja, se $A\hat{O}C=\frac{2\pi}{2}=\pi$ (o ângulo ABC está inscrito no arco AC). Portanto, o triângulo é retângulo para $\alpha=\frac{A\hat{O}C}{2}=\frac{\pi}{2}$.

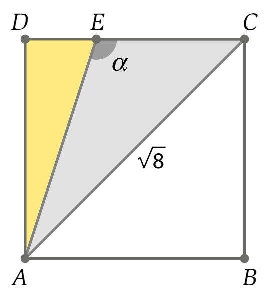
Neste caso, $A(0\ ,\ 1)$, $B(0\ ,\ -1)$ e $C(0\ ,\ -1)$, pelo que $\overline{AC}=2$, $\overline{AB}=\overline{BC}=\sqrt{1^2+1^2}=\sqrt{2}$ e o perímetro do triângulo $[ABC]$ é $2+2\sqrt{2}$.

PÁG. 80

Começar a preparar o exame

10.1

Para $\alpha\in\left]\frac{\pi}{2}\ ,\ \frac{3\pi}{4}\right[$, tem-se:



$$A_{[ACE]}=\frac{\overline{EC}\times\overline{AD}}{2}$$

$$\overline{AC}^2=\overline{AB}^2+\overline{BC}^2 \Leftrightarrow \overline{AC}^2=2\overline{AB}^2 \Leftrightarrow \left(\sqrt{8}\right)^2=2\overline{AB}^2 \Leftrightarrow \overline{AB}^2=4 \underset{\overline{AB}>0}{\Leftrightarrow} \overline{AB}=2$$

$$D\hat{E}A=\pi-\alpha$$

$$\frac{\overline{AD}}{\overline{DE}}=\operatorname{tg}(\pi-\alpha) \Leftrightarrow \frac{2}{\overline{DE}}=-\operatorname{tg}\alpha \Leftrightarrow \overline{DE}=-\frac{2}{\operatorname{tg}\alpha}\ ;$$

$$\overline{EC}=\overline{DC}-\overline{DE}=2+\frac{2}{\operatorname{tg}\alpha}$$

$$A_{[ACE]}=\frac{\left(2+\frac{2}{\operatorname{tg}\alpha}\right)\times 2}{2}=2+\frac{2}{\operatorname{tg}\alpha}$$

Para $\alpha=\frac{3\pi}{4}$, o triângulo reduz-se a um segmento de reta, pelo que a sua área é nula. Como se tem $2+\frac{2}{\operatorname{tg}\left(\frac{3\pi}{4}\right)}=2+\frac{2}{-1}=0$, verifica-se que a expressão $2+\frac{2}{\operatorname{tg}\alpha}$ também é válida para $\alpha=\frac{3\pi}{4}$.

Conclui-se, portanto que, para $\alpha\in\left]\frac{\pi}{2}\ ,\ \frac{3\pi}{4}\right]$, se tem $A(\alpha)=2+\frac{2}{\operatorname{tg}\alpha}$.

10.2 $\text{tg}^2\alpha + 1 = \frac{1}{\cos^2\alpha} \Leftrightarrow \text{tg}^2\alpha + 1 = \frac{1}{\left(-\sqrt{\frac{1}{10}}\right)^2} \Leftrightarrow \text{tg}^2\alpha + 1 = \frac{1}{\frac{1}{10}} \Leftrightarrow \text{tg}^2\alpha = 10 - 1 \Leftrightarrow$

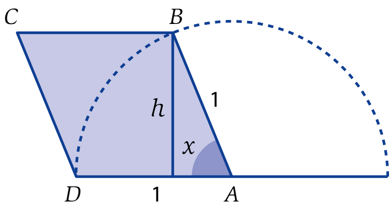
$$\underset{\alpha \in 2.^\circ Q}{\Leftrightarrow} \text{tg}\,\alpha = -\sqrt{9} \Leftrightarrow \text{tg}\,\alpha = -3$$

Logo, $A(\alpha) = 2 + \frac{2}{(-3)} = 2 - \frac{2}{3} = \frac{4}{3}$.

Portanto, $A_{[ADE]} = A_{[ACD]} - A_{[ACE]} = 2 \times \frac{2}{2} - \frac{4}{3} = 2 - \frac{4}{3} = \frac{2}{3}$.

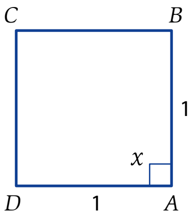
11.1

Para $x \in \left]0, \frac{\pi}{2}\right[$, tem-se:



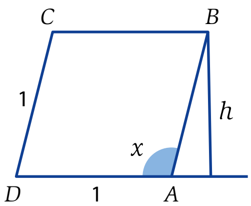
$\text{sen}\,x = \frac{h}{1} \Leftrightarrow h = \text{sen}\,x$ e $A_{[ABCD]} = 1 \times h = \text{sen}\,x$

Para $x = \frac{\pi}{2}$, tem-se:



$h = 1$ e $A_{[ABCD]} = 1 \times 1 = 1 = \text{sen}\left(\frac{\pi}{2}\right)$

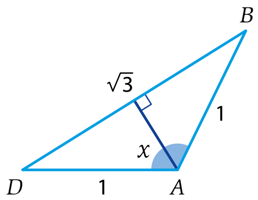
Para $x \in \left]\frac{\pi}{2}, \pi\right[$, tem-se:



$\text{sen}(\pi - x) = \frac{h}{1} \Leftrightarrow h = \text{sen}\,x$ e $A_{[ABCD]} = 1 \times h = \text{sen}\,x$

Logo, a área do losango é dada por $A(x) = \text{sen}\,x$, $x \in]0, \pi[$.

11.2 Se $\overline{BD}=\sqrt{3}$, então $x\in\left]\frac{\pi}{2},\pi\right[$ (Se $\overline{BD}$ fosse igual a $\sqrt{2}$, ter-se-ia $x=\frac{\pi}{2}$, como $\overline{BD}=\sqrt{3}>\sqrt{2}$, deduz-se que $x\in\left]\frac{\pi}{2},\pi\right[$.)



A altura do triângulo $[ABD]$, relativa ao lado $[BD]$ é $\sqrt{1^2-\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$.

A área do triângulo $[ABD]$ é $\frac{\sqrt{3}\times\frac{1}{2}}{2}=\frac{\sqrt{3}}{4}$, e a área do losango é $2\times\frac{\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$.

Como a área do losango é $\operatorname{sen} x$, conclui-se que $\operatorname{sen} x=\frac{\sqrt{3}}{2}$; assim, (*), como $x\in\left]\frac{\pi}{2},\pi\right[$, $x=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ e, portanto, $\cos x=\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$.

Por outro processo (a partir de (*)):

$\operatorname{sen}^2 x+\cos^2 x=1 \Leftrightarrow \cos^2 x=1-\left(\frac{\sqrt{3}}{2}\right)^2 \Leftrightarrow \cos^2 x=\frac{1}{4} \underset{\frac{\pi}{2}<x<\pi}{\Leftrightarrow} \cos x=-\frac{1}{2}$

Como $\operatorname{sen} x=\frac{\sqrt{3}}{2}$ e $\cos x=-\frac{1}{2}$, conclui-se que $x=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$.

12. $\operatorname{tg}\alpha=\frac{1}{\frac{1}{2}} \Leftrightarrow \operatorname{tg}\alpha=2$

$1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha} \Leftrightarrow 1+2^2=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{1}{5}$

$\operatorname{sen}^2\alpha+\cos^2\alpha=1 \Leftrightarrow \operatorname{sen}^2\alpha=1-\frac{1}{5} \Leftrightarrow \operatorname{sen}^2\alpha=\frac{4}{5} \underset{\alpha\in 1.^\circ\text{Q.}}{\Leftrightarrow} \operatorname{sen}\alpha=\frac{2\sqrt{5}}{5}$

$\operatorname{tg}\theta=\frac{1}{2}$, $1+\operatorname{tg}^2\theta=\frac{1}{\cos^2\theta} \Leftrightarrow 1+\left(\frac{1}{2}\right)^2=\frac{1}{\cos^2\theta} \Leftrightarrow \cos^2\theta=\frac{4}{5} \underset{\theta\in 3.^\circ\text{Q.}}{\Leftrightarrow} \cos\theta=-\frac{2\sqrt{5}}{5}$

$\operatorname{sen}\alpha+\cos(\pi-\theta)=\operatorname{sen}\alpha-\cos\theta=\frac{2\sqrt{5}}{5}-\left(-\frac{2\sqrt{5}}{5}\right)=\frac{4\sqrt{5}}{5}$

PÁG. 81

Começar a preparar o exame

13.

I. V (por exemplo, $x=\pi$).

II. F

Como $\alpha\in\left]-\pi,-\frac{\pi}{2}\right[$, conclui-se que $\cos\alpha<0$, $\operatorname{sen}\alpha<0$ e $\operatorname{tg}\alpha>0$.

Como $\operatorname{sen}\alpha\,\operatorname{sen}\beta<0$ e $\operatorname{sen}\alpha<0$, conclui-se que $\operatorname{sen}\beta>0$.

Como $\frac{\operatorname{tg}\alpha}{\cos\beta}<0$ e $\operatorname{tg}\alpha>0$, conclui-se que $\cos\beta<0$.

Como $\operatorname{sen}\beta>0$ e $\cos\beta<0$, conclui-se que $\beta\in 2.^\circ$ Q.

Opção correta: **(B)**

14. A área do quadrilátero $[OPQR]$ obtém-se adicionando as áreas dos triângulos $[OQR]$ e $[OPQ]$:

$$\frac{\overline{OR}\times\overline{OQ}}{2}+\frac{\overline{OQ}\times x_P}{2}=\frac{1\times\frac{1}{2}}{2}+\frac{\frac{1}{2}\times\cos\alpha}{2}=\frac{1+\cos\alpha}{4}.$$

Opção correta: **(B)**

15. $(\operatorname{sen}\alpha+\cos\alpha)^2=\frac{1}{3} \Leftrightarrow \operatorname{sen}^2\alpha+2\operatorname{sen}\alpha\cos\alpha+\cos^2\alpha=\frac{1}{3}$

$$\Leftrightarrow 2\operatorname{sen}\alpha\cos\alpha=\frac{1}{3}-1 \Leftrightarrow \operatorname{sen}\alpha\cos\alpha=-\frac{1}{3}$$

$$\operatorname{tg}\alpha+\frac{1}{\operatorname{tg}\alpha}=\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}\alpha\cos\alpha}=\frac{1}{-\frac{1}{3}}=-3$$

Opção correta: **(A)**

PÁG. 82

Começar a preparar o exame

16. $\frac{(\pi-\alpha)\times 2^2}{2}-\frac{2\times 2\operatorname{sen}\alpha}{2}=2(\pi-\alpha)-2\operatorname{sen}\alpha=2(\pi-\alpha-\operatorname{sen}\alpha)$

Opção correta: **(B)**

17.1 Tem-se $B(\cos\theta\,,\,\operatorname{sen}\theta)$, $A(\cos\theta\,,\,0)$ e $\overline{AB}=-\operatorname{sen}\theta$.

Equação da circunferência centrada em A e que contém o ponto B :

$$(x-\cos\theta)^2+(y-0)^2=(-\operatorname{sen}\theta)^2 \Leftrightarrow (x-\cos\theta)^2+y^2=\operatorname{sen}^2\theta$$

O ponto $C(0\,,\,y_C)$, com $y_C<0$, pertence à circunferência centrada em A :

$$(0-\cos\theta)^2+{y_C}^2=\operatorname{sen}^2\theta \Leftrightarrow \cos^2\theta+{y_C}^2=\operatorname{sen}^2\theta \Leftrightarrow {y_C}^2=\operatorname{sen}^2\theta-\cos^2\theta \Leftrightarrow$$

$$\Leftrightarrow {y_C}^2=\operatorname{sen}^2\theta-\left(1-\operatorname{sen}^2\theta\right) \Leftrightarrow {y_C}^2=2\operatorname{sen}^2\theta-1 \underset{y_C<0\,\wedge\,2\operatorname{sen}^2\theta-1>0}{\Leftrightarrow} y_C=-\sqrt{2\operatorname{sen}^2\theta-1}$$

Nota: $\theta\in\left]-\frac{\pi}{2}\,,\,-\frac{\pi}{4}\right[$, pelo que $-1<\operatorname{sen}\theta<-\frac{\sqrt{2}}{2} \Leftrightarrow \frac{1}{2}<\operatorname{sen}^2\theta<1 \Leftrightarrow$

$$\Leftrightarrow 1<2\operatorname{sen}^2\theta<2 \Leftrightarrow 0<2\operatorname{sen}^2\theta-1<1\ .$$

17.2 $-\sqrt{2\operatorname{sen}^2\theta-1}=-\frac{1}{\sqrt{3}} \Leftrightarrow 2\operatorname{sen}^2\theta-1=\frac{1}{3} \Leftrightarrow \operatorname{sen}^2\theta=\frac{4}{6} \Leftrightarrow \operatorname{sen}^2\theta=\frac{2}{3}$

$$\operatorname{sen}^2\theta+\cos^2\theta=1 \Leftrightarrow \cos^2\theta=1-\frac{2}{3} \Leftrightarrow \cos^2\theta=\frac{1}{3}\,,\ \operatorname{tg}^2\theta=\frac{\frac{2}{3}}{\frac{1}{3}}=2$$

$$\left(\operatorname{tg}\left(\theta-\frac{\pi}{2}\right)+\cos\theta\right)\left(\operatorname{tg}\left(\theta-\frac{\pi}{2}\right)-\cos\theta\right)=\operatorname{tg}^2\left(\theta-\frac{\pi}{2}\right)-\cos^2\theta=$$

$$=\operatorname{tg}^2\left(\theta-\frac{\pi}{2}\right)-\cos^2\theta=\left(-\operatorname{tg}\left(\frac{\pi}{2}-\theta\right)\right)^2-\cos^2\theta=\frac{1}{\operatorname{tg}^2\theta}-\cos^2\theta=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

PÁG. 83

Começar a preparar o exame

18.1 Tem-se $A(\cos\alpha\,,\,\operatorname{sen}\alpha)$ e $C(1\,,\,\operatorname{tg}\alpha)$; então:

$$A_{[ABC]}=\frac{\overline{AB}\times(y_A-y_C)}{2}=\frac{-2\cos\alpha\times(\operatorname{sen}\alpha-\operatorname{tg}\alpha)}{2}=-\cos\alpha\operatorname{sen}\alpha+\cos\alpha\operatorname{tg}\alpha=$$

$$=-\cos\alpha\operatorname{sen}\alpha+\cos\alpha\frac{\operatorname{sen}\alpha}{\cos\alpha}=-\cos\alpha\operatorname{sen}\alpha+\operatorname{sen}\alpha=\operatorname{sen}\alpha(1-\cos\alpha)$$

18.2 $(\operatorname{sen}\alpha-\cos\alpha)^2=1+\operatorname{sen}\alpha \Leftrightarrow \operatorname{sen}^2\alpha-2\operatorname{sen}\alpha\cos\alpha+\cos^2\alpha=1+\operatorname{sen}\alpha \Leftrightarrow$

$\Leftrightarrow -2\operatorname{sen}\alpha\cos\alpha-\operatorname{sen}\alpha=0 \Leftrightarrow \operatorname{sen}\alpha(2\cos\alpha+1)=0 \Leftrightarrow \operatorname{sen}\alpha=0 \vee 2\cos\alpha+1=0 \Leftrightarrow$

$\Leftrightarrow \operatorname{sen}\alpha=0 \vee \cos\alpha=-\frac{1}{2} \underset{\alpha\in\left]\frac{\pi}{2},\,\pi\right[}{\Leftrightarrow} \alpha=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

Área $=\operatorname{sen}\left(\frac{2\pi}{3}\right)\left(1-\cos\left(\frac{2\pi}{3}\right)\right)=\frac{\sqrt{3}}{2}\left(1+\frac{1}{2}\right)=\frac{3\sqrt{3}}{4}$

19. Tem-se $Q\left(\sqrt{6}\cos\alpha\,,\,\sqrt{6}\operatorname{sen}\alpha\right)$, $A\left(\sqrt{6}\cos\alpha\,,\,0\right)$

$P\left(-\sqrt{6}\,,\,-\sqrt{6}\operatorname{tg}\alpha\right)$, $C\left(0\,,\,-\sqrt{6}\operatorname{tg}\alpha\right)$ e $B\left(\sqrt{6}\cos\alpha\,,\,-\sqrt{6}\operatorname{tg}\alpha\right)$.

$$A_{[ABC]}=\sqrt{2} \Leftrightarrow \frac{\overline{AB}\times\overline{BC}}{2}=\sqrt{2} \Leftrightarrow \frac{\sqrt{6}\operatorname{tg}\alpha\times\left(-\sqrt{6}\cos\alpha\right)}{2}=\sqrt{2} \Leftrightarrow$$

$$\Leftrightarrow -6\operatorname{tg}\alpha\times\cos\alpha=2\sqrt{2} \Leftrightarrow \operatorname{sen}\alpha=-\frac{\sqrt{2}}{3}\,;$$

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1 \Leftrightarrow \cos^2\alpha=1-\left(-\frac{\sqrt{2}}{3}\right)^2 \Leftrightarrow \cos^2\alpha=\frac{7}{9} \underset{\alpha\in 3.^{\circ}\text{Q.}}{\Leftrightarrow} \cos\alpha=-\frac{\sqrt{7}}{3}\,;$$

$$\operatorname{tg}\alpha=\frac{\operatorname{sen}\alpha}{\cos\alpha}=\frac{-\frac{\sqrt{2}}{3}}{-\frac{\sqrt{7}}{3}}=\frac{\sqrt{2}}{\sqrt{7}}=\frac{\sqrt{14}}{7}\,;$$

$$\cos\left(\alpha+\frac{5\pi}{2}\right)-7\operatorname{tg}(\pi-\alpha)=\cos\left(\alpha+\frac{\pi}{2}\right)-7(-\operatorname{tg}\alpha)=-\operatorname{sen}\alpha+7\operatorname{tg}\alpha=$$

$$=-\left(-\frac{\sqrt{2}}{3}\right)+7\frac{\sqrt{14}}{7}=\frac{\sqrt{2}}{3}+\sqrt{14}\,.$$