6. Geometria analítica no plano

Volume 3

PÁG. 60

1.1 $\overline{AB}=\sqrt{(3+1)^2+(-2+2)^2}=4$

1.2 $\overline{CD}=\sqrt{(4-4)^2+(6+8)^2}=14$

1.3 $\overline{EF}=\sqrt{(-1-2)^2+(2-3)^2}=\sqrt{10}$

2.1 $\overline{AB}=\sqrt{(-1-3)^2+(2-12)^2}=\sqrt{116}$, $\overline{BC}=\sqrt{(3-9)^2+(12+2)^2}=\sqrt{232}$,

$\overline{AC}=\sqrt{(-1-9)^2+(2+2)^2}=\sqrt{116}$, o triângulo é isósceles.

2.2 $\overline{AB}^2+\overline{AC}^2=\sqrt{116}^2+\sqrt{116}^2=232$, $\overline{BC}^2=\sqrt{232}^2=232$, o triângulo é retângulo em A .

$\text{A}=\frac{\overline{AB}\times\overline{AC}}{2}=\frac{\sqrt{116}\times\sqrt{116}}{2}=58$

2.3 $D(x\ ,\ y)$,

$B=M_{[AD]} \Leftrightarrow (3\ ,\ 12)=\left(\frac{-1+x}{2}\ ,\ \frac{2+y}{2}\right) \Leftrightarrow \frac{-1+x}{2}=3 \wedge \frac{2+y}{2}=12 \Leftrightarrow x=7 \wedge y=22$, $D(7\ ,\ 22)$

3. $M_{[AC]}=\left(\frac{2+7}{2}\ ,\ \frac{-9+5}{2}\right)=\left(\frac{9}{2}\ ,\ -2\right)$ e $M_{[BD]}=\left(\frac{10-1}{2}\ ,\ \frac{-7+3}{2}\right)=\left(\frac{9}{2}\ ,\ -2\right)$

$\overline{AC}=\sqrt{(2-7)^2+(-9-15)^2}=\sqrt{221}$ e $\overline{BD}=\sqrt{(10+1)^2+(-7-3)^2}=\sqrt{221}$

As diagonais do quadrilátero bissetam-se e são iguais. Logo, o quadrilátero é um retângulo.

4.1 $C(x\ ,\ 0)$, com $x<0$

$\overline{AB}=\sqrt{(2+1)^2+(3-5)^2}=\sqrt{13}$, $\overline{BC}=\sqrt{(-1-x)^2+(5-0)^2}=\sqrt{x^2+2x+26}$,

$\overline{AC}=\sqrt{(2-x)^2+(3-0)^2}=\sqrt{x^2-4x+13}$

$\overline{AB}^2+\overline{BC}^2=\sqrt{13}^2+\sqrt{x^2+2x+26}^2=x^2+2x+39$, $\overline{AC}^2=\sqrt{x^2-4x+13}^2=x^2-4x+13$

$\overline{AB}^2+\overline{BC}^2=\overline{AC}^2 \Leftrightarrow x^2+2x+39=x^2-4x+13 \Leftrightarrow x=-\frac{13}{3}$

$C\left(-\frac{13}{3}\ ,\ 0\right)$

4.2 $D(0\ ,\ y)$: $\overline{AD}=\overline{CD}$

$\overline{AD}=\overline{CD} \Leftrightarrow \sqrt{(2-0)^2+(3-y)^2}=\sqrt{(-1-0)^2+(5-y)^2} \Rightarrow$

$\Rightarrow y^2-6y+13=y^2-10y+26 \Leftrightarrow y=\frac{13}{4}$

$D\left(0\ ,\ \frac{13}{4}\right)$

5. $A=\dfrac{\overline{AD}+\overline{BC}}{2}\times\overline{CD}$

$\overline{AD}=\sqrt{(-3+1)^2+(7+1)^2}=\sqrt{68}\ ,\ \ \overline{CD}=\sqrt{(3+1)^2+(0+1)^2}=\sqrt{17}$

$B(0\ ,\ y)\ ,\ \overline{BC}=\sqrt{(3-0)^2+(0-y)^2}=\sqrt{9+y^2}$

$\dfrac{\overline{AD}+\overline{BC}}{2}\times\overline{CD}=\dfrac{85}{2}\ \Leftrightarrow\ \dfrac{\sqrt{68}+\sqrt{9+y^2}}{2}\times\sqrt{17}=\dfrac{85}{2}\ \Leftrightarrow$

$\Leftrightarrow\ \left(2\sqrt{17}+\sqrt{9+y^2}\right)\sqrt{17}=85\ \Leftrightarrow\ \sqrt{17\left(9+y^2\right)}=85-34\ \Rightarrow\ 17\left(9+y^2\right)=51^2\ \Leftrightarrow$

$\Leftrightarrow\ 9+y^2=\dfrac{51^2}{17}\ \Leftrightarrow\ y^2=144\ \underset{y>0}{\Rightarrow}\ y=12$

PÁG. 61

6. $Q(x\ ,\ y)\ :\ x^2=4y$

$\overline{QP}=\sqrt{(x-0)^2+(y-1)^2}=\sqrt{4y+y^2-2y+1}=\sqrt{y^2+2y+1}=\sqrt{(y+1)^2}=y+1$

7.1 $x<3\vee y\geqslant 2$

7.2 $y\geqslant 0\wedge y\leqslant x$

7.3 $y\leqslant x\wedge y\geqslant -4\wedge x\leqslant 4$

7.4 $(x>-3\wedge y\leqslant 1)\vee(x=-3\wedge y=1)$

7.5 $y\geqslant -x\wedge x\leqslant 0$

7.6 $y\geqslant -x\wedge x\leqslant 2\wedge y\leqslant 3$

PÁG. 62

8. $A=18\ \Leftrightarrow\ \dfrac{(x-2)\times(x-2)}{2}=18\ \Leftrightarrow\ (x-2)^2=36\ \Leftrightarrow\ x-2=-6\vee x-2=6\ \Leftrightarrow\ x=-4\vee x=8$

$0\leqslant y\leqslant x\wedge 2\leqslant x\leqslant 8$

9. $A=64\ \Leftrightarrow\ \dfrac{\overline{OB}+\overline{AC}}{2}\times\overline{OA}=64\ \Leftrightarrow\ (4+(-x+4))(-x)=128\ \Leftrightarrow$

$\Leftrightarrow\ x^2-8x-128=0\ \Leftrightarrow\ x=\dfrac{8\pm\sqrt{(-8)^2-4(-128)}}{2}\ \Leftrightarrow\ x=\dfrac{8\pm 24}{2}\ \Leftrightarrow\ x=-8\vee x=16$

$-8\leqslant x\leqslant 0\wedge 0\leqslant y\leqslant -x+4$

10. $(y\geqslant x\wedge y\leqslant -x\wedge -4\leqslant x\leqslant 0)\vee(y\geqslant -x\wedge y\leqslant x\wedge 0\leqslant x\leqslant 4)$

11.1 $(x-3)^2+(y+2)^2=(x+1)^2+(y+2)^2\ \Leftrightarrow\ x^2-6x+9=x^2+2x+1\ \Leftrightarrow\ x=1$

11.2 $(x-4)^2+(y-6)^2=(x-4)^2+(y+8)^2\ \Leftrightarrow\ -12y+36=16y+64\ \Leftrightarrow$

$\Leftrightarrow\ -12y-16y=64-36\ \Leftrightarrow\ y=-1$

11.3 $(x+3)^2+(y-1)^2=(x-1)^2+(y+3)^2\ \Leftrightarrow\ x^2+6x+9+y^2-2y+1=x^2-2x+1+y^2+6y+9\ \Leftrightarrow$

$\Leftrightarrow\ 6x-2y=-2x+6y\ \Leftrightarrow\ y=x$

11.4 $(x+1)^2+(y-2)^2=(x-2)^2+(y-3)^2 \iff x^2+2x+1+y^2-4y+4=x^2-4x+4+y^2-6y+9 \iff$

$\iff 2x+1-4y=-4x-6y+9 \iff y=-3x+4$

12.1 $\overline{AB}=\sqrt{(-2-3)^2+(6+5)^2}=\sqrt{146}$

12.2 $M_{[AB]}=\left(\frac{-2+3}{2}, \frac{6-5}{2}\right)=\left(\frac{1}{2}, \frac{1}{2}\right)$

12.3 $(x+2)^2+(y-6)^2=(x-3)^2+(y+5)^2 \iff$

$\iff x^2+4x+4+y^2-12y+36=x^2-6x+9+y^2+10y+25 \iff$

$\iff -12y-10y=-6x-4x+34-40 \iff -22y=-10x-6 \iff 11y=5x+3$

12.4 $C(x, 0): \overline{AC}=\overline{BC}$

$\overline{AC}=\overline{BC} \iff \sqrt{(-2-x)^2+(6-0)^2}=\sqrt{(3-x)^2+(-5-0)^2} \implies$

$\implies 4+4x+x^2+36=9-6x+x^2+25 \iff 10x=34-40 \iff x=-\frac{3}{5}$

$C\left(-\frac{3}{5}, 0\right)$

PÁG. 63

13. $(x-3)^2+(y-4)^2=(x-5)^2+(y-6)^2 \iff$

$\iff x^2-6x+9+y^2-8y+16=x^2-10x+25+y^2-12y+36 \iff$

$\iff -8y+12y=-10x+6x+61-25 \iff y=-x+9$

$x \geqslant 2 \wedge y \geqslant 0 \wedge y \leqslant -x+9$

14.1 $4k+2(1-k)=1 \iff 2k=-1 \iff k=-\frac{1}{2}$

14.2 $\overline{AC}=\overline{BC}$, porque as coordenadas de C satisfazem a equação da mediatriz de $[AB]$; portanto, $[ABC]$ é isósceles.

14.3 $M_{[AB]}=\left(\frac{-4+6}{2}, \frac{y_A+y_B}{2}\right)=\left(1, \frac{y_A+y_B}{2}\right)$

$4\times 1+2\left(\frac{y_A+y_B}{2}\right)=1 \iff \frac{y_A+y_B}{2}=-\frac{3}{2}$

Logo, $M_{[AB]}=\left(1, -\frac{3}{2}\right)$

15. $Q(x, y)$, $M=M_{[PQ]} \iff (2, -3)=\left(\frac{1+x}{2}, \frac{6+y}{2}\right) \iff$

$\iff \frac{1+x}{2}=2 \wedge \frac{6+y}{2}=-3 \iff x=3 \wedge y=-12$

$(x-1)^2+(y-6)^2=(x-3)^2+(y+12)^2 \iff x^2-2x+1+y^2-12y+36=x^2-6x+9+y^2+24y+144 \iff$

$\iff -12y-24y=-6x+2x+153-37 \iff -36y=-4x+116 \iff y=\frac{1}{9}x-\frac{29}{9}$

16. $A(x_A\ ,\ 2x_A)\ ,\ x_A>0\ ;\ B(x_B\ ,\ x_B)\ ,\ x_B>0$ e $y_B>0$

$$(x-x_A)^2+(y-2x_A)^2=(x-x_B)^2+(y-y_B)^2 \Leftrightarrow$$

$$\Leftrightarrow x^2-2xx_A+x_A^2+y^2-4x_Ay+4x_A^2=x^2-2xx_B+x_B^2+y^2-2yy_B+y_B^2 \Leftrightarrow$$

$$\Leftrightarrow -4x_Ay+2yy_B=2xx_A-2xx_B-5x_A^2+x_B^2+y_B^2 \Leftrightarrow$$

$$\Leftrightarrow (-4x_A+2y_B)y=(2x_A-2x_B)x-5x_A^2+x_B^2+y_B^2 \Leftrightarrow$$

$$\Leftrightarrow y=\frac{2x_A-2x_B}{-4x_A+2y_B}x+\frac{-5x_A^2+x_B^2+y_B^2}{-4x_A+2y_B} \Leftrightarrow y=\frac{x_A-x_B}{-2x_A+y_B}x+\frac{-5x_A^2+x_B^2+y_B^2}{-4x_A+2y_B}$$

Como a bissetriz dos quadrantes ímpares é a mediatriz de $[AB]$, tem-se

$$\frac{x_A-x_B}{-2x_A+y_B}=1 \text{ e } \frac{-5x_A^2+x_B^2+y_B^2}{-4x_A+2y_B}=0$$

$$\frac{x_A-x_B}{-2x_A+y_B}=1 \Leftrightarrow x_A-x_B=-2x_A+y_B \Leftrightarrow 3x_A=x_B+y_B \quad \textbf{(I)}$$

$$\frac{-5x_A^2+x_B^2+y_B^2}{-4x_A+2y_B}=0 \Leftrightarrow -5x_A^2+x_B^2+y_B^2=0 \Leftrightarrow 5x_A^2=x_B^2+y_B^2$$

Por outro lado,

$$\overline{AB}=4 \Leftrightarrow \sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=4 \Leftrightarrow \sqrt{(x_A-x_B)^2+(2x_A-y_B)^2}=4 \Longrightarrow$$

$$\Longrightarrow x_B^2-2x_Ax_B+x_B^2+4x_A^2-4x_Ay_B+y_B^2=16 \Leftrightarrow 5x_A^2-2x_Ax_B-4x_Ay_B+x_B^2+y_B^2=16 \quad \textbf{(II)}$$

O ponto médio de $[AB]$,

$$M_{[AB]}=\left(\frac{x_A+x_B}{2}\ ,\ \frac{y_A+y_B}{2}\right)=\left(\frac{x_A+x_B}{2}\ ,\ \frac{2x_A+y_B}{2}\right) \text{ pertence à mediatriz de } [AB]:$$

$$\frac{2x_A+y_B}{2}=\frac{x_A+x_B}{2} \Leftrightarrow x_A=x_B-y_B$$

Substituindo em **(I)**, vem $3(x_B-y_B)=x_B+y_B \Leftrightarrow 3x_B-3y_B=x_B+y_B \Leftrightarrow x_B=2y_B$

Substituindo em **(I)**, vem $3x_A=2y_B+y_B \Leftrightarrow x_A=y_B$

Substituindo estas duas igualdades em **(II)**, vem

$$5y_B^2-2y_B2y_B-4y_By_B+(2y_B)^2+y_B^2=16 \Leftrightarrow$$

$$5y_B^2-4y_B^2-4y_B^2+4y_B^2+y_B^2=16 \Leftrightarrow 2y_B^2=16 \Leftrightarrow y_B^2=8 \underset{y_B>0}{\Longrightarrow} y_B=\sqrt{8}=2\sqrt{2}$$

Assim, $x_B=2\times2\sqrt{2}=4\sqrt{2}$, $x_A=2\sqrt{2}$ e $y_A=2\times2\sqrt{2}=4\sqrt{2}$.

Logo, $A(2\sqrt{2}\ ,\ 4\sqrt{2})$ e $B(4\sqrt{2}\ ,\ 2\sqrt{2})$.

17.1 $(x+3)^2+(y-2)^2=16 \Leftrightarrow (x-(-3))^2+(y-2)^2=4^2$

Centro: $(-3\ ,\ 2)$, raio: 4

17.2 $(x-9)^2+(y+8)^2=20 \Leftrightarrow (x-9)^2+(y-(-8))^2=\sqrt{20}^2$

Centro: $(9\ ,\ -8)$, raio: $\sqrt{20}=2\sqrt{5}$

17.3 $x^2+(y+1)^2=9 \Leftrightarrow (x-0)^2+(y-(-1))^2=3^2$

Centro: $(0\ ,\ -1)$, raio: 3

17.4 $x^2+y^2=8 \Leftrightarrow (x-0)^2+(y-0)^2=\sqrt{8}^2$

Centro: $(0\ ,\ 0)$, raio: $\sqrt{8}=2\sqrt{2}$

PÁG. 64

18.1 $(x-(-3))^2+(y-1)^2=2^2 \Leftrightarrow (x+3)^2+(y-1)^2=9$

18.2 Raio: $\sqrt{(4-0)^2+(-1-0)^2}=\sqrt{17}$

$(x-0)^2+(y-0)^2=\sqrt{17}^2 \Leftrightarrow x^2+y^2=17$

18.3 Centro: $M_{[AB]}=\left(\frac{1-1}{2}\ ,\ \frac{-4+2}{2}\right)=(0\ ,\ -1)$

Raio: $\frac{\overline{AB}}{2}=\frac{\sqrt{(1+1)^2+(-4-2)^2}}{2}=\frac{\sqrt{40}}{2}=\frac{2\sqrt{10}}{2}=\sqrt{10}$

$(x-0)^2+(y-(-1))^2=\sqrt{10}^2 \Leftrightarrow x^2+(y+1)^2=10$

18.4 $(x-2)^2+(y-(-4))^2=4^2 \Leftrightarrow (x-2)^2+(y+4)^2=16$

18.5 $(x-2)^2+(y-(-4))^2=2^2 \Leftrightarrow (x-2)^2+(y+4)^2=4$

19.1 $(x+3)^2+(y-2)^2\leqslant 16 \Leftrightarrow (x-(-3))^2+(y-2)^2\leqslant 4^2$

Círculo com centro de coordenadas $(-3\ ,\ 2)$ e raio 4 .

19.2 $(x+9)^2+(y-8)^2\leqslant 20 \Leftrightarrow (x-(-9))^2+(y-8)^2\leqslant\sqrt{20}^2$

Círculo com centro de coordenadas $(-9\ ,\ 8)$ e raio $\sqrt{20}=2\sqrt{5}$.

19.3 $x^2+(y+1)^2>25 \Leftrightarrow (x-0)^2+(y-(-1))^2>5^2$

Conjunto de pontos do plano exteriores à circunferência com centro de coordenadas $(0\ ,\ -1)$ e raio 5 .

19.4 $x^2+y^2<8 \Leftrightarrow (x-0)^2+(y-0)^2<\sqrt{8}^2$

Conjunto de pontos do plano interiores à circunferência com centro na origem e raio $\sqrt{8}=2\sqrt{2}$.

20.1 $(x-0)^2+(y-0)^2\leqslant 10^2 \Leftrightarrow x^2+y^2\leqslant 100$

20.2 $(x-(-1))^2+(y-(-1))^2\leqslant 1^2 \Leftrightarrow (x+1)^2+(y+1)^2\leqslant 1$

20.3 $(x-1)^2+(y-(-1))^2>\sqrt{3}^2 \Leftrightarrow (x-1)^2+(y+1)^2>3$

21.1 $(x+1)^2+(y-2)^2=9 \Leftrightarrow (x-(-1))^2+(y-2)^2=3^2$

Centro: $(-1\ ,\ 2)$, raio: 3

21.2 $(-2+1)^2+(5-2)^2=10>9$, o ponto pertence ao exterior da circunferência.

21.3 $(x+1)^2+(y-2)^2=9 \wedge y=0 \Leftrightarrow (x+1)^2+(0-2)^2=9 \wedge y=0 \Leftrightarrow$

$\Leftrightarrow (x+1)^2=5 \wedge y=0 \Leftrightarrow x+1=-\sqrt{5} \vee x+1=\sqrt{5} \wedge y=0 \Leftrightarrow x=-1-\sqrt{5} \vee x=-1+\sqrt{5} \wedge y=0$

$\left(-1-\sqrt{5}\ ,\ 0\right)$ e $\left(-1+\sqrt{5}\ ,\ 0\right)$

21.4 $(x+1)^2+(y-2)^2>9 \wedge x>0 \wedge y>0$

PÁG. 65

22.1 $A=\overline{AB}^2$ e $\overline{AC}^2=\overline{AD}^2+\overline{DC}^2=2\overline{AD}^2=2\overline{AB}^2$

$$\overline{AC}=\sqrt{\left(\frac{5}{2}-\frac{1}{2}\right)^2+\left(\frac{13}{2}-\frac{1}{2}\right)^2}=\sqrt{40}$$

$$A=\frac{\overline{AC}^2}{2}=\frac{\sqrt{40}^2}{2}=20$$

22.2 Centro: $M_{[AC]}=\left(\frac{\frac{5}{2}+\frac{1}{2}}{2}\ ,\ \frac{\frac{13}{2}+\frac{1}{2}}{2}\right)=\left(\frac{3}{2}\ ,\ \frac{7}{2}\right)$

Raio: $\frac{\overline{AC}}{2}=\frac{\sqrt{\left(\frac{5}{2}-\frac{1}{2}\right)^2+\left(\frac{13}{2}-\frac{1}{2}\right)^2}}{2}=\frac{\sqrt{40}}{2}=\frac{2\sqrt{10}}{2}=\sqrt{10}$

$$\left(x-\frac{3}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\sqrt{10}^2 \Leftrightarrow \left(x-\frac{3}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=10$$

22.3 O ponto B pertence à mediatriz de $[AC]$ porque as diagonais de um quadrado são perpendiculares.

$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{13}{2}\right)^2=\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2 \Leftrightarrow x^2-5x+\frac{25}{4}+y^2-13y+\frac{169}{4}=x^2-x+\frac{1}{4}+y^2-y+\frac{1}{4} \Leftrightarrow$

$\Leftrightarrow -13y+y=-x+5x+\frac{2}{4}-\frac{194}{4} \Leftrightarrow -12y=4x+-48 \Leftrightarrow y=-\frac{1}{3}x+4$

$\frac{9}{2}=-\frac{1}{3}x+4 \Leftrightarrow 27=-2x+24 \Leftrightarrow 2x=-3 \Leftrightarrow x=-\frac{3}{2}$

23. $A(-1\ ,\ 2)$, $B(-1\ ,\ -3)$, $C(4\ ,\ -3)$, $D(4\ ,\ 2)$

23.1 Centro: $M_{[AC]}=\left(\frac{-1+4}{2}\ ,\ \frac{2-3}{2}\right)=\left(\frac{3}{2}\ ,\ -\frac{1}{2}\right)$

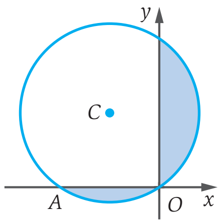
Raio: $\frac{\overline{AC}}{2}=\frac{\sqrt{(-1-4)^2+(2+3)^2}}{2}=\frac{\sqrt{50}}{2}=\frac{5\sqrt{2}}{2}$

23.2 $\left(x-\frac{3}{2}\right)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2=\left(\frac{\sqrt{50}}{2}\right)^2 \Leftrightarrow \left(x-\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{2}$

$\left(x-\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2\leqslant\frac{25}{2}\wedge x\geqslant 0\wedge y\geqslant 2$

24.1 $\overline{CO}=\sqrt{(-2-0)^2+(3-0)^2}=\sqrt{13}$

24.2



24.3 $(x+2)^2+(y-3)^2=13\wedge y=0 \Leftrightarrow (x+2)^2+(0-3)^2=13\wedge y=0 \Leftrightarrow$

$\Leftrightarrow (x+2)^2=4\wedge y=0 \Leftrightarrow x+2=-2\vee x+2=2\wedge y=0 \Leftrightarrow x=-4\vee x=0\wedge y=0$

$A(-4\ ,\ 0)$

$A=\frac{\overline{AO}\times y_C}{2}=\frac{4\times 3}{2}=6$

PÁG. 66

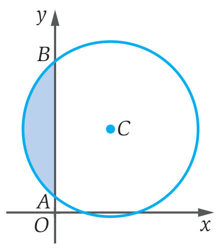
25.1 $(x-2)^2+(y-3)^2=10\wedge x=0 \Leftrightarrow (0-2)^2+(y-3)^2=10\wedge x=0 \Leftrightarrow$

$\Leftrightarrow (y-3)^2=6\wedge x=0 \Leftrightarrow y-3=-\sqrt{6}\vee y-3=\sqrt{6}\wedge x=0 \Leftrightarrow y=3-\sqrt{6}\vee y=3+\sqrt{6}\wedge x=0$

$A\left(0\ ,\ 3-\sqrt{6}\right)$ e $B\left(0\ ,\ 3+\sqrt{6}\right)$

$A=\frac{\overline{AB}\times x_C}{2}=\frac{\left[\left(3+\sqrt{6}\right)-\left(3-\sqrt{6}\right)\right]\times 2}{2}=2\sqrt{6}$

25.2



26.1 $-1\leqslant x\leqslant 4\wedge -4\leqslant y\leqslant 1$

26.2 Centro: $M_{[AC]}=\left(\frac{-1+4}{2}\ ,\ \frac{1-4}{2}\right)=\left(\frac{3}{2}\ ,\ -\frac{3}{2}\right)$

Raio: $\frac{\overline{AC}}{2}=\frac{\sqrt{(-1-4)^2+(1+4)^2}}{2}=\frac{\sqrt{50}}{2}=\frac{5\sqrt{2}}{2}$

$\left(x-\frac{3}{2}\right)^2+\left(y-\left(-\frac{3}{2}\right)\right)^2=\left(\frac{\sqrt{50}}{2}\right)^2 \Leftrightarrow \left(x-\frac{3}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{25}{4}$

27.1 $(x-2)^2+(y+1)^2 \leqslant 9 \wedge [(x \geqslant 0 \wedge y \geqslant x) \vee (x \leqslant 0 \wedge y \leqslant x)]$

27.2 $(5-2)^2+(0+1)^2=10 \neq 9$, o ponto não pertence à circunferência.

27.3 $(x-2)^2+(y+1)^2=9 \wedge y=x \Leftrightarrow (x-2)^2+(x+1)^2=9 \wedge y=x \Leftrightarrow$

$\Leftrightarrow x^2-4x+4+x^2+2x+1=9 \wedge y=x \Leftrightarrow 2x^2-2x-4=0 \wedge y=x \Leftrightarrow$

$2x^2-2x-4=0 \Leftrightarrow x^2-x-2=0 \Leftrightarrow x=\dfrac{1 \pm \sqrt{(-1)^2-4(-2)}}{2} \Leftrightarrow x=\dfrac{1 \pm 3}{2} \Leftrightarrow x=-1 \vee x=2$

$A(2, 2)$ e $B(-1, -1)$

27.4 $\overline{AC}=\sqrt{(2-2)^2+(2+1)^2}=3$, $\overline{BC}=\sqrt{(-1-2)^2+(-1+1)^2}=3$,

$\overline{AB}=\sqrt{(2+1)^2+(2+1)^2}=\sqrt{18}=3\sqrt{2}$

$\overline{AB}^2=\sqrt{18}^2=18$ e $\overline{AC}^2+\overline{CC}^2=3^2+3^2=18$, o triângulo é retângulo.

PÁG. 67

28.1 Equação da mediatriz do segmento de reta $[AB]$:

$(x-1)^2+(y-1)^2=(x+2)^2+(y-4)^2 \Leftrightarrow x^2-2x+1+y^2-2y+1=x^2+4x+4+y^2-8y+16 \Leftrightarrow$

$\Leftrightarrow -2y+8y=4x+2x+20-2 \Leftrightarrow 6y=6x+18 \Leftrightarrow y=x+3$

Interseção com a circunferência:

$x^2+(y-2)^2=13 \wedge y=x+3 \Leftrightarrow x^2+(x+3-2)^2=13 \wedge y=x+3 \Leftrightarrow$

$\Leftrightarrow x^2+(x+1)^2=13 \wedge y=x+3 \Leftrightarrow 2x^2+2x-12=0 \wedge y=x+3 \Leftrightarrow$

$2x^2+2x-12=0 \Leftrightarrow x^2+x-6=0 \Leftrightarrow x=\dfrac{-1 \pm \sqrt{1^2-4(-6)}}{2} \Leftrightarrow x=\dfrac{-1 \pm 5}{2} \Leftrightarrow x=-3 \vee x=2$

$P(-3, y_P)$ e $Q(2, y_Q)$

$y_P=-3+3=0$ e $y_Q=2+3=5$

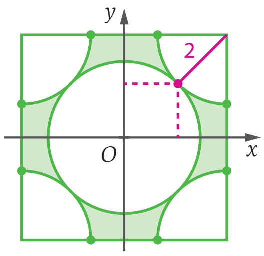
$P(-3, 0)$ e $Q(2, 5)$

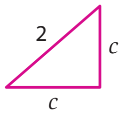
$\overline{PQ}=\sqrt{(-3-2)^2+(0-5)^2}=\sqrt{50}=5\sqrt{2}$

28.2 $x^2+(y-2)^2 \leqslant 13 \wedge y \geqslant -x+2$

29.1 Como o quadrado tem área igual a 36, a medida do seu lado é 6 e como está centrado na origem, as coordenadas dos seus vértices são $(3, 3)$, $(-3, 3)$, $(-3, -3)$ e $(3, -3)$.

29.2 As diagonais do quadrado estão contidas nas bissetrizes dos quadrantes.





$2^2 = c^2 + c^2 \Leftrightarrow 2c^2 = 4 \Leftrightarrow c^2 = 2 \underset{c>0}{\Longrightarrow} c = \sqrt{2}$

As coordenadas dos pontos de tangência da circunferência com os arcos são:

$(3-\sqrt{2}, 3-\sqrt{2})$, $(-3+\sqrt{2}, 3-\sqrt{2})$, $(-3+\sqrt{2}, -3+\sqrt{2})$ e $(3-\sqrt{2}, -3+\sqrt{2})$.

29.3 Raio: $r^2 = (3-\sqrt{2})^2 + (3-\sqrt{2})^2 \Leftrightarrow r^2 = 2(9 - 6\sqrt{2} + 2) \Leftrightarrow r^2 = 22 - 12\sqrt{2}$

Equação da circunferência: $x^2 + y^2 = 22 - 12\sqrt{2}$

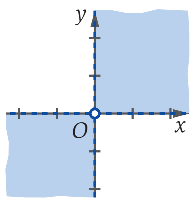
29.4 $x^2 + y^2 \geqslant 22 - 12\sqrt{2} \wedge$

$\wedge (x-3)^2 + (y-3)^2 \geqslant 4 \wedge (x-3)^2 + (y+3)^2 \geqslant 4 \wedge (x+3)^2 + (y+3)^2 \geqslant 4 \wedge (x+3)^2 + (y-3)^2 \geqslant 4 \wedge$

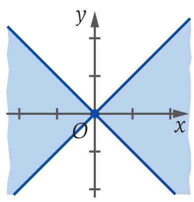
$\wedge x \geqslant -3 \wedge y \geqslant -3 \wedge x \leqslant 3 \wedge y \leqslant 3$

29.5 $A = 36 - \pi(22 - 12\sqrt{2}) - \pi \times 2^2 = 36 - 26\pi + 12\sqrt{2}\pi$

30.1 $xy > 0 \Leftrightarrow (x > 0 \wedge y > 0) \vee (x < 0 \wedge y < 0)$



30.2 $y^2 - x^2 \leqslant 0 \Leftrightarrow (y \leqslant x \wedge y \geqslant -x) \vee (y \geqslant x \wedge y \leqslant -x)$



PÁG. 68

31. $(x-4)^2+(y+1)^2=r^2 \Leftrightarrow x^2-8x+y^2+2y+17=r^2$ **(I)**

$(x-9)^2+(y-0)^2=r^2 \Leftrightarrow x^2-18x+y^2+81=r^2$ **(II)**

$(x-4)^2+(y-5)^2=r^2 \Leftrightarrow x^2-8x+y^2-10y+41=r^2$ **(III)**

De **(III)** e **(II)**, vem

$x^2-8x+y^2+2y+17=x^2-18x+y^2+81 \Leftrightarrow 10x+2y=64 \Leftrightarrow y=32-5x$ **(IV)**

De **(I)** e **(II)**, vem

$x^2-8x+y^2-10y+41=x^2-18x+y^2+81 \Leftrightarrow 10x-10y=40 \Leftrightarrow x-y=4$ **(V)**

De **(IV)** e **(V)**, vem

$x-(32-5x)=4 \Leftrightarrow x-32+5x=4 \Leftrightarrow x=6$

Substituindo em **(IV)**, vem $y=32-5\times 6=2$.

O centro da circunferência tem coordenadas $(6\ ,\ 2)$.

O raio da circunferência é tal que $(6-9)^2+(2-0)^2=r^2 \Leftrightarrow r^2=13$.

Equação da circunferência: $(x-6)^2+(y-2)^2=13$

32.1 $A(-3,3)$, $B(2,5)$, r é a mediatriz de $[AB]$:

$(x+3)^2+(y-3)^2=(x-2)^2+(y-5)^2 \Leftrightarrow$

$\Leftrightarrow x^2+6x+9+y^2-6y+9=x^2-4x+4+y^2-10y+25 \Leftrightarrow$

$\Leftrightarrow -6y+10y=-4x-6x+29-18 \Leftrightarrow 4y=-10x+11 \Leftrightarrow y=-\frac{5}{2}x+\frac{11}{4}$

32.2 Ordenada de C: $y=-\frac{5}{2}\times\frac{1}{2}+\frac{11}{4}=-\frac{5}{4}+\frac{11}{4}=\frac{6}{4}=\frac{3}{2}$

Raio: $\overline{AC}=\sqrt{\left(-3-\frac{1}{2}\right)^2+\left(3-\frac{3}{2}\right)^2}=\sqrt{\frac{49}{4}+\frac{9}{4}}=\sqrt{\frac{58}{4}}=\sqrt{\frac{29}{2}}$

Equação da circunferência: $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{29}{2}$

32.3 Ordenada de P: $y=-\frac{5}{2}x+\frac{11}{4}$

a. $Q(x\ ,\ -x)$

$\overline{BQ}=13 \Leftrightarrow \sqrt{(2-x)^2+(5+x)^2}=13 \Leftrightarrow \sqrt{2x^2+6x+29}=13 \Rightarrow$

$\Rightarrow 2x^2+6x+29=169 \Leftrightarrow 2x^2+6x-140=0 \Leftrightarrow x^2+3x-70=0 \Leftrightarrow$

$\Leftrightarrow x^2+3x-70=0 \Leftrightarrow x=\frac{-3\pm\sqrt{3^2-4(-70)}}{2} \Leftrightarrow x=\frac{-3\pm 17}{2} \Leftrightarrow x=-10 \vee x=7$

$Q(7\ ,\ -7)$ ou $Q(-10\ ,\ 10)$

b. $\overline{PQ}=5 \Leftrightarrow \sqrt{(x-x)^2+\left(-\frac{5}{2}x+\frac{11}{4}+x\right)^2}=5 \Leftrightarrow \sqrt{\left(-\frac{3}{2}x+\frac{11}{4}\right)^2}=5 \Rightarrow$

$\Rightarrow \left(-\frac{3}{2}x+\frac{11}{4}\right)^2=25 \Leftrightarrow \frac{9}{4}x^2-\frac{33}{4}x+\frac{121}{16}-25=0 \Leftrightarrow 36x^2-132x-279=0 \Leftrightarrow$

$\Leftrightarrow x=\frac{132\pm\sqrt{(-132)^2-4\times 36(-279)}}{2\times 36} \Leftrightarrow x=\frac{132\pm 240}{72} \Leftrightarrow x=-\frac{3}{2} \vee x=\frac{31}{6}$

$Q\left(-\frac{3}{2}, \frac{3}{2}\right)$

Ordenada de P: $y=-\frac{5}{2}\left(-\frac{3}{2}\right)+\frac{11}{4}=\frac{26}{4}=\frac{13}{2}$, $P\left(-\frac{3}{2}, \frac{13}{2}\right)$

32.4 $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 \leqslant \frac{29}{2} \wedge y \geqslant -x \wedge y \leqslant -\frac{5}{2}x+\frac{11}{4} \wedge x \leqslant 0$

PÁG. 69

33.1 $A(x_A, x_A)$, com $x_A>0$

$\overline{AO}=2\sqrt{2} \Leftrightarrow \sqrt{(x_A-0)^2+(x_A-0)^2}=2\sqrt{2} \Leftrightarrow \sqrt{2x_A^{\,2}}=2\sqrt{2} \Rightarrow$

$\Rightarrow 2x_A^2=8 \Leftrightarrow x_A^2=4 \underset{x_A>0}{\Rightarrow} x_A=2$

Logo, $A(2, 2)$ e a equação da circunferência é $(x-2)^2+(y-2)^2=8$.

33.2

a. $C(0, y_C)$, com $y_C>0$

$(0-2)^2+(y_C-2)^2=8 \Leftrightarrow (y_C-2)^2=4 \Leftrightarrow y_C-2=-2 \vee y_C-2=2 \Leftrightarrow y_C=0 \vee y_C=4$

Logo, $C(0, 4)$.

b. $B(-2, -1)$

$(x+2)^2+(y+1)^2=(x-0)^2+(y-4)^2 \Leftrightarrow$

$\Leftrightarrow x^2+4x+4+y^2+2y+1=x^2+y^2-8y+16 \Leftrightarrow$

$\Leftrightarrow 2y+8y+4x=16-5 \Leftrightarrow 10y+4x=11$

c. $10(a+1)+4\left(2a^2+\frac{3}{4}\right)=11 \Leftrightarrow 10a+10+8a^2+3-11=0 \Leftrightarrow$

$\Leftrightarrow 8a^2+10a+2=0 \Leftrightarrow 4a^2+5a+1=0 \Leftrightarrow$

$\Leftrightarrow a=\frac{-5\pm\sqrt{5^2-4\times 4}}{2\times 4} \Leftrightarrow a=\frac{-5\pm 3}{8} \Leftrightarrow a=-1 \vee a=-\frac{1}{4}$

33.3 Comecemos por determinar a equação reduzida da reta AB .

$m=\frac{2+1}{2+2}=\frac{3}{4}$, $2=\frac{3}{4}\times 2+b \Leftrightarrow b=2-\frac{3}{2} \Leftrightarrow b=\frac{1}{2}$ e $y=\frac{3}{4}x+\frac{1}{2}$

$(x+2)^2+(y+1)^2 \leqslant 10 \wedge (x-2)^2+(y-2)^2 \geqslant 8 \wedge y>\frac{3}{4}x+\frac{1}{2}$

7. Geometria analítica no espaço

Volume 3

PÁG. 70

1.1 $C(0\,,\ 0\,,\ 0)$, $B(4\,,\ 0\,,\ 0)$, $D(0\,,\ -4\,,\ 0)$, $H(0\,,\ 0\,,\ 4)$, $A(4\,,\ -4\,,\ 0)$, $G(4\,,\ 0\,,\ 4)$, $F(4\,,\ -4\,,\ 4)$, $E(0\,,\ -4\,,\ 4)$.

1.2 $A(2\,,\ -2\,,\ -2)$, $B(2\,,\ 2\,,\ -2)$, $G(2\,,\ 2\,,\ 2)$, $F(2\,,\ -2\,,\ 2)$, $C(-2\,,\ 2\,,\ -2)$, $D(-2\,,\ -2\,,\ -2)$, $H(-2\,,\ 2\,,\ 2)$, $E(-2\,,\ -2\,,\ 2)$.

1.3 $A(2\,,\ -4\,,\ -2)$, $B(2\,,\ 0\,,\ -2)$, $G(2\,,\ 0\,,\ 2)$, $F(2\,,\ -4\,,\ 2)$, $C(-2\,,\ 0\,,\ -2)$, $D(-2\,,\ -4\,,\ -2)$, $H(-2\,,\ 0\,,\ 2)$, $E(-2\,,\ -4\,,\ 2)$.

1.4 Como a aresta do cubo é 4 , a diagonal do quadrado é $\sqrt{4^2+4^2}=4\sqrt{2}$.
$A\left(2\sqrt{2}\,,\ 0\,,\ 0\right)$, $B\left(0\,,\ 2\sqrt{2}\,,\ 0\right)$, $C\left(-2\sqrt{2}\,,\ 0\,,\ 0\right)$, $D\left(0\,,\ -2\sqrt{2}\,,\ 0\right)$, $F\left(2\sqrt{2}\,,\ 0\,,\ 4\right)$, $G\left(0\,,\ 2\sqrt{2}\,,\ 4\right)$, $H\left(-2\sqrt{2}\,,\ 0\,,\ 4\right)$, $E\left(0\,,\ -2\sqrt{2}\,,\ 4\right)$.

2. **(B)**

PÁG. 71

3.1 $A(5\,,\ -6\,,\ 0)$, $B(5\,,\ 6\,,\ 0)$, $C(0\,,\ 6\,,\ 0)$, $D(0\,,\ -6\,,\ 0)$, $F(5\,,\ -6\,,\ 5)$, $G(5\,,\ 6\,,\ 5)$, $H(0\,,\ 6\,,\ 5)$, $E(0\,,\ -6\,,\ 5)$.

3.2 $ABC:\ z=0$, $ABF:\ x=5$, $ADE:\ y=-6$.

3.3 $FG:\ x=5\wedge z=5$, $HG:\ y=6\wedge z=5$, $AF:\ x=5\wedge y=-6$.

3.4 $[FEHG]:\ z=5\ \wedge\ 0\leqslant x\leqslant 5\ \wedge\ -6\leqslant y\leqslant 6$, $[CHGB]:\ y=6\ \wedge\ 0\leqslant x\leqslant 5\ \wedge\ 0\leqslant z\leqslant 5$.

3.5 $x=2\ \wedge\ -6\leqslant y\leqslant 6\ \wedge\ 0\leqslant z\leqslant 5$

3.6 Aresta $[DE]$.

4.1 $y=-2$

4.2 $z=-1$

4.3 Por exemplo, $x=1\wedge y=-2$.

4.4 $y=4\wedge z=-1$

4.5 $x=2\wedge z=-1$

5.1 $A(4\sqrt{2}\,,\ 0\,,\ -4)$, $F(4\sqrt{2}\,,\ 0\,,\ 4)$, $C(-4\sqrt{2}\,,\ 0\,,\ -4)$, $D(0\,,\ -4\sqrt{2}\,,\ -4)$, $E(0\,,\ -4\sqrt{2}\,,\ 4)$, $G(0\,,\ 4\sqrt{2}\,,\ 4)$.

5.2 $V = 8^3 = 512$

PÁG. 72

6.1 $O(0\,,\ 0\,,\ 0)$, $A(5\,,\ 0\,,\ 0)$, $C(0\,,\ 5\,,\ 0)$, $B(5\,,\ 5\,,\ 0)$, $G(0\,,\ 0\,,\ 5)$, $D(5\,,\ 0\,,\ 5)$, $F(0\,,\ 5\,,\ 5)$, $E(5\,,\ 5\,,\ 5)$.

6.2

a. $ADB:\ x=5$, $GDE:\ z=5$, $BCE:\ y=5$.

b. $AD:\ x=5 \wedge y=0$, $DE:\ x=5 \wedge z=5$, $BC:\ y=5 \wedge z=0$.

c. $x=5\ \wedge\ y5\ \wedge\ 0 \leqslant z \leqslant 5$

d. $z=0\ \wedge\ 0 \leqslant x \leqslant 5\ \wedge\ 0 \leqslant y \leqslant 5$

6.3 Equação do plano mediador de $[AF]$:

$(x-5)^2+(y-0)^2+(z-0)^2=(x-0)^2+(y-5)^2+(z-5)^2 \Leftrightarrow$

$\Leftrightarrow x^2-10x+25+y^2+z^2=x^2+y^2-10y+25+z^2-10z+25 \Leftrightarrow$

$\Leftrightarrow 10x-10y-10z+25=0 \Leftrightarrow 2x-2y-2z+5=0$

Os pontos da reta DG têm coordenadas $(x\,,\ 0\,,\ 5)$.

Substituindo na equação do plano mediador, $2x-2\times0-2\times5+5=0 \Leftrightarrow x=\frac{5}{2}$.

$\left(\frac{5}{2}\,,\ 0\,,\ 5\right)$

7.1 $\overline{AB}=\sqrt{(-1+1)^2+(2-2)^2+(2+1)^2}=3$

7.2 $\overline{CD}=\sqrt{(3+8)^2+(6-6)^2+(-4+4)^2}=11$

7.3 $\overline{EF}=\sqrt{(-1-2)^2+(2-3)^2+(0-1)^2}=\sqrt{11}$

8.1 $\overline{AB}=\sqrt{(2-5)^2+(1-2)^2+(-1-1)^2}=\sqrt{14}$

$\overline{BC}=\sqrt{(5-2)^2+(2+1)^2+(1-0)^2}=\sqrt{19}$, $\overline{AC}=\sqrt{(2-2)^2+(1+1)^2+(-1-0)^2}=\sqrt{5}$

O triângulo é escaleno.

8.2 $\overline{BC}^2=19$ e $\overline{AB}^2+\overline{AC}^2=14+5=19$, logo o triângulo é retângulo.

$A=\frac{\overline{AB}\times\overline{AC}}{2}=\frac{\sqrt{14}\times\sqrt{5}}{2}=\frac{\sqrt{70}}{2}$

9. $V = \overline{AB} \times \overline{BC} \times \overline{BG}$

$\overline{AB} = \sqrt{(2-1)^2 + (0-3)^2 + (-1+1)^2} = \sqrt{10}$

$\overline{BC} = \sqrt{(1-4)^2 + (3-4)^2 + (-1-3)^2} = \sqrt{26}$

$\overline{BG} = \sqrt{(1-7)^2 + (3-5)^2 + (-1+6)^2} = \sqrt{65}$.

$V = \sqrt{10} \times \sqrt{26} \times \sqrt{65} = 130$

PÁG. 73

10. $V = 32 \Leftrightarrow \frac{1}{3}\overline{AB} \times \overline{BC} \times \text{altura} = 32 \Leftrightarrow 4 \times 4 \times \text{altura} = 32 \times 3 \Leftrightarrow \text{altura} = \frac{32 \times 3}{4 \times 4} \Leftrightarrow \text{altura} = 6$

Como a base da pirâmide está contida no plano de equação $z = -1$, a cota do ponto V é $6 - 1 = 5$.

11.1 $E(x\,,\ 0\,,\ 0)$

$\overline{AB} = \sqrt{(1-3)^2 + (-1+4)^2 + (-1-0)^2} = \sqrt{14}$, $\overline{BE} = \sqrt{(3-x)^2 + (-4-0)^2 + (0-0)^2} = \sqrt{x^2 - 6x + 25}$,

$\overline{AE} = \sqrt{(1-x)^2 + (-1-0)^2 + (-1-0)^2} = \sqrt{x^2 - 2x + 3}$

$\overline{AE}^2 = \overline{AB}^2 + \overline{BE}^2 \Leftrightarrow x^2 - 2x + 3 = 14 + x^2 - 6x + 25 \Leftrightarrow -2x + 6x = 14 + 25 - 3 \Leftrightarrow x = 9$

Logo, $E(9\,,\ 0\,,\ 0)$.

11.2 $F(0\,,\ y\,,\ 0)$: $\overline{AF} = \overline{BF}$

$\overline{AF} = \overline{BF} \Leftrightarrow \sqrt{(1-0)^2 + (-1-y)^2 + (-1-0)^2} = \sqrt{(3-0)^2 + (-4-y)^2 + (0-0)^2} \Rightarrow$

$\Rightarrow y^2 + 2y + 3 = y^2 + 8y + 25 \Leftrightarrow 2y - 8y = 25 - 3 \Leftrightarrow y = -\frac{11}{3}$

Logo, $F\left(0\,,\ -\frac{11}{3}\,,\ 0\right)$.

11.3 Verdadeira, pois o plano ACD é definido por $y = -1$, portanto é perpendicular a Oy .

11.4 $x = 3 \wedge z = 0$

12. Designando por l a medida do lado do quadrado, tem-se $l^2 = 8$.

Sendo $[PQ]$ a diagonal do quadrado,

tem-se $\overline{PQ}^2 = l^2 + l^2 \Leftrightarrow \overline{PQ}^2 = 2l^2 \Leftrightarrow \overline{PQ}^2 = 2 \times 8 \Leftrightarrow \overline{PQ}^2 = 16 \underset{\overline{PQ} > 0}{\Rightarrow} \overline{PQ} = 4$

$\overline{PQ} = 4 \Leftrightarrow \sqrt{(2-2)^2 + (-1-1)^2 + (a+1-1+a)^2} = 4 \Rightarrow$

$\Rightarrow 4a^2 + 4 = 16 \Leftrightarrow a^2 = 3 \Leftrightarrow a = -\sqrt{3} \vee a = \sqrt{3}$

13.1 $x = \frac{3-1}{2} \Leftrightarrow x = 1$

13.2 $y = \frac{6-8}{2} \Leftrightarrow y = -1$

13.3 $z = \frac{2+6}{2} \Leftrightarrow z = 4$

13.4 $(x+1)^2 + (y-2)^2 + (z-0)^2 = (x+1)^2 + (y-3)^2 + (z-1)^2 \Leftrightarrow$

$\Leftrightarrow -4y + 4 = -6y + 9 - 2z + 1 \Leftrightarrow -4y + 6y + 2z = 9 + 1 - 4 \Leftrightarrow 2y + 2z = 6 \Leftrightarrow y + z = 3$

13.5 $(x+3)^2+(y-1)^2+(z-2)^2=(x-1)^2+(y+3)^2+(z-2)^2 \Leftrightarrow$

$\Leftrightarrow 6x+9-2y+1=-2x+1+6y+9 \Leftrightarrow 6x+2x-2y-6y=0 \Leftrightarrow 8x-8y=0 \Leftrightarrow x-y=0$

PÁG. 74

14.1

a. $\overline{AB}=\sqrt{(-2-3)^2+(6+2)^2+(1-0)^2}=\sqrt{90}=3\sqrt{10}$

b. $M_{[AB]}=\left(\frac{-2+3}{2}, \frac{6-2}{2}, \frac{1+0}{2}\right)=\left(\frac{1}{2}, 2, \frac{1}{2}\right)$

c. $C(x, y, z) : B=M_{[AC]}$

$B=M_{[AC]} \Leftrightarrow (3, -2,0)=\left(\frac{-2+x}{2}, \frac{6+y}{2}, \frac{1+z}{2}\right) \Leftrightarrow$

$\Leftrightarrow 3=\frac{-2+x}{2} \wedge -2=\frac{6+y}{2} \wedge 0=\frac{1+z}{2} \Leftrightarrow x=8 \wedge y=-10 \wedge z=-1$

Logo, $C(8, -10, -1)$.

14.2 $(x+2)^2+(y-6)^2+(z-1)^2=(x-3)^2+(y+2)^2+(z-0)^2 \Leftrightarrow$

$\Leftrightarrow 4x+4-12y+36-2z+1=-6x+9+4y+4 \Leftrightarrow 4x+6x-12y-4y-2z=9-36-1 \Leftrightarrow$

$\Leftrightarrow 10x-16y-2z=-28 \Leftrightarrow 5x-8y-z=-14$

15.1 $(x+3)^2+(y-2)^2+z^2=16 \Leftrightarrow (x-(-3))^2+(y-2)^2+(z-0)^2=4^2$

Centro: $(-3, 2, 0)$, raio: 4

15.2 $(x-9)^2+(y-8)^2+(z+1)^2=20 \Leftrightarrow (x-9)^2+(y-8)^2+(z-(-1))^2=\sqrt{20}^2$

Centro: $(9, 8, -1)$, raio: $\sqrt{20}=2\sqrt{5}$

15.3 $x^2+(y+1)^2+(z-1)^2=9 \Leftrightarrow (x-0)^2+(y-(-1))^2+(z-1)^2=3^2$

Centro: $(0, -1, 1)$, raio: 3

15.4 $x^2+y^2+z^2=8 \Leftrightarrow (x-0)^2+(y-0)^2+(z-0)^2=\sqrt{8}^2$

Centro: $(0, 0, 0)$, raio: $\sqrt{8}=2\sqrt{2}$

16.1 $(x-(-2))^2+(y-1)^2+(z-1)^2=2^2 \Leftrightarrow (x+2)^2+(y-1)^2+(z-1)^2=4$

16.2 Raio: $\overline{AB}=\sqrt{(-2-4)^2+(1-3)^2+(1-0)^2}=\sqrt{41}$

$(x-(-2))^2+(y-1)^2+(z-1)^2=\sqrt{41}^2 \Leftrightarrow (x+2)^2+(y-1)^2+(z-1)^2=41$

16.3 Centro: $M_{[AB]}=\left(\frac{-2+4}{2}, \frac{1+3}{2}, \frac{1+0}{2}\right)=\left(1, 2, \frac{1}{2}\right)$, raio: $\frac{\overline{AB}}{2}=\frac{\sqrt{41}}{2}$

$(x-1)^2+(y-2)^2+\left(z-\frac{1}{2}\right)^2 \leqslant \left(\frac{\sqrt{41}}{2}\right)^2 \Leftrightarrow (x-1)^2+(y-2)^2+\left(z-\frac{1}{2}\right)^2 \leqslant \frac{41}{4}$

16.4 $(x-(-2))^2+(y-1)^2+(z-1)^2=2^2 \Leftrightarrow (x+2)^2+(y-1)^2+(z-1)^2=4$

17.1 $(x+3)^2+(y-2)^2+z^2\leqslant 16 \iff (x-(-3))^2+(y-2)^2+(z-0)^2\leqslant 4^2$

Esfera com centro de coordenadas $(-3\,,\ 2\,,\ 0)$ e raio 4 .

17.2 $(x-9)^2+(y+8)^2+(z-1)^2\leqslant 20 \iff (x-9)^2+(y-(-8))^2+(z-1)^2\leqslant \sqrt{20}^2$

Esfera com centro de coordenadas $(9\,,\ -8\,,\ 1)$ e raio $\sqrt{20}=2\sqrt{5}$.

17.3 $x^2+(y+1)^2+z^2>25 \iff (x-0)^2+(y-(-1))^2+(z-0)^2>5^2$

Conjunto de pontos do espaço exteriores à superfície esférica com centro de coordenadas $(0\,,\ -1\,,\ 0)$ e raio 5 .

17.4 $x^2+y^2+z^2<8 \iff (x-0)^2+(y-0)^2+(z-0)^2<\sqrt{8}^2$

Conjunto de pontos do espaço interiores à superfície esférica com centro na origem e raio $\sqrt{8}=2\sqrt{2}$.

PÁG. 75

18.1 $(x+1)^2+(y-3)^2+z^2=10 \iff (x-(-1))^2+(y-3)^2+(z-0)^2=\sqrt{10}^2$

Centro: $(-1\,,\ 3\,,\ 0)$, raio: $\sqrt{10}$

18.2 $(0+1)^2+(0-3)^2+0^2=10 \iff 10=10$ proposição verdadeira, logo a origem do referencial pertence à superfície esférica.

18.3 $(2+1)^2+(3-3)^2+(-2)^2=13>10$, o ponto é exterior à superfície esférica.

18.4

a. $z=-\sqrt{10}\wedge z=\sqrt{10}$

b. $x=-1-\sqrt{10}\wedge x=-1+\sqrt{10}$

c. $y=3-\sqrt{10}\wedge y=3+\sqrt{10}$

19.1 $\overline{AB}=\sqrt{(-1-1)^2+(2-0)^2+(3-3)^2}=\sqrt{8}=2\sqrt{2}$,

$\overline{BC}=\sqrt{(1-2)^2+(0+1)^2+(3-3)^2}=\sqrt{2}$, $\overline{AC}=\sqrt{(-1-2)^2+(2+1)^2+(3-3)^2}=\sqrt{18}=3\sqrt{2}$

$P=2\sqrt{2}+\sqrt{2}+3\sqrt{2}=6\sqrt{2}$

19.2 $P(0\,,\ y\,,\ 0)$, $y>0$

$\overline{AP}=\sqrt{35} \iff \sqrt{(-1-0)^2+(2-y)^2+(3-0)^2}=\sqrt{35} \implies$

$\implies y^2-4y+14=35 \iff y^2-4y-21=0 \iff y=\dfrac{4\pm\sqrt{(-4)^2-4(-21)}}{2} \iff$

$\iff y=\dfrac{4\pm 10}{2} \iff y=-3\vee y=7 \underset{y>0}{\iff} y=7$

Logo, $P(0\,,\ 7\,,\ 0)$.

19.3 Centro: $M_{[BC]}=\left(\dfrac{1+2}{2}\,,\ \dfrac{0-1}{2}\,,\ \dfrac{3+3}{2}\right)=\left(\dfrac{3}{2}\,,\ -\dfrac{1}{2}\,,\ 3\right)$, raio: $\dfrac{\overline{BC}}{2}=\dfrac{\sqrt{2}}{2}$

$\left(x-\dfrac{3}{2}\right)^2+\left(y-\left(-\dfrac{1}{2}\right)\right)^2+(z-3)^2\leqslant\left(\dfrac{\sqrt{2}}{2}\right)^2 \iff \left(x-\dfrac{3}{2}\right)^2+\left(y+\dfrac{1}{2}\right)^2+(z-3)^2\leqslant\dfrac{1}{2}$

20.1 Círculo com centro de coordenadas $(-3\ ,\ 2\ ,\ 6)$ e raio 4, contido no plano de equação $z=6$.

20.2 Círculo com centro de coordenadas $(3\ ,\ 1\ ,\ 0)$ e raio 3, contido no plano de equação $x=3$.

20.3 $(x-2)^2+(y+2)^2+(z+3)^2=4 \wedge y=-1 \Leftrightarrow$

$\Leftrightarrow (x-2)^2+(-1+2)^2+(z+3)^2=4 \wedge y=-1 \Leftrightarrow (x-2)^2+(z+3)^2=3 \wedge y=-1$

Circunferência com centro de coordenadas $(2\ ,\ -1\ ,\ -3)$ e raio $\sqrt{3}$,
contida no plano de equação $y=-1$.

21.1 $(x+3)^2+(y-2)^2+(z-6)^2 \leqslant 16 \wedge z=6 \Leftrightarrow$

$\Leftrightarrow (x+3)^2+(y-2)^2+(6-6)^2 \leqslant 16 \wedge z=6 \Leftrightarrow (x+3)^2+(y-2)^2 \leqslant 16 \wedge z=6$

$A=16\pi$

21.2 $x^2+(y-1)^2+z^2 \leqslant 9 \wedge x=\sqrt{5} \Leftrightarrow$

$\Leftrightarrow \sqrt{5}^2+(y-1)^2+z^2 \leqslant 9 \wedge x=\sqrt{5} \Leftrightarrow (y-1)^2+z^2 \leqslant 4 \wedge x=\sqrt{5}$

$A=4\pi$

21.3 $(x-1)^2+(y+3)^2+(z-1)^2 \leqslant 12 \wedge y=-1 \Leftrightarrow$

$\Leftrightarrow (x-1)^2+(-1+3)^2+(z-1)^2 \leqslant 12 \wedge y=-1 \Leftrightarrow (x-1)^2+(z-1)^2 \leqslant 8 \wedge y=-1$

$A=8\pi$

21.4 $x^2+(y-1)^2+z^2 \leqslant 10 \wedge y=1 \Leftrightarrow$

$\Leftrightarrow x^2+(1-1)^2+z^2 \leqslant 10 \wedge y=1 \Leftrightarrow x^2+z^2 \leqslant 10 \wedge y=1$

$A=10\pi$

PÁG. 76

22.1 O ponto R tem coordenadas do tipo $(3\ ,\ y\ ,\ -1)$ e pertence ao plano mediador de $[PQ]$: $3+y+2(-1)=4 \Leftrightarrow y=3$.

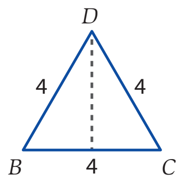
22.2 **(C)**

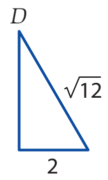
23.1 $[AB]$: $x=4 \wedge 0 \leqslant y \leqslant 4 \wedge z=0$, $[BC]$: $0 \leqslant x \leqslant 4 \wedge y=4 \wedge z=0$

23.2 $x=2 \wedge y=2$

23.3 BDO é perpendicular a $[AC]$ e contém o seu ponto médio, pelo que é o plano mediador de $[AC]$.

23.4 A superfície esférica que contém os oito vértices do sólido tem centro no centro do quadrado $[ABCD]$ e o raio é igual à altura da pirâmide $[OABCD]$.





A altura do triângulo $[BCD]$ é $\sqrt{4^2-2^2}=\sqrt{12}$ e a altura da pirâmide $[OABCD]$ é

$\sqrt{\sqrt{12}^2-2^2}=\sqrt{8}=2\sqrt{2}$.

Equação da superfície esférica: $(x-2)^2+(y-2)^2+z^2=8$

24.1 $V(0\,,\ 0\,,\ 2)$ e $\overline{VE}=\sqrt{12}$

24.2 Os pontos F e H são os pontos de interseção da superfície esférica com o eixo Oy: $0^2+y^2+(0-2)^2=12 \Leftrightarrow y^2=8 \Leftrightarrow y=-\sqrt{8} \vee y=\sqrt{8} \Leftrightarrow y=-2\sqrt{2} \vee y=2\sqrt{2}$

Logo, $H(0\,,\ -2\sqrt{2}\,,\ 0)$.

24.3 $(x-2\sqrt{2})^2+y^2+z^2=x^2+(y+2\sqrt{2})^2+z^2 \Leftrightarrow -4\sqrt{2}x+8=4\sqrt{2}y+8 \Leftrightarrow y=-x$

24.4 $V=\frac{1}{3}\times\overline{EF}^2\times 2$

$\overline{EF}^2=\overline{OE}^2+\overline{OF}^2 \Leftrightarrow \overline{EF}^2=(2\sqrt{2})^2+(2\sqrt{2})^2 \Leftrightarrow \overline{EF}^2=16$

$V=\frac{1}{3}\times 16\times 2=\frac{32}{3}$

PÁG. 77

25.1 $A=30\times 30\times 2+30\times 15\times 4=3600$

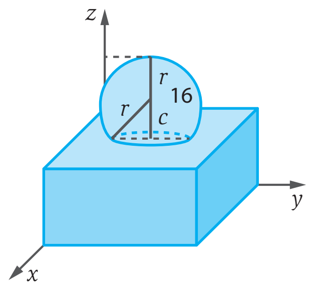
25.2 $x^2+y^2+z^2=(x-30)^2+(y-30)^2+(z-15)^2 \Leftrightarrow$

$\Leftrightarrow -60x+900-60y+900-30z+225=0 \Leftrightarrow -60x-60y-30z+2025=0 \Leftrightarrow$

$\Leftrightarrow 4x+4y+2z-135=0$

25.3 $0\leqslant x\leqslant 30 \wedge y=0 \wedge 0\leqslant z\leqslant 15$

25.4



$\begin{cases} r^2=c^2+8^2 \\ c+r=16 \end{cases} \Leftrightarrow \begin{cases} (16-c)^2=c^2+8^2 \\ r=16-c \end{cases} \Leftrightarrow \begin{cases} c=6 \\ r=10 \end{cases}$

$(x-15)^2+(y-15)^2+(z-21)^2\leqslant 100 \wedge z\geqslant 15$

26.1 O raio da superfície esférica é $1+\sqrt{3}-1=\sqrt{3}$ e a equação da superfície esférica é $(x-1)^2+(y-1)^2+(z-1)^2=3$.

Os pontos da superfície esférica que têm as três coordenadas iguais satisfazem a condição $(x-1)^2+(x-1)^2+(x-1)^2=3 \Leftrightarrow 3(x-1)^2=3 \Leftrightarrow (x-1)^2=1 \Leftrightarrow$

$\Leftrightarrow x-1=-1 \vee x-1=1 \Leftrightarrow x=0 \vee x=2$

Logo, as coordenadas desses pontos são $(0\,,\ 0\,,\ 0)$ e $(2\,,\ 2\,,\ 2)$.

26.2 O comprimento do segmento de reta é $2\sqrt{3}$, portanto, o dobro do raio $\sqrt{3}$; logo, é um diâmetro.

26.3 O diâmetro da superfície esférica é a diagonal espacial do cubo.

Assim, $a^2+a^2+a^2=(2\sqrt{3})^2 \Leftrightarrow 3a^2=12 \Leftrightarrow a^2=4 \underset{a>0}{\Longrightarrow} a=2$ e $V=2^3=8$.

PÁG. 78

27.1 $V=V_{cubo}+V_{cilindro}$

Designemos por a a medida da aresta do cubo.

O volume do cubo é a^3.

A diagonal facial do cubo é $\sqrt{a^2+a^2}=\sqrt{2a^2}=\sqrt{2}\,a$.

O volume do cilindro é $\pi\times\left(\frac{\sqrt{2}\,a}{2}\right)^2\times a=\frac{\pi\times a^3}{2}$.

Assim, $a^3+\frac{\pi\times a^3}{2}=32(\pi+2) \Leftrightarrow \frac{2a^3+\pi a^3}{2}=32(\pi+2) \Leftrightarrow 2a^3+\pi a^3=2\times 32(\pi+2) \Leftrightarrow$

$\Leftrightarrow a^3(2+\pi)=2\times 32(\pi+2) \Leftrightarrow a^3=\frac{2\times 32(\pi+2)}{2+\pi} \Leftrightarrow a^3=64 \Leftrightarrow a=4$

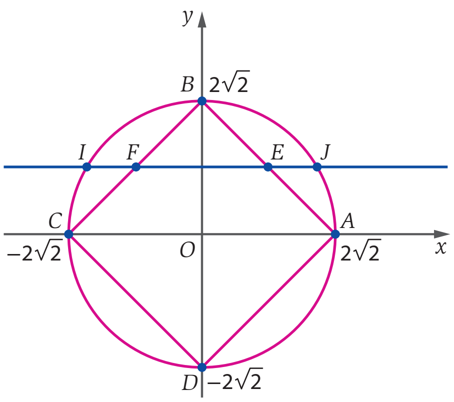
27.2 $A(2\sqrt{2},\ 0,\ 0)$, $B(0,\ 2\sqrt{2},\ 0)$, $C(-2\sqrt{2},\ 0,\ 0)$, $D(0,\ -2\sqrt{2},\ 0)$,
$E(2\sqrt{2},\ 0,\ 4)$, $F(0,\ 2\sqrt{2},\ 4)$, $G(-2\sqrt{2},\ 0,\ 4)$, $H(0,\ -2\sqrt{2},\ 4)$

27.3

a. $x=0 \wedge y=-2\sqrt{2} \wedge 0\leqslant z\leqslant 4$

b. $x^2+y^2\leqslant 8 \wedge z=-4$

27.4



$E(\sqrt{2},\ \sqrt{2})$, $F(-\sqrt{2},\ \sqrt{2})$ e $\overline{EF}=2\sqrt{2}$

$x^2+y^2=(2\sqrt{2})^2 \wedge y=\sqrt{2} \Leftrightarrow x^2=6 \wedge y=\sqrt{2} \Leftrightarrow x=-\sqrt{6} \vee x=\sqrt{6} \wedge y=\sqrt{2}$

$I(-\sqrt{6},\ \sqrt{2})$, $J(\sqrt{6},\ \sqrt{2})$ e $\overline{IJ}=2\sqrt{6}$

A interseção do cubo com o plano de equação $y=\sqrt{2}$ é um retângulo de altura 4 e base $2\sqrt{2}$ e a interseção do cilindro com o plano de equação $y=\sqrt{2}$ é um retângulo de altura 4 e base $2\sqrt{6}$.

A área da figura que resulta da interseção do sólido com o plano de equação $y=\sqrt{2}$ é
$4\times 2\sqrt{2}+4\times 2\sqrt{6}=8\sqrt{2}+8\sqrt{6}=8\left(\sqrt{2}+\sqrt{6}\right)$.

27.5 O plano BCE é o plano mediador de $[AF]$:

$$\left(x-2\sqrt{2}\right)^2+y^2+z^2=x^2+\left(y-2\sqrt{2}\right)^2+(z-4)^2 \Leftrightarrow$$

$$\Leftrightarrow -4\sqrt{2}x+8=-4\sqrt{2}y+8-8z+16 \Leftrightarrow -4\sqrt{2}x+4\sqrt{2}y+8z-16=0 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{2}x-\sqrt{2}y-2z+4=0$$

28.1 $V(2\ ,\ 2\ ,\ 8)$

Coordenadas de A :

$$(x-2)^2+(0-2)^2+(0-8)^2=72 \Leftrightarrow (x-2)^2=72-4-64 \Leftrightarrow (x-2)^2=4 \Leftrightarrow$$

$$\Leftrightarrow x-2=-2 \vee x-2=2 \Leftrightarrow x=0 \vee x=4\ ,\ A(4\ ,\ 0\ ,\ 0)$$

$$V_{[VOABC]}=\frac{1}{3}\times 4^2\times 8=\frac{128}{3}$$

28.2 $V_{[VDEFG]}=\frac{1}{8}V_{[VOABC]} \Rightarrow \text{altura}_{[VDEFG]}=\frac{1}{2}\text{altura}_{[VOABC]}=\frac{1}{2}\times 8=4$

A base da pirâmide $[VDEFG]$ é o quadrado que resulta da interseção da pirâmide $[VOABC]$ com o plano de equação $z=4$.

Os vértices do quadrado $[DEFG]$ são os pontos médios dos segmentos de reta formados pelo vértice V e cada um dos vértices do quadrado $[OABC]$:

$$D=M_{[VA]}=\left(\frac{2+4}{2}\ ,\ \frac{2+0}{2}\ ,\ \frac{8+0}{2}\right)=(3\ ,\ 1\ ,\ 4)\ ,\ E=M_{[VB]}=\left(\frac{2+4}{2}\ ,\ \frac{2+4}{2}\ ,\ \frac{8+0}{2}\right)=(3\ ,\ 3\ ,\ 4)\ ,$$

$$F=M_{[VC]}=\left(\frac{2+0}{2}\ ,\ \frac{2+4}{2}\ ,\ \frac{8+0}{2}\right)=(1\ ,\ 3\ ,\ 4)\ ,\ G=M_{[VO]}=\left(\frac{2+0}{2}\ ,\ \frac{2+0}{2}\ ,\ \frac{8+0}{2}\right)=(1\ ,\ 1\ ,\ 4)$$

PÁG. 79

28.3 A esfera descrita no enunciado tem centro de coordenadas $(2\ ,\ 2\ ,\ 2)$ e raio $4\div 2=2$: $(x-2)^2+(y-2)^2+(z-2)^2\leqslant 4$

28.4 A linha descrita pelo ponto V quando a pirâmide dá uma volta completa em torno da aresta $[AO]$ é uma circunferência com centro no ponto médio da aresta $[AO]$ e com raio igual a $\overline{VM_{[AO]}}$ contida no plano de equação $x=2$:

$$M_{[AO]}=(2\ ,\ 0\ ,\ 0) \text{ e } \overline{VM_{[AO]}}=\sqrt{(2-2)^2+(2-0)^2+(8-0)^2}=\sqrt{68}$$

$$(x-2)^2+y^2+z^2=68 \wedge x=2$$

29.1 $(1+1)^2+(1+4)^2+(2-3)^2=30$ e $(-3+1)^2+(-9+4)^2+(4-3)^2=30$

$$\overline{AB}=\sqrt{(1+3)^2+(1+9)^2+(2-4)^2}=\sqrt{120}=2\sqrt{30}$$

29.2

a. $(x-1)^2+(y-1)^2+(z-2)^2=(x+3)^2+(y+9)^2+(z-4)^2 \Leftrightarrow$

$$\Leftrightarrow -2x+1-2y+1-4z+4=6x+9+18y+81-8z+16 \Leftrightarrow$$

$$\Leftrightarrow -2x-6x-2y-18y-4z+8z=9+81+16-1-1-4 \Leftrightarrow$$

$$\Leftrightarrow -8x-20y+4z=100 \Leftrightarrow -2x-5y+z=25$$

b. $-2x-5y+z=25 \wedge x=1 \wedge z=2 \Leftrightarrow -2\times 1-5y+2=25 \wedge x=1 \wedge z=2 \Leftrightarrow y=-5 \wedge x=1 \wedge z=2$
$(1\ ,\ -5\ ,\ 2)$

c. $-2(-1)-5(-4)+3=25$

d. Círculo contido no plano α, centrado no ponto de coordenadas $(-1\ ,\ -4\ ,\ 3)$ e raio $\sqrt{30}$, perímetro: $2\pi\sqrt{30}$.

PÁG. 80

30.1 $V=64 \Leftrightarrow \frac{\overline{AB}\times\overline{BC}}{2}\times\overline{AE}=64 \Leftrightarrow \frac{\overline{AB}^2}{2}\times\overline{AE}=64$

$\overline{AC}^2=\overline{AB}^2+\overline{BC}^2 \Leftrightarrow (4\sqrt{2})^2=2\overline{AB}^2 \Leftrightarrow \overline{AB}^2=16$

$\frac{\overline{AB}^2}{2}\times\overline{AE}=64 \Leftrightarrow \frac{16}{2}\times\overline{AE}=64 \Leftrightarrow \overline{AE}=8$

O ponto B tem a mesma ordenada e a mesma cota que o ponto A e tem abcissa $2-4=-2$, logo $B(-2\ ,\ 5\ ,\ -1)$.

O ponto C tem a mesma abcissa e a mesma ordenada que o ponto B e tem cota $-1+4=3$, logo $C(-2\ ,\ 5\ ,\ 3)$.

O ponto D tem a mesma abcissa e a mesma cota que o ponto C e tem ordenada $5-8=-3$, logo $D(-2\ ,\ -3\ ,\ 3)$.

O ponto E tem a mesma abcissa e a mesma cota que o ponto A e tem ordenada $5-8=-3$, logo $E(2\ ,\ -3\ ,\ -1)$.

O ponto F tem a mesma abcissa e a mesma cota que o ponto B e tem a mesma ordenada que o ponto E, logo $F(-2\ ,\ -3\ ,\ -1)$.

30.2

a. $y=-3$

b. $x=-2 \wedge y=-3$

c. $x=-2 \wedge y=5 \wedge -1\leqslant z\leqslant 3$

d. $x=-2 \wedge -3\leqslant y\leqslant 5 \wedge -1\leqslant z\leqslant 3$

e. $y=\frac{-3+5}{2}=1$

30.3

a. O plano ACD é o plano mediador do segmento de reta formado pelo ponto B e o seu simétrico em relação ao plano ACD:

$(x-2)^2+(y-5)^2+(z-3)^2=(x+2)^2+(y-5)^2+(z+1)^2 \Leftrightarrow$

$\Leftrightarrow -4x+4-6z+9=4x+4+2z+1 \Leftrightarrow -4x-4x-6z-2z=1-9 \Leftrightarrow$

$\Leftrightarrow -8x-8z=-8 \Leftrightarrow x+z=1$

b. $x+z=1 \wedge x=0 \wedge y=0 \Leftrightarrow z=1 \wedge x=0 \wedge y=0$, $(0\ ,\ 0\ ,\ 1)$

30.4 Centro: $\left(\frac{2-2}{2}, \frac{5-3}{2}, \frac{3-1}{2}\right)=(0,1,1)$,

raio: $\frac{\overline{EC}}{2}=\frac{\sqrt{(2+2)^2+(-3-5)^2+(-1-3)^2}}{2}=\frac{\sqrt{96}}{2}=\frac{4\sqrt{6}}{2}=2\sqrt{6}$, $x^2+(y-1)^2+(z-1)^2=24$

Planos tangentes à superfície esférica e paralelos ao plano yOz: $x=-2\sqrt{6}$ e $x=2\sqrt{6}$

30.5 Centro: $(-1,3,2)$, raio: 6

a. $x+z=1 \wedge x=-1 \wedge y=3 \Leftrightarrow z=2 \wedge x=-1 \wedge y=3$, $(-1, 3, 2)$

b. $A=27\pi \Leftrightarrow \pi r^2=27\pi \Leftrightarrow r^2=27$

(A) $(x+1)^2+(y-3)^2+(z-2)^2 \leqslant 36 \wedge x=-1 \Leftrightarrow (y-3)^2+(z-2)^2 \leqslant 36 \wedge x=-1$, raio: 6

(B) $(x+1)^2+(y-3)^2+(z-2)^2 \leqslant 36 \wedge y=1 \Leftrightarrow (x+1)^2+(z-2)^2 \leqslant 32 \wedge y=1$, raio: $\sqrt{32}$

(C) $(x+1)^2+(y-3)^2+(z-2)^2 \leqslant 36 \wedge x=3 \Leftrightarrow (y-3)^2+(z-2)^2 \leqslant 20 \wedge x=3$, raio: $\sqrt{20}$

(D) $(x+1)^2+(y-3)^2+(z-2)^2 \leqslant 36 \wedge z=5 \Leftrightarrow (x+1)^2+(y-3)^2 \leqslant 27 \wedge z=5$, raio: $\sqrt{27}$

8. Vetores no plano

PÁG. 81

1.1

a. $E+\overrightarrow{FG}=E+\overrightarrow{ED}=D$

b. $\overrightarrow{PG}+\overrightarrow{CA}=\overrightarrow{PG}+\overrightarrow{GI}=\overrightarrow{PI}$

c. $H-\overrightarrow{NL}=H+\overrightarrow{LN}=H+\overrightarrow{HF}=F$

d. $\frac{2}{3}\overrightarrow{PL}+H=\overrightarrow{PM}+H=J$

e. $\frac{1}{2}\overrightarrow{EP}-2\overrightarrow{GF}=\overrightarrow{EF}+\overrightarrow{FH}=\overrightarrow{EH}$

f. $I+\overrightarrow{KH}=D$

1.2 $\left\|\overrightarrow{EH}\right\|^2=\left\|\overrightarrow{EC}\right\|^2+\left\|\overrightarrow{CH}\right\|^2 \Leftrightarrow \left\|\overrightarrow{EH}\right\|^2=8^2+4^2 \Leftrightarrow \left\|\overrightarrow{EH}\right\|^2=80 \underset{\left\|\overrightarrow{EH}\right\|>0}{\Rightarrow} \left\|\overrightarrow{EH}\right\|=\sqrt{80}=4\sqrt{5}$

1.3 Por exemplo, $\overrightarrow{MN}$.

1.4 Por exemplo, $\overrightarrow{AB}$ e $\overrightarrow{IJ}$.

2.1 $\left\|2\vec{u}\right\|=2\left\|\vec{u}\right\|=2\times 3=6$, $\left\|-\vec{u}\right\|=\left\|\vec{u}\right\|=3$, $\left\|\frac{7}{3}\vec{u}\right\|=\frac{7}{3}\left\|\vec{u}\right\|=\frac{7}{3}\times 3=7$,

$\left\|-\frac{4}{5}\vec{u}\right\|=\frac{4}{5}\left\|\vec{u}\right\|=\frac{4}{5}\times 3=\frac{12}{5}$

2.2

a. $\left\|\vec{w}\right\|=12=4\times 3=4\left\|\vec{u}\right\|$, $\vec{w}=4\vec{u}$

b. $\left\|\vec{v}\right\|=1=\frac{1}{3}\times 3=\frac{1}{3}\left\|\vec{u}\right\|$, $\vec{v}=-\frac{1}{3}\vec{u}$

c. $\vec{t}=-\frac{2}{3}\vec{u}$ ou $\vec{t}=-\frac{4}{3}\vec{u}$

3. $\vec{w}=5\vec{u}-2\vec{v}-3\left(\vec{u}-\frac{2}{3}\vec{v}\right)=5\vec{u}-2\vec{v}-3\vec{u}+2\vec{v}=2\vec{u}$

4.1

a. $\frac{3}{2}\overrightarrow{CG}+\overrightarrow{LH}=\frac{3}{2}\overrightarrow{CG}-\frac{1}{2}\overrightarrow{CG}=\overrightarrow{CG}$

b. $D+\overrightarrow{CJ}+\overrightarrow{IE}=D+\overrightarrow{CJ}+\overrightarrow{JH}=D+\overrightarrow{CH}=K$

4.2 Seja F o ponto de interseção das diagonais do losango $[BCDE]$. Tem-se

$\left\|\overrightarrow{CF}\right\|^2=\left\|\overrightarrow{CB}\right\|^2-\left\|\overrightarrow{BF}\right\|^2 \Leftrightarrow \left\|\overrightarrow{CF}\right\|^2=3^2-\left(\frac{3}{2}\right)^2 \Leftrightarrow \left\|\overrightarrow{CF}\right\|^2=\frac{27}{4} \underset{\left\|\overrightarrow{CF}\right\|>0}{\Rightarrow} \left\|\overrightarrow{CF}\right\|=\sqrt{\frac{27}{4}}=\frac{3\sqrt{3}}{2}$.

Logo, $\left\|\overrightarrow{CE}\right\|=2\left\|\overrightarrow{CF}\right\|=2\times\frac{3\sqrt{3}}{2}=3\sqrt{3}$.

PÁG. 82

5. (A)

$\|\vec{u}+\vec{v}\| \leqslant \|\vec{u}\|+\|\vec{v}\| \Leftrightarrow \|\vec{u}+\vec{v}\| \leqslant 4+3 \Leftrightarrow \|\vec{u}+\vec{v}\| \leqslant 7$

6. $X = B - \frac{1}{2}\overrightarrow{AD} = B + \frac{1}{2}\overrightarrow{DA}$ é o ponto médio de $[BD]$;

$Y = C - \overrightarrow{DF} + \frac{1}{2}\overrightarrow{FA} = C + \overrightarrow{FD} + \frac{1}{2}\overrightarrow{FA} = E + \frac{1}{2}\overrightarrow{FA}$ é o ponto médio de $[DE]$;

$Z = A - 2\left(\overrightarrow{CF} + \frac{3}{4}\overrightarrow{DF}\right) = A - 2\overrightarrow{CF} - \frac{6}{4}\overrightarrow{DF} = A + 2\overrightarrow{FC} + \frac{3}{2}\overrightarrow{FD} = A + \overrightarrow{AC} + \frac{3}{2}\overrightarrow{FD} = C + \frac{3}{2}\overrightarrow{FD}$

é o ponto médio de $[BE]$.

Assim, $A_{[XYZ]} = \frac{1}{16}A_{[ABC]} = \frac{1}{16} \times 16 = 1$.

7. Tem-se $\overrightarrow{PS} = \overrightarrow{PB} + \overrightarrow{BS} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$ e

$\overrightarrow{QR} = \overrightarrow{QD} + \overrightarrow{DR} = \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$.

Logo, $\overrightarrow{PS} = \overrightarrow{QR}$.

Por outro lado, $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{AD} = \frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{AD}\right) = \frac{1}{2}\overrightarrow{BD}$ e

$\overrightarrow{SR} = \overrightarrow{SC} + \overrightarrow{CR} = \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{CD} = \frac{1}{2}\left(\overrightarrow{BC} + \overrightarrow{CD}\right) = \frac{1}{2}\overrightarrow{BD}$.

Logo, $\overrightarrow{PQ} = \overrightarrow{SR}$.

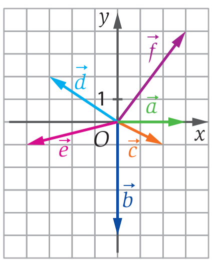
Assim, $[PQRS]$ tem os lados opostos paralelos e com o mesmo comprimento, pelo que é um losango.

Por outro lado, tem-se $\overrightarrow{PR} = \overrightarrow{BC}$, $\overrightarrow{SQ} = \overrightarrow{CD}$ e $\|\overrightarrow{BC}\| = \|\overrightarrow{CD}\|$, donde $\|\overrightarrow{PR}\| = \|\overrightarrow{SQ}\|$.

Logo, como o losango tem as diagonais iguais, é um quadrado.

8. $\vec{a}(4\ ,\ 0)$, $\vec{b}(-2\ ,\ -3)$, $\vec{c}(0\ ,\ 2)$, $\vec{d}(3\ ,\ 2)$, $\vec{e}(-2\ ,\ 1)$, $\vec{f}(3\ ,\ -1)$

9.1



9.2 $-\vec{a}(-3\ ,\ 0)$, $-\vec{b}(0\ ,\ 5)$, $-\vec{d}(3\ ,\ -2)$

9.3 $\vec{a}+\vec{b} = (3\ ,\ 0) + (0\ ,\ -5) = (3\ ,\ -5)$, $2\vec{a} = 2\,(3\ ,\ 0) = (6\ ,\ 0)$,

$-3\vec{c} = -3\,(2\ ,\ -1) = (-6\ ,\ 3)$,

$\vec{e} + 2\vec{d} = (-4\ ,\ -1) + 2\,(-3\ ,\ 2) = (-4\ ,\ -1) + (-6\ ,\ 4) = (-10\ ,\ 3)$,

$\vec{a} - \left(3\vec{e} + \vec{f}\right) = (3\ ,\ 0) - [3\,(-4\ ,\ -1) + (3\ ,\ 4)] = (3\ ,\ 0) - [(-12\ ,\ -3) + (3\ ,\ 4)] =$

$= (3\ ,\ 0) - (-9\ ,\ 1) = (12\ ,\ -1)$

PÁG. 83

10.1 $\overrightarrow{BC}(-3-2\ ,\ 0+3)=(-5\ ,\ 3)$

10.2

a. $C-3\vec{v}=(-3\ ,\ 0)-3(-2\ ,\ 1)=(3\ ,\ -3)$ ponto

b. $A-B+2\vec{v}=(4\ ,\ -2)-(2\ ,\ -3)+2(-2\ ,\ 1)=(-2\ ,\ 3)$ vetor

10.3

a. $\vec{u}=\overrightarrow{CD} \Leftrightarrow (k\ ,\ 4)=(3\ ,\ 4) \Leftrightarrow k=3$

b. Os vetores $\vec{u}$ e $\vec{v}$ são colineares se $\exists \lambda \in \mathbb{R}: \vec{u}=\lambda\vec{v}$.

$\vec{u}=\lambda\vec{v} \Leftrightarrow (k\ ,\ 4)=\lambda(-2\ ,\ 1) \Leftrightarrow k=-2\lambda \wedge 4=\lambda \Leftrightarrow k=-8$

10.4

a. $\|\vec{v}\|=\sqrt{(-2)^2+1^2}=\sqrt{5}$

b. $\|\overrightarrow{BC}\|=\sqrt{(-5)^2+3^2}=\sqrt{34}$

c. $\vec{v}+\overrightarrow{BC}=(-2\ ,\ 1)+(-5\ ,\ 3)=(-7\ ,\ 4)\ ,\ \|\vec{v}+\overrightarrow{BC}\|=\sqrt{(-7)^2+4^2}=\sqrt{65}$

10.5 $\|\vec{u}\|=6 \Leftrightarrow \sqrt{k^2+4^2}=6 \Rightarrow k^2+16=36 \Leftrightarrow k^2=20 \Leftrightarrow k=-\sqrt{20} \vee k=\sqrt{20} \Leftrightarrow$

$\Leftrightarrow k=-2\sqrt{5} \vee k=2\sqrt{5}$

11. **(A)**

12. $D=C+\overrightarrow{BA}=C-\overrightarrow{AB}=(8\ ,\ 0)-(-4\ ,\ -3)=(12\ ,\ 3)$

13.1 $(5-2)^2+(1+1)^2=13$ e $(4-2)^2+(2+1)^2=13$

13.2 $Q(x\ ,\ y):\ \overline{AQ}^2=\overline{AB}^2+\overline{BQ}^2$

$\overline{AQ}=\sqrt{(x-5)^2+(y-1)^2}\ ,\ \overline{AB}=\sqrt{(5-4)^2+(1-2)^2}=\sqrt{2}\ ,\ \overline{BQ}=\sqrt{(x-4)^2+(y-2)^2}$

$(x-5)^2+(y-1)^2=2+(x-4)^2+(y-2)^2 \Leftrightarrow -10x+25-2y+1=2-8x+16-4y+4 \Leftrightarrow$

$\Leftrightarrow -10x+8x-2y+4y=2+16+4-25-1 \Leftrightarrow -2x+2y=-4 \Leftrightarrow x-y=2$

$(x-2)^2+(x-2+1)^2=13 \Leftrightarrow x^2-4x+4+x^2-2x+1-13=0 \Leftrightarrow$

$\Leftrightarrow 2x^2-6x-8=0 \Leftrightarrow x^2-3x-4=0 \Leftrightarrow x=\dfrac{3\pm\sqrt{(-3)^2-4(-4)}}{2} \Leftrightarrow x=\dfrac{3\pm 5}{2} \Leftrightarrow x=-1 \vee x=4$

$-1-y=2 \vee 4-y=2 \Leftrightarrow y=-3 \vee y=2$

Logo, $Q(-1\ ,\ -3)$.

13.3 $C(-1\ ,\ y)\ ,\ y>0$

$(-1-2)^2+(y+1)^2=13 \Leftrightarrow (y+1)^2=4 \Leftrightarrow y+1=-2 \vee y+1=2 \Leftrightarrow y=-3 \vee y=1$

$C(-1\ ,\ 1)$

$D=C+\overrightarrow{BE}=C-\overrightarrow{EB}=(-1\ ,\ 1)-(-1\ ,\ 5)=(0\ ,\ -4)$

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14. $\vec{v}=\lambda\vec{u}$, $\lambda\in\mathbb{R}$, $\vec{v}=\lambda(-2\,,\ 4)=(-2\lambda\,,\ 4\lambda)$

$\|\vec{v}\|=2 \Leftrightarrow \sqrt{(-2\lambda)^2+(4\lambda)^2}=2 \Rightarrow 20\lambda^2=4 \Leftrightarrow \lambda^2=\frac{4}{20} \Leftrightarrow \lambda=-\frac{\sqrt{5}}{5}\vee\lambda=\frac{\sqrt{5}}{5}$

$\vec{v}=-\frac{\sqrt{5}}{5}(-2\,,\ 4)\vee\vec{v}=\frac{\sqrt{5}}{5}(-2\,,\ 4) \Leftrightarrow \vec{v}=\left(\frac{2\sqrt{5}}{5}\,,\ -\frac{4\sqrt{5}}{5}\right)\vee\vec{v}=\left(-\frac{2\sqrt{5}}{5}\,,\ \frac{4\sqrt{5}}{5}\right)$

Como $\vec{v}$ tem o mesmo sentido de $\vec{u}$, $\vec{v}=\left(-\frac{2\sqrt{5}}{5}\,,\ \frac{4\sqrt{5}}{5}\right)$.

15. $\|\overrightarrow{AB}\|=\sqrt{(-2)^2+1^2}=\sqrt{5}$ e $\|\overrightarrow{BC}\|=\sqrt{3^2+1^2}=\sqrt{10}$

$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=(-2+3\,,\ 1+1)=(1\,,\ 2) \Rightarrow \|\overrightarrow{AC}\|=\sqrt{1^2+2^2}=\sqrt{5}$

O triângulo é isósceles.

16.1 $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$; $\overrightarrow{AD}(2\,,\ -2)$ e $\overrightarrow{BC}(4\,,\ -4)$

são colineares; logo, os lados $[AD]$ e $[BC]$ são paralelos, ou seja, $[ABCD]$ é um trapézio.

16.2 $A=\frac{\|\overrightarrow{AD}\|+\|\overrightarrow{BC}\|}{2}\times\|\overrightarrow{CD}\|=\frac{\sqrt{2^2+(-2)^2}+\sqrt{4^2+(-4)^2}}{2}\times\sqrt{2^2+2^2}=$

$=\frac{2\sqrt{2}+4\sqrt{2}}{2}\times 2\sqrt{2}=(\sqrt{2}+2\sqrt{2})\times 2\sqrt{2}=4+8=12$

17.1 $\overrightarrow{BD}=\overrightarrow{BQ}+\overrightarrow{QC}+\overrightarrow{CR}+\overrightarrow{RD}=2\overrightarrow{QC}+2\overrightarrow{CR}=2\left(\overrightarrow{QC}+\overrightarrow{CR}\right)=2\overrightarrow{QR}$. Logo, $[BD]$ e $[QR]$ são paralelos.

Por outro lado, $\overrightarrow{BD}=\overrightarrow{BP}+\overrightarrow{PA}+\overrightarrow{AS}+\overrightarrow{SD}=2\overrightarrow{PA}+2\overrightarrow{AS}=2\left(\overrightarrow{PA}+\overrightarrow{AS}\right)=2\overrightarrow{PS}$.

Logo, $[BD]$ e $[PS]$ são paralelos. Portanto, $[QR]$ e $[PS]$ também são paralelos.

Analogamente se mostra que $[PQ]$ e $[SR]$ são paralelos e, portanto, $[PQRS]$ é um paralelogramo.

17.2 $S=P+\overrightarrow{QR}=(2\,,\ 4)+[(6\,,\ 3)-(6\,,\ 7)]=(2\,,\ 0)$

$P=M_{[AB]} \Leftrightarrow (2\,,\ 4)=\left(\frac{0+x}{2}\,,\ \frac{2+y}{2}\right) \Leftrightarrow \frac{x}{2}=2\wedge\frac{2+y}{2}=4 \Leftrightarrow x=4\wedge y=6\,,\ B(4\,,\ 6)$

$C=Q+\overrightarrow{BQ}=(6\,,\ 7)+[(6\,,\ 7)-(4\,,\ 6)]=(8\,,\ 8)$

$D=R+\overrightarrow{CR}=(6\,,\ 3)+[(6\,,\ 3)-(8\,,\ 8)]=(4\,,\ -2)$

18.1

a. $A-2\vec{v}=(-1\,,\ 3)-2(1\,,\ -5)=(-3\,,\ 13)$ ponto

b. $B-A+3\vec{v}=(2\,,\ -5)-(-1\,,\ 3)+3(1\,,\ -5)=(6\,,\ -23)$ vetor

18.2 Os vetores $\vec{u}$ e $\vec{v}$ são colineares se $\exists\lambda\in\mathbb{R}:\vec{u}=\lambda\vec{v}$.

$\vec{u}=\lambda\vec{v} \Leftrightarrow (-5\,,\ k)=\lambda(1\,,\ -5) \Leftrightarrow -5=\lambda\wedge k=-5\lambda \Leftrightarrow k=25$

18.3 $\overrightarrow{AB}-\vec{v}=[(2\,,\ -5)-(-1\,,\ 3)]-(1\,,\ -5)=(2\,,\ -3)$, $\|\overrightarrow{AB}-\vec{v}\|=\sqrt{2^2+(-3)^2}=\sqrt{13}$

18.4 $m=\frac{-5}{1}=-5$, $3=-5(-1)+b \Leftrightarrow b=-2$, $y=-5x-2$

PÁG. 85

19.1 $\overrightarrow{PQ}(2-1\ ,\ 5+3)=(1\ ,\ 8)$

19.2 Reta vertical: $(0\ ,\ 1)$

19.3 Reta horizontal: $(1\ ,\ 0)$

19.4 $y=-x\ ,\ (1\ ,\ -1)$

20.1 $(1\ ,\ -2)$

20.2 $(1\ ,\ -1)$

20.3 $(3\ ,\ 2)$

20.4 $x+3y=2 \Leftrightarrow y=-\frac{1}{3}x+\frac{2}{3}\ ,\ (3\ ,\ -1)$

21. As retas são paralelas se os seus vetores diretores são colineares:

$\frac{k}{2}=\frac{k+1}{-1} \Leftrightarrow -k=2k+2 \Leftrightarrow k=-\frac{2}{3}$

22.1 $A_{[ABCD]}=\left\|\overrightarrow{BC}\right\|^2=2^2+(-3)^2=13$

22.2 $(x\ ,\ y)=(8\ ,\ 3)+\lambda(2\ ,\ -3)\ ,\ \lambda\in\mathbb{R}$

22.3 $\vec{u}=\lambda\overrightarrow{BC}\ ,\ \lambda\in\mathbb{R}\ ,\ \vec{u}=\lambda(2\ ,\ -3)=(2\lambda\ ,\ -3\lambda)$

$\|\vec{u}\|=\sqrt{52} \Leftrightarrow \sqrt{(2\lambda)^2+(3\lambda)^2}=\sqrt{52} \Rightarrow 13\lambda^2=52 \Leftrightarrow \lambda^2=4 \Leftrightarrow \lambda=-2 \vee \lambda=2$

$\vec{u}=-2(2\ ,\ -3) \vee \vec{u}=2(2\ ,\ -3) \Leftrightarrow \vec{u}=(-4\ ,\ 6) \vee \vec{u}=(4\ ,\ -6)$

Como $\vec{u}$ tem sentido oposto a $\overrightarrow{BC}$, $\vec{u}=(-4\ ,\ 6)$.

22.4 $A=D-\overrightarrow{BC}=(8\ ,\ 3)-(2\ ,\ -3)=(6\ ,\ 6)$, logo pertence à reta de equação $y=x$, que é a bissetriz dos quadrantes ímpares.

23.1 $(x\ ,\ y)=(0\ ,\ 2)+\lambda(3\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

23.2 $(x\ ,\ y)=(4\ ,\ -1)+\lambda(1\ ,\ 0)\ ,\ \lambda\in\mathbb{R}$

23.3 $\overrightarrow{AB}(4-0\ ,\ -1-2)=(4\ ,\ -3)\ ,\ (x\ ,\ y)=(0\ ,\ 2)+\lambda(4\ ,\ -3)\ ,\ \lambda\in\mathbb{R}$

23.4 $(x\ ,\ y)=(4\ ,\ -1)+\lambda(1\ ,\ 3)\ ,\ \lambda\in\mathbb{R}$

PÁG. 86

24.1 Pontos: $(3\ ,\ -2)$ e $(3\ ,\ -2)+(0\ ,\ 1)=(3\ ,\ -1)$;
vetores diretores: $(0\ ,\ 1)$ e $-2(0\ ,\ 1)=(0\ ,\ -2)$

24.2 Pontos: $(-1\ ,\ 4)$ e $(-1\ ,\ 4)+(1\ ,\ 0)=(0\ ,\ 4)$;
vetores diretores: $(2\ ,\ 0)$ e $-\frac{1}{2}(2\ ,\ 0)=(-1\ ,\ 0)$

24.3 Pontos: $(2\ ,\ -1)$ e $(2\ ,\ -1)+(3\ ,\ -1)=(5\ ,\ -2)$;
vetores diretores: $(3\ ,\ -1)$ e $-2(3\ ,\ -1)=(-6\ ,\ 2)$

24.4 Pontos: $(1\ ,\ 3)$ e $(1\ ,\ 3)+(-3\ ,\ 4)=(-2\ ,\ 7)$;
vetores diretores: $(-3\ ,\ 4)$ e $-2(-3\ ,\ 4)=(6\ ,\ -8)$

25.1 $(x\ ,\ y)=(1\ ,\ 0)+\lambda(0\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

25.2 $(x\ ,\ y)=(0\ ,\ 2)+\lambda(1\ ,\ 0)\ ,\ \lambda\in\mathbb{R}$

25.3 $m=1$ e $b=1\ ,\ (x\ ,\ y)=(0\ ,\ 1)+\lambda(1\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

25.4 $m=-2$ e $b=5\ ,\ (x\ ,\ y)=(0\ ,\ 5)+\lambda(1\ ,\ -2)\ ,\ \lambda\in\mathbb{R}$

25.5 $m=\frac{1}{3}$ e $b=-4\ ,\ (x\ ,\ y)=(0\ ,\ -4)+\lambda(3\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

25.6 $x+y=8 \Leftrightarrow y=-x+8\ ,\ m=-1$ e $b=8\ ,\ (x\ ,\ y)=(0\ ,\ 8)+\lambda(1\ ,\ -1)\ ,\ \lambda\in\mathbb{R}$

26. (C)

(C) ou (D), tendo em conta o vetor diretor.

(C) porque o ponto $(0\ ,\ 2)$ não pertence ao eixo das abcissas.

27. Reta a :

$(1\ ,\ 3)$ e $(7\ ,\ 5)\ ,\ (7\ ,\ 5)-(1\ ,\ 3)=(6\ ,\ 2)\ ,\ (x\ ,\ y)=(4\ ,\ 4)+\lambda(3\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

Reta b :

$(7\ ,\ 0)$ e $(0\ ,\ 7)\ ,\ (0\ ,\ 7)-(7\ ,\ 0)=(-7\ ,\ 7)\ ,\ (x\ ,\ y)=(5\ ,\ 2)+\lambda(1\ ,\ -1)\ ,\ \lambda\in\mathbb{R}$

Reta c :

$y=-2\ ,\ (x\ ,\ y)=(0\ ,\ -2)+\lambda(1\ ,\ 0)\ ,\ \lambda\in\mathbb{R}$

Reta d :

$x=-1\ ,\ (x\ ,\ y)=(-1\ ,\ 0)+\lambda(0\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

28.1 $m=\frac{2}{1}=2\ ,\ -2=2\times 4+b \Leftrightarrow b=-10\ ,\ y=2x-10$

28.2 $m=\frac{1}{4}\ ,\ -1=\frac{1}{4}\times 3+b \Leftrightarrow b=-1-\frac{3}{4} \Leftrightarrow b=-\frac{7}{4}\ ,\ y=\frac{1}{4}x-\frac{7}{4}$

28.3 $m=\frac{-3}{3}=-1\ ,\ -2=-1\times 1+b \Leftrightarrow b=-1\ ,\ y=-x-1$

28.4 $m=\frac{5}{-1}=-5\ ,\ 0=-5\times 3+b \Leftrightarrow b=15\ ,\ y=-5x+15$

PÁG. 87

29.1

a. $(4\ ,\ y)=(2\ ,\ -5)+\lambda(1\ ,\ -2)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}4=2+\lambda\\ y=-5-2\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases}\lambda=2\\ y=-5-2\times 2\end{cases} \Leftrightarrow \begin{cases}\lambda=2\\ y=-9\end{cases}$

b. $(x\ ,\ 10)=(2\ ,\ -5)+\lambda(1\ ,\ -2)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}x=2+\lambda\\ 10=-5-2\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases}x=2-\frac{15}{2}\\ \lambda=-\frac{15}{2}\end{cases} \Leftrightarrow \begin{cases}x=-\frac{11}{2}\\ \lambda=-\frac{15}{2}\end{cases}$

c. Interseção com Ox :

$(x\,,\ 0)=(2\,,\ -5)+\lambda(1\,,\ -2)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=2+\lambda \\ 0=-5-2\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x=2-\frac{5}{2} \\ \lambda=-\frac{5}{2} \end{cases} \Leftrightarrow \begin{cases} x=-\frac{1}{2} \\ \lambda=-\frac{5}{2} \end{cases},\ \left(-\frac{1}{2}\,,\ 0\right)$

Interseção com Oy :

$(0\,,\ y)=(2\,,\ -5)+\lambda(1\,,\ -2)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} 0=2+\lambda \\ y=-5-2\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \lambda=-2 \\ y=-5-2(-2) \end{cases} \Leftrightarrow \begin{cases} \lambda=-2 \\ y=-1 \end{cases},\ (0\,,\ -1)$

29.2 $(-6\,,\ 20)=(2\,,\ -5)+\lambda(1\,,\ -2)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} -6=2+\lambda \\ 20=-5-2\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=-8 \\ \lambda=-\frac{25}{2} \end{cases}$

O ponto não pertence à reta.

29.3 $(x\,,\ y)=(4\,,\ -6)+\lambda(1\,,\ -2)\,,\ \lambda\in\mathbb{R}$

30. (B) ou (C) porque têm vetores diretores colineares.

Para ser estritamente paralela, (B) porque o ponto da reta dada pertence à reta de (C).

31.1

a. $(-3\,,\ y)=(1\,,\ -4)+\lambda(1\,,\ 2)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} -3=1+\lambda \\ y=-4+2\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \lambda=-4 \\ y=-4+2(-4) \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=-4 \\ y=-12 \end{cases},\ (-3\,,\ -12)$

b. $(x\,,\ -x+1)=(1\,,\ -4)+\lambda(1\,,\ 2)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=1+\lambda \\ -x+1=-4+2\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x=1+\lambda \\ x=5-2\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} 1+\lambda=5-2\lambda \\ \underline{\qquad\qquad} \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=\frac{4}{3} \\ x=1+\frac{4}{3} \end{cases} \Leftrightarrow \begin{cases} x=\frac{7}{3} \\ y=-\frac{7}{3}+1 \end{cases},\ \left(\frac{7}{3}\,,\ -\frac{4}{3}\right)$

31.2 As retas são paralelas se os seus vetores diretores são colineares.

$\frac{k-1}{1}=\frac{k}{2} \Leftrightarrow k=2k-2 \Leftrightarrow k=2$

32.1

a. $(x\,,\ -2)=(2\,,\ -3)+\lambda(3\,,\ 1)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=2+3\lambda \\ -2=-3+\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=5 \\ \lambda=1 \end{cases},\ (5\,,\ -2)$

b. $(-3\,,\ y)=(2\,,\ -3)+\lambda(3\,,\ 1)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} -3=2+3\lambda \\ y=-3+\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \lambda=-\frac{5}{3} \\ y=-3-\frac{5}{3} \end{cases} \Leftrightarrow \begin{cases} \lambda=-\frac{5}{3} \\ y=-\frac{14}{3} \end{cases},\ \left(-3\,,\ -\frac{14}{3}\right)$

c. $(x\,,\,-x+1)=(2\,,\,-3)+\lambda(3\,,\,1)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}x=2+3\lambda\\ -x+1=-3+\lambda\end{cases},\ \lambda\in\mathbb{R}\Leftrightarrow$

$\Leftrightarrow \begin{cases}x=2+3\lambda\\ x=4-\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}2+3\lambda=4-\lambda\\ \_\_\_\_\_\_\end{cases},\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}\lambda=\frac{1}{2}\\ x=4-\frac{1}{2}\end{cases}\Leftrightarrow \begin{cases}x=\frac{7}{2}\\ y=-\frac{7}{2}+1\end{cases},\ \left(\frac{7}{2}\,,\,-\frac{5}{2}\right)$

32.2 Interseção com Ox :

$(x\,,\,0)=(2\,,\,-3)+\lambda(3\,,\,1)\,,\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}x=2+3\lambda\\ 0=-3+\lambda\end{cases},\ \lambda\in\mathbb{R}\Leftrightarrow$

$\Leftrightarrow \begin{cases}x=2+3\times 3\\ \lambda=3\end{cases}\Leftrightarrow \begin{cases}x=11\\ \lambda=3\end{cases},\ A(11\,,\,0)$

Interseção com Oy :

$(0\,,\,y)=(2\,,\,-3)+\lambda(3\,,\,1)\,,\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}0=2+3\lambda\\ y=-3+\lambda\end{cases},\ \lambda\in\mathbb{R}\Leftrightarrow$

$\Leftrightarrow \begin{cases}\lambda=-\frac{2}{3}\\ y=-3-\frac{2}{3}\end{cases}\Leftrightarrow \begin{cases}\lambda=-\frac{2}{3}\\ y=-\frac{11}{3}\end{cases},\ B\left(0\,,\,-\frac{11}{3}\right)$

$A_{[AOB]}=\frac{\overline{OA}\times\overline{OB}}{2}=\frac{11\times\frac{11}{3}}{2}=\frac{121}{6}$

32.3 As retas são paralelas se os seus vetores diretores são colineares.

$\frac{k}{3}=\frac{k+3}{1}\Leftrightarrow k=3k+9\Leftrightarrow k=-\frac{9}{2}$

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33.1 $(x\,,\,y)=(4\,,\,2)+\lambda(1\,,\,3)\,,\ \lambda\in\mathbb{R}$

33.2 $A=D+\overrightarrow{CB}=(4\,,\,2)+(-1\,,\,-3)=(3\,,\,-1)$

33.3 A mediatriz de $[DC]$ é a reta de equação $x=7$.

A distância do ponto D à mediatriz é $7-4=3$.

Logo, $\|\overrightarrow{DC}\|=2\times 3=6$.

34.1 $(x\,,\,y)=(0\,,\,0)+\lambda(-3\,,\,4)\,,\ \lambda\in\mathbb{R}$

34.2 $(-12\,,\,16)=(4\,,\,-7)+\lambda(-3\,,\,4)\,,\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}-12=4-3\lambda\\ 16=-7+4\lambda\end{cases},\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}\lambda=\frac{16}{3}\\ \lambda=\frac{23}{4}\end{cases}$

O ponto não pertence à reta.

34.3 Como $[AB]$ é um diâmetro da circunferência e $\overline{AB}=10$,

conclui-se que o raio da circunferência é 5 .

$A(x\,,\,5)\,,\ (x\,,\,5)=(4\,,\,-7)+\lambda(-3\,,\,4)\,,\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}x=4-3\lambda\\ 5=-7+4\lambda\end{cases},\ \lambda\in\mathbb{R}\Leftrightarrow \begin{cases}x=4-3\times 3\\ \lambda=3\end{cases},$

$A(-5\,,\,5)$

$C(x\ ,\ y)$ centro da circunferência

$(x\ ,\ y)=(4\ ,\ -7)+\lambda(-3\ ,\ 4)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=4-3\lambda \\ y=-7+4\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=\frac{x-4}{-3} \\ \lambda=\frac{y+7}{4} \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \frac{x-4}{-3}=\frac{y+7}{4} \Leftrightarrow 4(x-4)=-3(y+7) \Leftrightarrow y=-\frac{4}{3}x-\frac{5}{3}$

$\overline{AC}=5 \Leftrightarrow \sqrt{(x+5)^2+(y-5)^2}=5 \Rightarrow (x+5)^2+\left(-\frac{4}{3}x-\frac{5}{3}-5\right)^2=25 \Leftrightarrow$

$\Leftrightarrow (x+5)^2+\left(-\frac{4}{3}x-\frac{20}{3}\right)^2=25 \Leftrightarrow x^2+10x+25+\frac{16}{9}x^2+\frac{160}{9}x+\frac{400}{9}=25 \Leftrightarrow$

$\Leftrightarrow 25x^2+250x+400=0 \Leftrightarrow x^2+10x+16=0 \Leftrightarrow x=\frac{-10\pm\sqrt{10^2-4\times16}}{2} \Leftrightarrow$

$\Leftrightarrow x=\frac{-10\pm6}{2} \Leftrightarrow x=-8 \vee x=-2$

$y=-\frac{4}{3}(-8)-\frac{5}{3} \vee y=-\frac{4}{3}(-2)-\frac{5}{3} \Leftrightarrow y=9 \vee y=1$

$C(-8\ ,\ 9) \vee C(-2\ ,\ 1)$

Como a abcissa do ponto C é maior do que a abcissa do ponto A e a ordenada do ponto C é menor do que a ordenada do ponto A, conclui-se que $C(-2\ ,\ 1)$.

34.4 $AB:\ m=\frac{4}{-3}=-\frac{4}{3}\ ,\ -7=-\frac{4}{3}\times4+b \Leftrightarrow b=-7+\frac{16}{3} \Leftrightarrow b=-\frac{5}{3}\ ,\ y=-\frac{4}{3}x-\frac{5}{3}$

$x\leqslant0 \wedge y\geqslant-\frac{4}{3}x-\frac{5}{3} \wedge (x+2)^2+(y-1)^2\leqslant25$

35. $(0\ ,\ y)=(0\ ,\ 3)+\lambda(3\ ,\ -1)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} 0=0+3\lambda \\ y=3-\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=0 \\ y=3 \end{cases},\ A(0\ ,\ 3)$

$(0\ ,\ y)=(0\ ,\ -3)+\lambda(2\ ,\ 1)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} 0=0+2\lambda \\ y=-3+\lambda \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=0 \\ y=-3 \end{cases},\ B(0\ ,\ -3)$

$r:\ m=-\frac{1}{3}\ ,\ y=-\frac{1}{3}x+3$

$s:\ m=\frac{1}{2}\ ,\ y=\frac{1}{2}x-3$

$\begin{cases} y=-\frac{1}{3}x+3 \\ y=\frac{1}{2}x-3 \end{cases} \Leftrightarrow \begin{cases} -\frac{1}{3}x+3=\frac{1}{2}x-3 \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} -\frac{1}{3}x-\frac{1}{2}x=-3-3 \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} -\frac{5}{6}x=-6 \\ \_\_\_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} x=\frac{36}{5} \\ y=\frac{1}{2}\times\frac{36}{5}-3 \end{cases} \Leftrightarrow \begin{cases} x=\frac{36}{5} \\ y=\frac{3}{5} \end{cases},\ C\left(\frac{36}{5}\ ,\ \frac{3}{5}\right)$

$A_{[ABC]}=\frac{\overline{AB}\times|x_C|}{2}=\frac{6\times\frac{36}{5}}{2}=\frac{216}{10}=\frac{108}{5}$

PÁG. 89

36. $C(3\,,\,-5)$

$$\begin{cases}(x-3)^2+(y+5)^2=104\\ y=x\end{cases} \Leftrightarrow \begin{cases}(x-3)^2+(x+5)^2=104\\ y=x\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}x^2-6x+9+x^2+10x+25=104\\ \_\_\_\_\_\_\end{cases} \Leftrightarrow \begin{cases}2x^2+4x-70=0\\ \_\_\_\_\_\_\end{cases} \Leftrightarrow \begin{cases}x^2+2x-35=0\\ \_\_\_\_\_\_\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}x=\dfrac{-2\pm\sqrt{2^2-4(-35)}}{2}\\ \_\_\_\_\_\_\end{cases} \Leftrightarrow \begin{cases}x=\dfrac{-2\pm 12}{2}\\ \_\_\_\_\_\_\end{cases} \Leftrightarrow \begin{cases}x=5\vee x=-7\\ y=5\vee y=-7\end{cases},\ P(5\,,\,5)$$

$\overrightarrow{CP}(5-3\,,\,5+5)=(2\,,\,10)$

$(x\,,\,y)=(3\,,\,-5)+\lambda(1\,,\,5)\,,\ \lambda\in\mathbb{R}$

37. O ponto de interseção da reta com o eixo Oy tem coordenadas $(0\,,\,5)$.

$C(x\,,\,y)$

$\sqrt{(x-0)^2+(y-5)^2}=2\sqrt{2} \Rightarrow x^2+(y-5)^2=8$

$$(x\,,\,y)=(0\,,\,5)+\lambda(-1\,,\,1)\,,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}x=0-\lambda\\ y=5+\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}\lambda=-x\\ \lambda=y-5\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}-x=y-5\\ \_\_\_\_\_\_\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}x=5-y\\ \_\_\_\_\_\_\end{cases}$$

$(5-y)^2+(y-5)^2=8 \Leftrightarrow 2y^2-20y+42=0 \Leftrightarrow y^2-10y+21=0 \Leftrightarrow$

$$\Leftrightarrow y=\frac{10\pm\sqrt{(-10)^2-4\times 21}}{2} \Leftrightarrow y=\frac{10\pm 4}{2} \Leftrightarrow y=3\vee y=7$$

$x=5-3\vee x=5-7 \Leftrightarrow x=2\vee x=-2$

$C(2\,,\,3)$

$x\leqslant 0\wedge(x-2)^2+(y-3)^2\leqslant 8$

38. **(B)**

$r:\ y+a^2x=x+6 \Leftrightarrow y=(1-a^2)x+6$

O declive da reta t é $\dfrac{-6}{2}=-3$.

Como as retas são paralelas, $1-a^2=-3 \Leftrightarrow a^2=4 \Leftrightarrow a=-2\vee a=2$.

$y+a^2\times 0=0+6 \Leftrightarrow y=6$ e $a+4\neq 6 \Leftrightarrow a\neq 2$

Logo, $a=-2$. (B)

39.1 $C = B + \overrightarrow{BC} = (0\ ,\ 2) + (7\ ,\ -3) = (7\ ,\ -1)$

39.2 Mediatriz de $[AC]$: $(x+3)^2 + (y-1)^2 = (x-7)^2 + (y+1)^2 \Leftrightarrow$

$\Leftrightarrow 6x + 9 - 2y + 1 = -14x + 49 + 2y + 1 \Leftrightarrow -2y - 2y = -14x - 6x + 49 - 9 \Leftrightarrow$

$\Leftrightarrow -4y = -20x + 40 \Leftrightarrow y = 5x - 10$

Por exemplo: $(x\ ,\ y) = (0\ ,\ -10) + \lambda(1\ ,\ 5)\ ,\ \lambda \in \mathbb{R}$

39.3 $AB:\ m = \frac{2-1}{0+3} = \frac{1}{3}\ ,\ y = \frac{1}{3}x + 2\ ,\ BC:\ m = \frac{-1-2}{7-0} = -\frac{3}{7}\ ,\ y = -\frac{3}{7}x + 2$

$CD:\ m = \frac{-3+1}{1-7} = \frac{1}{3}\ ,\ -3 = \frac{1}{3} \times 1 + b \Leftrightarrow b = -3 - \frac{1}{3} \Leftrightarrow b = -\frac{10}{3}\ ,\ y = \frac{1}{3}x - \frac{10}{3}$

$y \leqslant \frac{1}{3}x + 2 \wedge y \leqslant -\frac{3}{7}x + 2 \wedge y \geqslant \frac{1}{3}x - \frac{10}{3} \wedge y \geqslant -x - 2$

PÁG. 90

40.1 Como a reta AB é paralela ao eixo Ox, tem equação $y = -1$.

$\overrightarrow{AC} = \left(\frac{13}{2}\ ,\ 3\right) \Leftrightarrow C = \left(\frac{13}{2}\ ,\ 3\right) + (-2\ ,\ -1) = \left(\frac{9}{2}\ ,\ 2\right)$

Como a reta CD é paralela à reta AB, tem equação $y = 2$.

Assim, uma equação vetorial da reta CD é $(x\ ,\ y) = \left(\frac{9}{2}\ ,\ 2\right) + \lambda(1\ ,\ 0)\ ,\ \lambda \in \mathbb{R}$.

40.2 $B(x\ ,\ -x)$ e $A(-2\ ,\ -1)$, logo $B(1\ ,\ -1)$

$D = C + \overrightarrow{CD} = C + \overrightarrow{BA} = \left(\frac{9}{2}\ ,\ 2\right) + (-3\ ,\ 0) = \left(\frac{3}{2}\ ,\ 2\right)$

Raio da circunferência: $\overline{AD} = \sqrt{\left(\frac{3}{2} + 2\right)^2 + (2+1)^2} = \sqrt{\frac{85}{4}}$

$\left(x - \frac{3}{2}\right)^2 + (y-2)^2 = \frac{85}{4}$

40.3

a. $\overrightarrow{BC} = \left(\frac{9}{2} - 1\ ,\ 2+1\right) = \left(\frac{7}{2}\ ,\ 3\right)$

$\frac{k\sqrt{2}}{\frac{7}{2}} = \frac{(k-1)\sqrt{2}}{3} \Leftrightarrow 3k\sqrt{2} = \frac{7(k-1)\sqrt{2}}{2} \Leftrightarrow 6k\sqrt{2} = 7(k-1)\sqrt{2} \Leftrightarrow k = 7$

b. Se $k = 4$, $\vec{u} = (4\sqrt{2}\ ,\ 3\sqrt{2})$.

$\vec{v} = \lambda(4\sqrt{2}\ ,\ 3\sqrt{2}) = (4\sqrt{2}\lambda\ ,\ 3\sqrt{2}\lambda)\ ,\ \lambda \in \mathbb{R}$

$\|\vec{v}\| = \sqrt{2} \Leftrightarrow \sqrt{(4\sqrt{2}\lambda)^2 + (3\sqrt{2}\lambda)^2} = \sqrt{2} \Rightarrow 50\lambda^2 = 2 \Leftrightarrow \lambda^2 = \frac{2}{50} \Leftrightarrow \lambda = -\frac{1}{5} \vee \lambda = \frac{1}{5}$

$\vec{v} = -\frac{1}{5}(4\sqrt{2}\ ,\ 3\sqrt{2}) \vee \vec{v} = \frac{1}{5}(4\sqrt{2}\ ,\ 3\sqrt{2}) \Leftrightarrow \vec{v} = \left(-\frac{4\sqrt{2}}{5}\ ,\ -\frac{3\sqrt{2}}{5}\right) \vee \vec{v} = \left(\frac{4\sqrt{2}}{5}\ ,\ \frac{3\sqrt{2}}{5}\right)$

Como $\vec{v}$ tem o mesmo sentido de $\vec{u}$, $\vec{v} = \left(\frac{4\sqrt{2}}{5}\ ,\ \frac{3\sqrt{2}}{5}\right)$.

40.4 $M = M_{[CD]}\left(\frac{\frac{9}{2}+\frac{3}{2}}{2}\ ,\ \frac{2+2}{2}\right) = (3\ ,\ 2)$

$\overrightarrow{AB} = (1+2\ ,\ -1+1) = (3\ ,\ 0)\ ,\ \overrightarrow{DB} = \left(1-\frac{3}{2}\ ,\ -1-2\right) = \left(-\frac{1}{2}\ ,\ -3\right)\ ,$

$\overrightarrow{MA} = (-2-3\ ,\ -1-2) = (-5\ ,\ -3)$

$\overrightarrow{AB} + y\overrightarrow{DB} = x\overrightarrow{MA} \Leftrightarrow (3\ ,\ 0) + y\left(-\frac{1}{2}\ ,\ -3\right) = x(-5\ ,\ -3) \Leftrightarrow$

$\Leftrightarrow 3-\frac{1}{2}y = -5x \wedge 0-3y = -3x \Leftrightarrow y = 10x+6 \wedge x = y \Leftrightarrow x = 10x+6 \wedge x = y \Leftrightarrow x = -\frac{2}{3} \wedge y = -\frac{2}{3}$

40.5 $\overrightarrow{BC} = \left(\frac{9}{2}-1\ ,\ 2+1\right) = \left(\frac{7}{2}\ ,\ 3\right)\ ,\ m = \frac{3}{\frac{7}{2}} = \frac{6}{7}$

$-1 = \frac{6}{7}\times 1 + b \Leftrightarrow b = -1-\frac{6}{7} \Leftrightarrow b = -\frac{13}{7}$

$BC:\ y = \frac{6}{7}x - \frac{13}{7}$

$\left(x-\frac{3}{2}\right)^2 + (y-2)^2 \leqslant \frac{85}{4} \wedge \left(\left(y \geqslant 2 \wedge y \leqslant \frac{6}{7}x-\frac{13}{7}\right) \vee \left(y \leqslant -1 \wedge y \geqslant \frac{6}{7}x-\frac{13}{7}\right)\right)$

9. Vetores no espaço

PÁG. 91

1.1 $[F, G]$, $[A, B]$, $[D, C]$ e $[E, H]$

1.2

a. $A + \overrightarrow{AB} = B$

b. $D + \overrightarrow{CH} = D + \overrightarrow{DE} = E$

c. $D + \overrightarrow{DE} = E$

d. $\overrightarrow{DC} + \overrightarrow{CG} = \overrightarrow{DG}$

e. $\overrightarrow{DC} + \overrightarrow{HF} = \overrightarrow{DC} + \overrightarrow{CA} = \overrightarrow{DA}$

f. $\overrightarrow{FG} + \overrightarrow{GB} = \overrightarrow{FB} = \overrightarrow{EC}$

1.3 $\overrightarrow{FD}$

1.4 $\overrightarrow{BH}$ e $\overrightarrow{EA}$

1.5

a. $\|\overrightarrow{AC}\|^2 = \|\overrightarrow{AB}\|^2 + \|\overrightarrow{BC}\|^2 \Leftrightarrow \|\overrightarrow{AC}\|^2 = 3^2 + 3^2 \Leftrightarrow \|\overrightarrow{AC}\|^2 = 18 \underset{\|\overrightarrow{AC}\| > 0}{\Leftrightarrow} \|\overrightarrow{AC}\| = \sqrt{18} = 3\sqrt{2}$

b. $\|\overrightarrow{AH}\|^2 = \|\overrightarrow{AB}\|^2 + \|\overrightarrow{BC}\|^2 + \|\overrightarrow{CH}\|^2 \Leftrightarrow \|\overrightarrow{AH}\|^2 = 3^2 + 3^2 + 3^2 \Leftrightarrow$

$\Leftrightarrow \|\overrightarrow{AH}\|^2 = 27 \underset{\|\overrightarrow{AH}\| > 0}{\Leftrightarrow} \|\overrightarrow{AH}\| = \sqrt{27} = 3\sqrt{3}$

2.1

a. $\overrightarrow{DA}$

b. $\overrightarrow{AB}$ e $\overrightarrow{BC}$, por exemplo.

c. $\overrightarrow{AB}$ e $\overrightarrow{EF}$, por exemplo.

d. $\overrightarrow{AB}$

2.2

a. $A + \overrightarrow{BC} = D$

b. $A - \frac{1}{2}\overrightarrow{CA} = A + \frac{1}{2}\overrightarrow{AC} = O$

c. $\overrightarrow{FO} + \overrightarrow{AO} = \overrightarrow{FO} + \overrightarrow{OC} = \overrightarrow{FC}$

d. $\overrightarrow{CE} - (E - F) = \overrightarrow{CE} - \overrightarrow{FE} = \overrightarrow{CE} + \overrightarrow{EF} = \overrightarrow{CF}$

PÁG. 92

3.1

a. $Q+\overrightarrow{XT}=P$

b. $X-\overrightarrow{UT}=X+\overrightarrow{TU}=X+\overrightarrow{XV}=V$

c. $\overrightarrow{RX}+\overrightarrow{SP}=\overrightarrow{RX}+\overrightarrow{XV}=\overrightarrow{RV}$

d. $\overrightarrow{TV}-\overrightarrow{SR}=\overrightarrow{TV}+\overrightarrow{RS}=\overrightarrow{TV}+\overrightarrow{VU}=\overrightarrow{TU}$

e. $U-Q-\overrightarrow{SP}=\overrightarrow{QU}+\overrightarrow{PS}=\overrightarrow{QU}+\overrightarrow{UT}=\overrightarrow{QT}$

f. $S+\left(-\overrightarrow{TU}-\overrightarrow{VT}\right)=S+\left(\overrightarrow{UT}+\overrightarrow{TV}\right)=S+\overrightarrow{UV}=S+\overrightarrow{SR}=R$

3.2 $\overrightarrow{XP}$

3.3 $V=20 \Leftrightarrow \|\overrightarrow{QP}\|\times\|\overrightarrow{PS}\|\times\|\overrightarrow{ST}\|=20 \Leftrightarrow 5\times\|\overrightarrow{PS}\|\times\|\overrightarrow{PS}\|=20 \Leftrightarrow$

$\Leftrightarrow \|\overrightarrow{PS}\|^2=4 \underset{\|\overrightarrow{PS}\|>0}{\Leftrightarrow} \|\overrightarrow{PS}\|=2$

$\|\overrightarrow{QT}\|^2=\|\overrightarrow{QP}\|^2+\|\overrightarrow{PS}\|^2+\|\overrightarrow{ST}\|^2 \Leftrightarrow \|\overrightarrow{QT}\|^2=5^2+\|\overrightarrow{PS}\|^2+\|\overrightarrow{PS}\|^2 \Leftrightarrow$

$\Leftrightarrow \|\overrightarrow{QT}\|^2=5^2+2^2+2^2 \Leftrightarrow \|\overrightarrow{QT}\|^2=33 \underset{\|\overrightarrow{QT}\|>0}{\Leftrightarrow} \|\overrightarrow{QT}\|=\sqrt{33}$

4.1

a. $E-\overrightarrow{BG}=E+\overrightarrow{GB}=E+\overrightarrow{ED}=D$

b. $\overrightarrow{EB}-\overrightarrow{EC}=\overrightarrow{EB}+\overrightarrow{CE}=\overrightarrow{CE}+\overrightarrow{EB}=\overrightarrow{CB}$

4.2 $\overrightarrow{FH}+\overrightarrow{CA}=\vec{0}$ e $\|\overrightarrow{FH}+\overrightarrow{CA}\|=\|\vec{0}\|=0$

4.3 $V=64 \Leftrightarrow \|\overrightarrow{AB}\|^3=64 \Leftrightarrow \|\overrightarrow{AB}\|=4$

$\|\overrightarrow{BD}\|^2=\|\overrightarrow{BA}\|^2+\|\overrightarrow{AD}\|^2 \Leftrightarrow \|\overrightarrow{BD}\|^2=4^2+4^2 \Leftrightarrow \|\overrightarrow{BD}\|^2=32 \underset{\|\overrightarrow{BD}\|>0}{\Leftrightarrow} \|\overrightarrow{BD}\|=\sqrt{32}=4\sqrt{2}$

5. $\vec{t}\,(2,\ 0,\ 0)$, $\vec{s}\,(0,\ 0,\ -5)$, $\vec{u}\,(-2,\ 2,\ 0)$, $\vec{w}\,(2,\ 0,\ -5)$, $\vec{v}\,(-2,\ 2,\ 5)$

6.1 $V=\|\overrightarrow{AB}\|^2=\sqrt{0^2+1^2+0^2}^{\,2}=1$

6.2

a. $\overrightarrow{BA}\,(0,\ -1,\ 0)$

b. $\overrightarrow{BC}\,(-1,\ 0,\ 0)$

c. $\overrightarrow{GC}\,(0,\ 0,\ -1)$

d. $\overrightarrow{AB}+\overrightarrow{BG}=\overrightarrow{AG}\,(-1,\ 1,\ 1)$

PÁG. 93

7.1 $B(1\ ,\ 7\ ,\ 0)\ ,\ D(0\ ,\ 2\ ,\ 0)\ ,\ C(0\ ,\ 7\ ,\ 0)\ ,\ E(0\ ,\ 2\ ,\ 1)\ ,\ H(0\ ,\ 7\ ,\ 1)\ ,$
$F(1\ ,\ 2\ ,\ 1)\ ,\ G(1\ ,\ 7\ ,\ 1)$

7.2 $\overrightarrow{AH}(0-1\ ,\ 7-2\ ,\ 1-0)=(-1\ ,\ 5\ ,\ 1)\ ,\ (x\ ,\ y\ ,\ z)=(1\ ,\ 2\ ,\ 0)+\lambda(-1\ ,\ 5\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$

7.3 $\overrightarrow{FB}(1-1\ ,\ 7-2\ ,\ 0-1)=(0\ ,\ 5\ ,\ -1)\ ,\ \vec{u}=\lambda(0\ ,\ 5\ ,\ -1)=(0\ ,\ 5\lambda\ ,\ -\lambda)\ ,\ \lambda\in\mathbb{R}$

$\|\vec{u}\|=10 \Leftrightarrow \sqrt{0^2+(5\lambda)^2+(-\lambda)^2}=10 \Rightarrow 26\lambda^2=100 \Leftrightarrow$

$\Leftrightarrow \lambda^2=\frac{100}{26} \Leftrightarrow \lambda=\pm\sqrt{\frac{100}{26}} \Leftrightarrow \lambda=\pm\frac{10\sqrt{26}}{26}$

$\vec{u}=-\frac{10\sqrt{26}}{26}(0\ ,\ 5\ ,\ -1) \vee \vec{u}=\frac{10\sqrt{26}}{26}(0\ ,\ 5\ ,\ -1) \Leftrightarrow$

$\Leftrightarrow \vec{u}=\left(0\ ,\ -\frac{25\sqrt{26}}{13}\ ,\ \frac{5\sqrt{26}}{13}\right) \vee \vec{u}=\left(0\ ,\ \frac{25\sqrt{26}}{13}\ ,\ -\frac{5\sqrt{26}}{13}\right)$

Como $\vec{u}$ tem sentido oposto a $\overrightarrow{FB}$, $\vec{u}=\left(0\ ,\ -\frac{25\sqrt{26}}{13}\ ,\ \frac{5\sqrt{26}}{13}\right)$.

8.1

a. $\overrightarrow{ID}(-2\ ,\ 2\ ,\ -2)$

b. $\overrightarrow{AK}(-2\ ,\ 4\ ,\ 2)$

c. $2\overrightarrow{CD}(-4\ ,\ 0\ ,\ 0)$

d. $\frac{1}{2}\overrightarrow{AC}-\overrightarrow{KF}=\overrightarrow{AB}+\overrightarrow{FK}=(0\ ,\ 2\ ,\ 0)+(0\ ,\ 4\ ,\ 2)=(0\ ,\ 6\ ,\ 2)$

8.2

a. $(x\ ,\ y\ ,\ z)=(2\ ,\ 0\ ,\ 0)+\lambda(-2\ ,\ 0\ ,\ 2)\ ,\ \lambda\in\mathbb{R}$

b. $(x\ ,\ y\ ,\ z)=(2\ ,\ -2\ ,\ 2)+\lambda(-2\ ,\ -2\ ,\ 2)\ ,\ \lambda\in\mathbb{R}$

8.3 $A=2\times 2^2+4\times 4\times 2=40$

9.1 $\overrightarrow{GB}-\frac{1}{2}\overrightarrow{FB}=\overrightarrow{GB}+\frac{1}{2}\overrightarrow{BF}=\frac{1}{2}\overrightarrow{HD}$

9.2 $\overrightarrow{GA}+\overrightarrow{AE}-2\overrightarrow{BC}=\overrightarrow{GE}+2\overrightarrow{CB}=$

$=[(-3\ ,\ -4\ ,\ 2)-(3\ ,\ -4\ ,\ -2)]+2[(3\ ,\ 4\ ,\ -2)-(-3\ ,\ 4\ ,\ -2)]=(6\ ,\ 0\ ,\ 4)$

PÁG. 94

10.1

a. $3\vec{u}-2\vec{v}=3(1\ ,\ -2\ ,\ -4)-2(-2\ ,\ 1\ ,\ 3)=(7\ ,\ -8\ ,\ -18)$ vetor

b. $A-2\vec{v}+\vec{u}=(1\ ,\ 0\ ,\ 1)-2(-2\ ,\ 1\ ,\ 3)+(1\ ,\ -2\ ,\ -4)=(6\ ,\ -4\ ,\ -9)$ ponto

c. $\frac{2}{3}\vec{u}-\frac{1}{3}(3\vec{u}-\vec{v})=-\frac{1}{3}\vec{u}+\frac{1}{3}\vec{v}=-\frac{1}{3}(1\ ,\ -2\ ,\ -4)+\frac{1}{3}(-2\ ,\ 1\ ,\ 3)=\left(-1\ ,\ 1\ ,\ \frac{7}{3}\right)$ vetor

d. $-\overrightarrow{v}+\left(\overrightarrow{BA}-2\overrightarrow{u}\right)=-(-2\,,\;1\,,\;3)+[(1-1\,,\;0-1\,,\;1+3)-2\,(1\,,\;-2\,,\;-4)]=(0\,,\;2\,,\;9)$ vetor

e. $-2\overrightarrow{AB}+3\left(\overrightarrow{u}-2\overrightarrow{BA}\right)=3\overrightarrow{u}-4\overrightarrow{BA}=3\,(1\,,\;-2\,,\;-4)-4\,(0\,,\;-1\,,\;4)=(3\,,\;-2\,,\;-28)$ vetor

f. $-3\overrightarrow{v}+2\,(B-A)=-3\overrightarrow{v}+2\overrightarrow{AB}=-3\,(-2\,,\;1\,,\;3)+2\,(0\,,\;1\,,\;-4)=(6\,,\;-1\,,\;-17)$ vetor

10.2 $C\,(x\,,\;y\,,\;z)$: $\overrightarrow{AC}=2\,(\overrightarrow{v}-\overrightarrow{u})$

$\overrightarrow{AC}=2\,(\overrightarrow{v}-\overrightarrow{u})\;\Leftrightarrow\;(x-1\,,\;y-0\,,\;z-1)=2\,(-2-1\,,\;1+2\,,\;3+4)\;\Leftrightarrow$

$\Leftrightarrow\;(x-1\,,\;y\,,\;z-1)=(-6\,,\;6\,,\;14)\;\Leftrightarrow$

$\Leftrightarrow\;x-1=-6\wedge y=6\wedge z-1=14\;\Leftrightarrow\;x=-5\wedge y=6\wedge z=15$

$C\,(-5\,,\;6\,,\;15)$

10.3

a. $\left\|\overrightarrow{AB}\right\|=\sqrt{0^2+1^2+(-4)^2}=\sqrt{17}$

b. $\left\|\overrightarrow{u}+\overrightarrow{v}\right\|=\left\|(-1\,,\;-1\,,\;-1)\right\|=\sqrt{(-1)^2+(-1)^2+(-1)^2}=\sqrt{3}$

11.1 $\left\|\overrightarrow{u}\right\|=\sqrt{0^2+0^2+(-10)^2}=10$

11.2 $\left\|\overrightarrow{v}\right\|=\sqrt{(-3)^2+0^2+(-1)^2}=\sqrt{10}$

11.3 $\left\|\overrightarrow{w}\right\|=\sqrt{1^2+2^2+3^2}=\sqrt{14}$

12. $\left\|\overrightarrow{v}\right\|=\sqrt{(-3)^2+3^2+3^2}=\sqrt{27}$

$\left\|\overrightarrow{u}\right\|=\sqrt{27}\;\Leftrightarrow\;\sqrt{(-3)^2+k^2+(k-1)^2}=\sqrt{27}\;\Rightarrow\;2k^2-2k+10=27\;\Leftrightarrow\;2k^2-2k-17=0\;\Leftrightarrow$

$\Leftrightarrow\;k=\dfrac{2\pm\sqrt{(-2)^2-4\times2\,(-17)}}{2\times2}\;\Leftrightarrow\;k=\dfrac{2\pm\sqrt{140}}{4}\;\Leftrightarrow\;k=\dfrac{2\pm2\sqrt{35}}{4}\;\Leftrightarrow\;k=\dfrac{1\pm\sqrt{35}}{2}$

13. $\left\|\overrightarrow{AB}\right\|=\sqrt{(-2)^2+(-2)^2+(-6)^2}=\sqrt{44}$

$\overrightarrow{u}=\lambda\overrightarrow{AB}=\lambda\,(-2\,,\;-2\,,\;-6)\,,\;\lambda\in\mathbb{R}$

$\left\|\overrightarrow{u}\right\|=5\;\Leftrightarrow\;\sqrt{(-2\lambda)^2+(-2\lambda)^2+(-6\lambda)^2}=5\;\Rightarrow\;44\lambda^2=25\;\Leftrightarrow\;\lambda^2=\dfrac{25}{44}\;\Leftrightarrow$

$\Leftrightarrow\;\lambda=\pm\sqrt{\dfrac{25}{44}}\;\Leftrightarrow\;\lambda=\pm\dfrac{5\sqrt{11}}{22}$

$\overrightarrow{u}=-\dfrac{5\sqrt{11}}{22}(-2\,,\;-2\,,\;-6)\vee\overrightarrow{u}=\dfrac{5\sqrt{11}}{22}(-2\,,\;-2\,,\;-6)\;\Leftrightarrow$

$\Leftrightarrow\;\overrightarrow{u}=\left(\dfrac{5\sqrt{11}}{11}\,,\;\dfrac{5\sqrt{11}}{11}\,,\;\dfrac{15\sqrt{11}}{11}\right)\vee\overrightarrow{u}=\left(-\dfrac{5\sqrt{11}}{11}\,,\;-\dfrac{5\sqrt{11}}{11}\,,\;-\dfrac{15\sqrt{11}}{11}\right)$

Como $\overrightarrow{u}$ tem o mesmo sentido que $\overrightarrow{AB}$, $\overrightarrow{u}=\left(-\dfrac{5\sqrt{11}}{11}\,,\;-\dfrac{5\sqrt{11}}{11}\,,\;-\dfrac{15\sqrt{11}}{11}\right)$.

14. $\vec{u}$ e $\vec{s}$; $\vec{v}$ e $\vec{t}$

15. $\frac{k}{-3}=\frac{1}{\frac{k}{2}} \Leftrightarrow k^2=-6$ impossível,

logo não existe nenhum valor de k para o qual os vetores sejam colineares.

PÁG. 95

16. $Q(x\ ,\ y\ ,\ z)$: $\|\overrightarrow{PQ}\|=6$ e $\overrightarrow{PQ}=\lambda\vec{a} \Leftrightarrow (x-1\ ,\ y\ ,\ z+1)=(\lambda\ ,\ -2\lambda\ ,\ \lambda)\ ,\ \lambda\in\mathbb{R}$

$\|\overrightarrow{PQ}\|=6 \Leftrightarrow \sqrt{\lambda^2+(-2\lambda)^2+\lambda^2}=6 \Rightarrow 6\lambda^2=36 \Leftrightarrow \lambda^2=6 \Leftrightarrow \lambda=-\sqrt{6} \vee \lambda=\sqrt{6}$

Se $\lambda=-\sqrt{6}$, $\overrightarrow{PQ}=\left(-\sqrt{6}\ ,\ -2\left(-\sqrt{6}\right)\ ,\ -\sqrt{6}\right)=\left(-\sqrt{6}\ ,\ 2\sqrt{6}\ ,\ -\sqrt{6}\right)$;

se $\lambda=\sqrt{6}$, $\overrightarrow{PQ}=\left(\sqrt{6}\ ,\ -2\sqrt{6}\ ,\ \sqrt{6}\right)$.

Como $\overrightarrow{PQ}$ tem o mesmo sentido que $\vec{a}$, conclui-se que $\lambda=\sqrt{6}$ e

$(x-1\ ,\ y\ ,\ z+1)=\left(\sqrt{6}\ ,\ -2\sqrt{6}\ ,\ \sqrt{6}\right) \Leftrightarrow (x-1\ ,\ y\ ,\ z+1)=\left(\sqrt{6}\ ,\ -2\sqrt{6}\ ,\ \sqrt{6}\right) \Leftrightarrow$

$\Leftrightarrow x-1=\sqrt{6} \wedge y=-2\sqrt{6} \wedge z+1=\sqrt{6} \Leftrightarrow x=1+\sqrt{6} \wedge y=-2\sqrt{6} \wedge z=-1+\sqrt{6}$.

Logo, $Q\left(1+\sqrt{6}\ ,\ -2\sqrt{6}\ ,\ -1+\sqrt{6}\right)$.

17.1 $\overrightarrow{AB}(2-1\ ,\ 0-1\ ,\ -1-1)=(1\ ,\ -1\ ,\ -2)$,

$(x\ ,\ y\ ,\ z)=(1\ ,\ 1\ ,\ 1)+\lambda(1\ ,\ -1\ ,\ -2)\ ,\ \lambda\in\mathbb{R}$

17.2 $\overrightarrow{AB}\left(\frac{5}{2}+\frac{1}{2}\ ,\ 1-0\ ,\ \frac{1}{2}-\frac{1}{2}\right)=(3\ ,\ 1\ ,\ 0)$,

$(x\ ,\ y\ ,\ z)=\left(-\frac{1}{2}\ ,\ 0\ ,\ \frac{1}{2}\right)+\lambda(3\ ,\ 1\ ,\ 0)\ ,\ \lambda\in\mathbb{R}$

18.1 $(1\ ,\ 0\ ,\ -2)$

18.2 O ponto tem coordenadas da forma $(1+\lambda\ ,\ 0-2\lambda\ ,\ -2+3\lambda)\ ,\ \lambda\in\mathbb{R}$.

Como o ponto tem ordenada 2 , vem $0-2\lambda=2 \Leftrightarrow \lambda=-1$.

Logo, o ponto tem coordenadas $(0\ ,\ 2\ ,\ -5)$.

18.3 $(11\ ,\ -20\ ,\ 28)=(1\ ,\ 1\ ,\ 1)+\lambda(1\ ,\ -1\ ,\ -2)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}11=1+\lambda\\ -20=0-2\lambda\\ 28=-2+3\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases}11=1+10\\ \lambda=10\\ 28=-2+3\times 10\end{cases} \Leftrightarrow \begin{cases}11=11\\ \lambda=10\\ 28=28\end{cases}$, o ponto pertence à reta.

18.4 $(x\ ,\ y\ ,\ z)=(0\ ,\ 0\ ,\ 0)+\lambda(1\ ,\ -2\ ,\ 3)\ ,\ \lambda\in\mathbb{R}$, por exemplo.

18.5 $(x\ ,\ 0\ ,\ z)=(1\ ,\ 1\ ,\ 1)+\lambda(1\ ,\ -1\ ,\ -2)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}x=1+\lambda\\ 0=0-2\lambda\\ z=-2+3\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow$

$\Leftrightarrow \begin{cases}x=1+0\\ \lambda=0\\ z=-2+3\times 0\end{cases} \Leftrightarrow \begin{cases}x=1\\ \lambda=0\\ z=-2\end{cases}$, a reta não interseta o eixo das abcissas.

19. A reta e o eixo admitem ambos o vetor diretor de coordenadas $(0\ ,\ 3\ ,\ 0)$.

20. $(x\ ,\ y\ ,\ z)=(0\ ,\ 0\ ,\ 0)+\lambda(0\ ,\ 0\ ,\ 1)\ ,\ \lambda\in\mathbb{R}$, por exemplo.

21.1 $(x\ ,\ y\ ,\ z)=(0\ ,\ 0\ ,\ 0)+\lambda(-6\ ,\ 8\ ,\ 10)\ ,\ \lambda\in\mathbb{R}$, por exemplo.

21.2 Medida da aresta do cubo:

$$\|\overrightarrow{AE}\|^2=\sqrt{(-6)^2+8^2+10^2}^{\,2}=200$$

$$\|\overrightarrow{AE}\|^2=\|\overrightarrow{AD}\|^2+\|\overrightarrow{DE}\|^2 \Leftrightarrow 200=2\|\overrightarrow{AD}\|^2 \Leftrightarrow \|\overrightarrow{AD}\|^2=100 \underset{\|\overrightarrow{AD}\|>0}{\Leftrightarrow} \|\overrightarrow{AD}\|=10$$

$$B+\overrightarrow{AE}=H \Leftrightarrow H=(14\ ,\ -7\ ,\ 0)+(-6\ ,\ 8\ ,\ 10) \Leftrightarrow H(8\ ,\ 1\ ,\ 10)$$

Como a face $[ABCD]$ está contida no plano xOy , os vértices A , B , C e D têm cota nula.

Como a aresta do cubo é 10 , os vértices E , F , G e H têm cota 10 .

Os vértices D e E pertencem à reta ED , logo têm abcissa nula e ordenada -5 .

Assim, $D(0\ ,\ -5\ ,\ 0)$ e $E(0\ ,\ -5\ ,\ 10)$.

$$\overrightarrow{AE}(-6\ ,\ 8\ ,\ 10) \Leftrightarrow A=(0\ ,\ -5\ ,\ 10)-(-6\ ,\ 8\ ,\ 10) \Leftrightarrow A(6\ ,\ -13\ ,\ 0)$$

$$\overrightarrow{BD}(0-14\ ,\ -5+7\ ,\ 0-0)=(-14\ ,\ 2\ ,\ 0)$$

$$G+\overrightarrow{GE}=E \Leftrightarrow G=(0\ ,\ -5\ ,\ 10)-(-14\ ,\ 2\ ,\ 0) \Leftrightarrow G(14\ ,\ -7\ ,\ 10)$$

$$\overrightarrow{BG}(14-14\ ,\ -7+7\ ,\ 10-0)=(0\ ,\ 0\ ,\ 10)$$

$$A+\overrightarrow{AF}=F \Leftrightarrow F=(6\ ,\ -13\ ,\ 0)+(0\ ,\ 0\ ,\ 10) \Leftrightarrow F(6\ ,\ -13\ ,\ 10)$$

$$\overrightarrow{FH}(8-6\ ,\ 1+13\ ,\ 10-10)=(2\ ,\ 14\ ,\ 0)$$

$$A+\overrightarrow{AC}=C \Leftrightarrow C=(6\ ,\ -13\ ,\ 0)+(2\ ,\ 14\ ,\ 0) \Leftrightarrow C(8\ ,\ 1\ ,\ 0)$$

PÁG. 96

22.1 Coordenadas dos vértices do prisma:

$O(0\ ,\ 0\ ,\ 0)$

$CE:\ (x\ ,\ y\ ,\ z)=(0\ ,\ 5\ ,\ -2)+\lambda(0\ ,\ 10\ ,\ 4)\ ,\ \lambda\in\mathbb{R}$

Interseção com Oy :

$$(0\ ,\ y\ ,\ 0)=(0\ ,\ 5\ ,\ -2)+\lambda(0\ ,\ 10\ ,\ 4)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}0=0+0\\ y=5+10\lambda\\ 0=-2+4\lambda\end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}0=0+0\\ y=5+10\times\frac{1}{2}\\ \lambda=\frac{1}{2}\end{cases},$$

$E(0\ ,\ 10\ ,\ 0)$

Interseção com Oz :

$(0\ ,\ 0\ ,\ z)=(0\ ,\ 5\ ,\ -2)+\lambda(0\ ,\ 10\ ,\ 4)\ ,\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases}0=0+0\\0=5+10\lambda\ ,\ \lambda\in\mathbb{R}\\z=-2+4\lambda\end{cases} \Leftrightarrow \begin{cases}0=0+0\\\lambda=-\frac{1}{2}\\z=-2+4\left(-\frac{1}{2}\right)\end{cases}$,

$C(0\ ,\ 0\ ,\ -4)$

$B(0\ ,\ 10\ ,\ -4)$

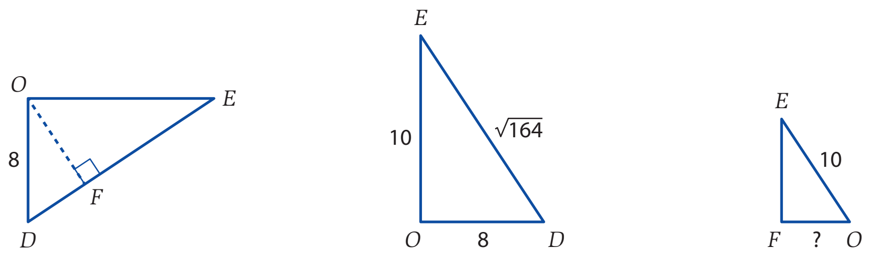
$\|\overrightarrow{OD}\|=\frac{4}{5}\|\overrightarrow{OE}\|=\frac{4}{5}\times 10=8$

$D(8\ ,\ 0\ ,\ 0)$ e $A(8\ ,\ 0\ ,\ -4)$

a. $(x\ ,\ y\ ,\ z)=(0\ ,\ 10\ ,\ -4)+\lambda(8\ ,\ -10\ ,\ 4)\ ,\ \lambda\in\mathbb{R}$, por exemplo

b. $x^2+(y-10)^2+z^2=16$

c.



$\frac{\overline{OF}}{8}=\frac{10}{\sqrt{164}} \Leftrightarrow \overline{OF}=\frac{80}{\sqrt{164}}=\frac{80}{2\sqrt{41}}=\frac{40\sqrt{41}}{41}$

22.2 $V=\frac{\|\overrightarrow{OD}\|\times\|\overrightarrow{OE}\|}{2}\times\|\overrightarrow{AD}\| \Leftrightarrow \frac{\|\overrightarrow{OD}\|\times\|\overrightarrow{OE}\|}{2}\times 2=20 \Leftrightarrow \|\overrightarrow{OD}\|\times\|\overrightarrow{OE}\|=20 \Leftrightarrow$

$\Leftrightarrow x\times\frac{5}{4}x=20 \Leftrightarrow x^2=\frac{80}{5} \Leftrightarrow x^2=16 \underset{x>0}{\Leftrightarrow} x=4$

$D(4\ ,\ 0\ ,\ 0)$ e $E(0\ ,\ 5\ ,\ 0)$, $\overrightarrow{DE}(-4\ ,\ 5\ ,\ 0)$

$A(4\ ,\ 0\ ,\ -2)$

A reta AB é paralela à rera DE .

$(x\ ,\ y\ ,\ z)=(4\ ,\ 0\ ,\ -2)+\lambda(-4\ ,\ 5\ ,\ 0)\ ,\ \lambda\in\mathbb{R}$

23.1 $D=A+\overrightarrow{BC}=(14\ ,\ -7\ ,\ 4)+(10-16\ ,\ -6+4\ ,\ 13-10)=(8\ ,\ -9\ ,\ 7)$

$F=B+\overrightarrow{AE}=(16\ ,\ -4\ ,\ 10)+(8-14\ ,\ 5+7\ ,\ 0-4)=(10\ ,\ 8\ ,\ 6)$

$G=C+\overrightarrow{AE}=(10\ ,\ -6\ ,\ 13)+(8-14\ ,\ 5+7\ ,\ 0-4)=(4\ ,\ 6\ ,\ 9)$

$H=C+\overrightarrow{BA}=(4\ ,\ 6\ ,\ 9)+(14-16\ ,\ -7+4\ ,\ 4-10)=(2\ ,\ 3\ ,\ 3)$

23.2 $V = \|\overrightarrow{AB}\|^2 \times \|\overrightarrow{AE}\|$

$\|\overrightarrow{AB}\|^2 = \sqrt{(16-14)^2 + (-4+7)^2 + (10-4)^2}^{\,2} = 49$

$\|\overrightarrow{AE}\| = \sqrt{(8-14)^2 + (5+7)^2 + (0-4)^2} = 14$

$V = 49 \times 14 = 686$

23.3 $(x\,,\ y\,,\ z) = (14\,,\ -7\,,\ 4) + \lambda\,(2\,,\ 3\,,\ 6)\,,\ \lambda \in \mathbb{R}\,,$ por exemplo

23.4 Centro: $M_{[CE]}\left(\frac{10+8}{2}\,,\ \frac{-6+5}{2}\,,\ \frac{13+0}{2}\right) = \left(9\,,\ -\frac{1}{2}\,,\ \frac{13}{2}\right)$

Raio: $\frac{\|\overrightarrow{CE}\|}{2} = \frac{\sqrt{(8-10)^2 + (5+6)^2 + (0-13)^2}}{2} = \frac{\sqrt{294}}{2}$

$(x-9)^2 + \left(y+\frac{1}{2}\right)^2 + \left(z-\frac{13}{2}\right)^2 = \frac{147}{2}$

23.5 DBF é o plano mediador de $[AC]$.

$(x-14)^2 + (y+7)^2 + (z-4)^2 = (x-10)^2 + (y+6)^2 + (z-13)^2 \Leftrightarrow$

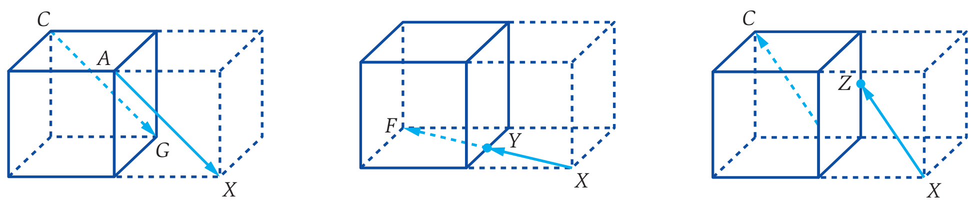
$\Leftrightarrow -28x + 196 + 14y + 49 - 8z + 16 = -20x + 100 + 12y + 36 - 26z + 169 \Leftrightarrow$

$\Leftrightarrow -28x + 20x + 14y - 12y - 8z + 26z = -196 - 49 - 16 + 100 + 36 + 169 \Leftrightarrow$

$\Leftrightarrow -8x + 2y + 18z = 44 \Leftrightarrow 4x - y - 9z = -22$

PÁG. 97

24.1



24.2 $W(x\,,\ y\,,\ z)\,,\ X(1\,,\ 3\,,\ -1)\,,\ B(-1\,,\ 1\,,\ 1)$

$\overline{XW} = \overline{XB} \Leftrightarrow (x-1)^2 + (y-3)^2 + (z+1)^2 = (-1-1)^2 + (1-3)^2 + (1+1)^2 \Leftrightarrow$

$\Leftrightarrow (x-1)^2 + (y-3)^2 + (z+1)^2 = 12$ superfície esférica de centro no ponto X que contém o ponto B

24.3 O plano mediador de $[BD]$ é o plano CAH.

$(x+1)^2 + (y-1)^2 + (z-1)^2 = (x-1)^2 + (y+1)^2 + (z-1)^2 \Leftrightarrow$

$\Leftrightarrow 2x + 1 - 2y + 1 = -2x + 1 + 2y + 1 \Leftrightarrow 2x + 2x = 2y + 2y \Leftrightarrow x = y$

Interseção da reta com o plano:

$(x\,,\ y\,,\ z) = (1\,,\ -1\,,\ -1) + \lambda\,(0\,,\ 1\,,\ 1)\,,\ \lambda \in \mathbb{R} \wedge x = y \Leftrightarrow$

$\begin{cases} x = 1 + 0 \\ y = -1 + \lambda \\ z = -1 + \lambda \\ x = y \end{cases},\ \lambda \in \mathbb{R} \Leftrightarrow \begin{cases} x = 1 \\ 1 = -1 + \lambda \\ z = -1 + \lambda \\ y = 1 \end{cases},\ \lambda \in \mathbb{R} \Leftrightarrow \begin{cases} x = 1 \\ \lambda = 2 \\ z = -1 + 2 \\ y = 1 \end{cases},\ A(1\,,\ 1\,,\ 1)$

25. (D)

$B(x_B\ ,\ 0\ ,\ z_B)\ ,\ D(0\ ,\ y_D\ ,\ 0)$

$\overrightarrow{BE}(-3\ ,\ 8\ ,\ 1)\ \Leftrightarrow\ x_E - x_B = -3 \wedge y_E - 0 = 8 \wedge z_E - z_B = 1$

$\overrightarrow{DE}(1\ ,\ 7\ ,\ 5)\ \Leftrightarrow\ x_E - 0 = 1 \wedge y_E - y_D = 7 \wedge z_E - 0 = 1\ \Leftrightarrow\ x_E = 1 \wedge 8 - y_D = 7 \wedge z_E = 1$

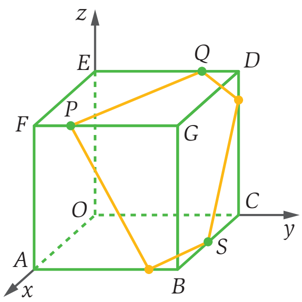
$E(1\ ,\ 8\ ,\ 5)$

$1 - x_B = -3 \wedge 5 - z_B = 1\ \Leftrightarrow\ x_B = 4 \wedge z_B = 4$

Logo, $B(4\ ,\ 0\ ,\ 4)$.

PÁG. 98

26.1



$P(10\ ,\ 3\ ,\ 10)\ ,\ Q(0\ ,\ 7\ ,\ 10)\ ,\ S(5\ ,\ 10\ ,\ 0)$

$\overrightarrow{PQ}(0-10\ ,\ 7-3\ ,\ 10-10) = (-10\ ,\ 4\ ,\ 0)\ ,$

$TS:\ (x\ ,\ y\ ,\ z) = (5\ ,\ 10\ ,\ 0) + \lambda(-10\ ,\ 4\ ,\ 0)\ ,\ \lambda \in \mathbb{R}$

O ponto T resulta da interseção das retas TS e AB :

$(x\ ,\ y\ ,\ z) = (5\ ,\ 10\ ,\ 0) + \lambda(-10\ ,\ 4\ ,\ 0)\ ,\ \lambda \in \mathbb{R} \wedge x = 10 \wedge z = 0\ \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x = 5 - 10\lambda \\ y = 10 + 4\lambda \\ z = 0 + 0 \\ x = 10 \\ z = 0 \end{cases},\ \lambda \in \mathbb{R}\ \Leftrightarrow \begin{cases} 10 = 5 - 10\lambda \\ y = 10 + 4\lambda \\ z = 0 \\ x = 10 \\ z = 0 \end{cases},\ \lambda \in \mathbb{R}\ \Leftrightarrow \begin{cases} \lambda = -\frac{1}{2} \\ y = 10 + 4\left(-\frac{1}{2}\right) \\ z = 0 \\ x = 10 \end{cases},\ T(10\ ,\ 8\ ,\ 0)$$

$\overrightarrow{PT}(10-10\ ,\ 8-3\ ,\ 0-10) = (0\ ,\ 5\ ,\ -10)\ ,$

$QR:\ (x\ ,\ y\ ,\ z) = (0\ ,\ 7\ ,\ 10) + \lambda(0\ ,\ 5\ ,\ -10)\ ,\ \lambda \in \mathbb{R}$

O ponto R resulta da interseção das retas QR e CD :

$(x\ ,\ y\ ,\ z) = (0\ ,\ 7\ ,\ 10) + \lambda(0\ ,\ 5\ ,\ -10)\ ,\ \lambda \in \mathbb{R} \wedge x = 0 \wedge y = 10\ \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x = 0 + 0 \\ y = 7 + 5\lambda \\ z = 10 - 10\lambda \\ x = 0 \\ y = 10 \end{cases},\ \lambda \in \mathbb{R}\ \Leftrightarrow \begin{cases} x = 0 \\ 10 = 7 + 5\lambda \\ z = 10 - 10\lambda \\ x = 0 \\ y = 10 \end{cases},\ \lambda \in \mathbb{R}\ \Leftrightarrow \begin{cases} x = 0 \\ \lambda = \frac{3}{5} \\ z = 10 - 10\left(\frac{3}{5}\right) \\ y = 10 \end{cases},\ R(0\ ,\ 10\ ,\ 4)$$

26.2 $(x, y, z)=(10, 3, 10)+\lambda(-10, 4, 0), \lambda\in\mathbb{R} \wedge y=0 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x=10-10\lambda \\ y=3+4\lambda \\ z=10+0\lambda \\ y=0 \end{cases}, \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=10-10\lambda \\ 0=3+4\lambda \\ z=10 \\ y=0 \end{cases}, \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} x=10-10\left(-\frac{3}{4}\right) \\ \lambda=-\frac{3}{4} \\ z=10 \\ y=0 \end{cases}, I\left(\frac{35}{2}, 0, 10\right)$$

$\overline{EC}=\sqrt{(0-0)^2+(10-0)^2+(0-10)^2}=\sqrt{200}, \overline{EC}^2=200$

$\overline{IC}=\sqrt{\left(0-\frac{35}{2}\right)^2+(10-0)^2+(0-10)^2}=\sqrt{\frac{2025}{4}}, \overline{IC}^2=\frac{2025}{4}$

$\overline{EI}=\sqrt{\left(\frac{35}{2}-0\right)^2+(0-0)^2+(10-10)^2}=\sqrt{\frac{1225}{4}}, \overline{EI}^2=\frac{1225}{4}$

$\frac{2025}{4}=\frac{1225}{4}+200 \Leftrightarrow \overline{IC}^2=\overline{EI}^2+\overline{EC}^2$, o triângulo $[EIC]$ é retângulo em E

$$A=\frac{\overline{EI}\times\overline{EC}}{2}=\frac{\sqrt{\frac{1225}{4}}\times\sqrt{200}}{2}=\frac{\frac{35}{2}\times 10\sqrt{2}}{2}=\frac{175\sqrt{2}}{2}$$

27.1 $(x, y, z)=(5, 0, 0)+k(-5, 5, 4), k\in\mathbb{R}\wedge y=0\wedge z=0 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x=5-5k \\ y=0+5k \\ z=0+4k \\ y=0 \\ z=0 \end{cases}, k\in\mathbb{R} \Leftrightarrow \begin{cases} x=5-5k \\ 0=0+5k \\ 0=0+4k \\ y=0 \\ z=0 \end{cases}, k\in\mathbb{R} \Leftrightarrow \begin{cases} x=5 \\ k=0 \\ k=0 \\ y=0 \\ z=0 \end{cases}, A(5, 0, 0)$$

$(x, y, z)=(0, -10, 16)+k(0, 5, -4), k\in\mathbb{R}\wedge x=0\wedge z=0 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x=0+0k \\ y=-10+5k \\ z=16-4k \\ x=0 \\ z=0 \end{cases}, k\in\mathbb{R} \Leftrightarrow \begin{cases} 0=0+0k \\ y=-10+5k \\ 0=16-4k \\ x=0 \\ z=0 \end{cases}, k\in\mathbb{R} \Leftrightarrow \begin{cases} 0=0 \\ y=-10+5\times 4 \\ k=4 \\ x=0 \\ z=0 \end{cases}, B(0, 10, 0)$$

Plano mediador de $[AB]$:

$(x-5)^2+y^2+z^2=x^2+(y-10)^2+z^2 \Leftrightarrow -10x+25=-20y+100 \Leftrightarrow$

$\Leftrightarrow -10x+20y+25-100=0 \Leftrightarrow 2x-4y+15=0$

Substituindo na equação do plano α, x e y por zero, $2\times 0-4\times 0+15=0 \Leftrightarrow 15=0$ impossível.

Logo, o plano α não interseta o eixo Oz.

27.2

$$P(x\,,\ y\,,\ 4):\ P\in r\ \Leftrightarrow \begin{cases} x=0+0k \\ y=-10+5k\,,\ k\in\mathbb{R} \\ 4=16-4k \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y=-10+5\times 3\,,\ P(0\,,\ 5\,,\ 4) \\ k=3 \end{cases}$$

$$P\in s\ \Leftrightarrow \begin{cases} 0=5-5k \\ 5=0+5k\,,\ k\in\mathbb{R} \\ 4=0+4k \end{cases} \Leftrightarrow \begin{cases} k=1 \\ k=1 \\ k=1 \end{cases}$$

$$OP:\ (x\,,\ y\,,\ z)=(0\,,\ 0\,,\ 0)+k(0\,,\ 5\,,\ 4)\,,\ k\in\mathbb{R}$$

$$(x\,,\ y\,,\ z)=(0\,,\ 0\,,\ 0)+k(0\,,\ 5\,,\ 4)\,,\ k\in\mathbb{R}\wedge 2x-4y+15=0\ \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x=0+0k \\ y=0+5k \\ z=0+4k \\ x=2y-\frac{15}{2} \end{cases},\ k\in\mathbb{R} \Leftrightarrow \begin{cases} 2y-\frac{15}{2}=0 \\ y=5k \\ z=4k \\ x=2y-\frac{15}{2} \end{cases},\ k\in\mathbb{R} \Leftrightarrow \begin{cases} y=\frac{15}{4} \\ \frac{15}{4}=5k \\ z=4k \\ x=2y-\frac{15}{2} \end{cases},\ k\in\mathbb{R} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y=\frac{15}{4} \\ k=\frac{3}{4} \\ z=4\left(\frac{3}{4}\right) \\ x=2\left(\frac{15}{4}\right)-\frac{15}{2} \end{cases} \Leftrightarrow \begin{cases} y=\frac{15}{4} \\ k=\frac{3}{4} \\ z=3 \\ x=0 \end{cases},\ \left(0\,,\ \frac{15}{4}\,,\ 3\right)$$

27.3 $(x\,,\ y\,,\ z)=(0\,,\ -10\,,\ 16)+k(0\,,\ 5\,,\ -4)\,,\ k\in\mathbb{R}\wedge x=0\wedge y=0\ \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x=0+0k \\ y=-10+5k \\ z=16-4k \\ x=0 \\ y=0 \end{cases},\ k\in\mathbb{R} \Leftrightarrow \begin{cases} 0=0+0k \\ 0=-10+5k \\ z=16-4k \\ x=0 \\ y=0 \end{cases},\ k\in\mathbb{R} \Leftrightarrow \begin{cases} 0=0 \\ k=2 \\ z=16-4\times 2\,,\ C(0\,,\ 0\,,\ 8) \\ x=0 \\ y=0 \end{cases}$$

$$V=\frac{1}{3}\times\frac{\overline{OA}\times\overline{OC}}{2}\times|y_P|=\frac{1}{3}\times\frac{5\times 8}{2}\times 5=\frac{100}{3}$$

PÁG. 99

28.1 $\overrightarrow{BC}(6\,,\ -4\,,\ -10)\ \Rightarrow\ C=B+(6\,,\ -4\,,\ -10)=$

$=(0\,,\ -3\,,\ 0)+(6\,,\ -4\,,\ -10)=(6\,,\ -7\,,\ -10)$

$\overrightarrow{AC}(6-5\,,\ -7-0\,,\ -10-0)=(1\,,\ -7\,,\ -10)$

$(x\,,\ y\,,\ z)=(5\,,\ 0\,,\ 0)+\lambda(1\,,\ -7\,,\ -10)\,,\ \lambda\in\mathbb{R}$

$(x\,,\ y\,,\ z)=(5\,,\ 0\,,\ 0)+\lambda(1\,,\ -7\,,\ -10)\,,\ \lambda\in\mathbb{R}\wedge x=0\ \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x=5+\lambda \\ y=0-7\lambda \\ z=0-10\lambda \\ x=0 \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} 0=5+\lambda \\ y=-7\lambda \\ z=-10\lambda \\ x=0 \end{cases},\ \lambda\in\mathbb{R} \Leftrightarrow \begin{cases} \lambda=-5 \\ y=-7(-5) \\ z=-10(-5) \\ x=0 \end{cases},\ (0\,,\ 35\,,\ 50)$$

28.2 $\overrightarrow{AC} - \overrightarrow{BC} = \overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB} = (-5\ ,\ -3\ ,\ 0)$

$\lambda^2 - 7\lambda + 6 = 0 \Leftrightarrow \lambda = \dfrac{7 \pm \sqrt{(-7)^2 - 4 \times 6}}{2} \Leftrightarrow \lambda = \dfrac{7 \pm 5}{2} \Leftrightarrow \lambda = 1 \vee \lambda = 6$

Se $\lambda = 1$, $\vec{u}\left(\dfrac{5}{3}\ ,\ 1\ ,\ 0\right)$ e $\|\vec{u}\| = \sqrt{\left(\dfrac{5}{3}\right)^2 + 1^2} = \sqrt{\dfrac{34}{9}} = \dfrac{\sqrt{34}}{3}$.

Se $\lambda = 6$, $\vec{u}(10\ ,\ 6\ ,\ 0)$ e $\|\vec{u}\| = \sqrt{10^2 + 6^2} = \sqrt{136} = 2\sqrt{34}$.

28.3

a. Raio: $\overline{AC} = \sqrt{(6-5)^2 + (-7-0)^2 + (-10-0)^2} = \sqrt{150}$

$(x-6)^2 + (y+7)^2 + (z+10)^2 \leqslant 150$

b. O plano AOB é o plano $z = 0$.

$(x-6)^2 + (y+7)^2 + (z+10)^2 \leqslant 150 \wedge z = 0 \Leftrightarrow (x-6)^2 + (y+7)^2 + (0+10)^2 \leqslant 150 \wedge z = 0 \Leftrightarrow$

$\Leftrightarrow (x-6)^2 + (y+7)^2 \leqslant 50 \wedge z = 0$

A secção definida na esfera pela interseção segundo o plano AOB é o círculo de centro no ponto de coordenadas $(6\ ,\ -7\ ,\ 0)$ e raio $\sqrt{50}$ contido no plano $z = 0$.

$\text{P} = 2\pi\sqrt{50}$ e $\text{A} = \pi\sqrt{50}^2 = 50\pi$

28.4 $\text{V} = \dfrac{1}{3} \times \dfrac{|\overline{OA}| \times |\overline{OB}|}{2} \times |z_c| = \dfrac{1}{3} \times \dfrac{5 \times 3}{2} \times 10 = 25$

PÁG. 100

29.1 $A(x_A\ ,\ y_A\ ,\ 0)$, $C(x_C\ ,\ 0\ ,\ z_C)$, $V(x_V\ ,\ y_V\ ,\ z_V)$

Como o ponto A pertence à reta AF , as suas coordenadas são da forma
$(-6-5k\ ,\ -11-7k\ ,\ 2+k)$, $k \in \mathbb{R}$.

Assim, $2 + k = 0 \Leftrightarrow k = -2$ e $A(4\ ,\ 3\ ,\ 0)$.

$\overrightarrow{AV}\left(-\dfrac{5}{2}\ ,\ -\dfrac{13}{2}\ ,\ \dfrac{9}{2}\right) \Rightarrow x_V - 4 = -\dfrac{5}{2} \wedge y_V - 3 = -\dfrac{13}{2} \wedge z_V - 0 = \dfrac{9}{2}$, $V\left(\dfrac{3}{2}\ ,\ -\dfrac{7}{2}\ ,\ \dfrac{9}{2}\right)$

$\overrightarrow{CV}\left(\dfrac{5}{2}\ ,\ -\dfrac{7}{2}\ ,\ \dfrac{1}{2}\right) \Rightarrow \dfrac{3}{2} - x_C = \dfrac{5}{2} \wedge -\dfrac{7}{2} - y_C = -\dfrac{7}{2} \wedge \dfrac{9}{2} - z_C = \dfrac{1}{2}$, $C(-1\ ,\ 0\ ,\ 4)$

29.2

a. O plano ADE é paralelo ao plano que contém a face $[BCFG]$ e contém o ponto A : $x = 4$.

b. $y = -\dfrac{7}{2} \wedge z = \dfrac{9}{2}$

c. $y = -1$

29.3

a. $A - \overrightarrow{CF} + \overrightarrow{AE} = A + \overrightarrow{AE} + \overrightarrow{FC} = E + \overrightarrow{FC} = E + \overrightarrow{ED} = D$

b. $\overrightarrow{AB} + 2 \times \left(\overrightarrow{AH} - \frac{1}{2}\overrightarrow{DE} \right) + \overrightarrow{GD} = \overrightarrow{AB} + \overrightarrow{AH} + \overrightarrow{GD} = \overrightarrow{AG} + \overrightarrow{GD} = \overrightarrow{AD}$

29.4

a. $P = H + \frac{1}{2}\overrightarrow{CE} - \overrightarrow{BH} - \overrightarrow{FG} = H + \overrightarrow{HB} + \overrightarrow{GF} + \frac{1}{2}\overrightarrow{CE} = C + \frac{1}{2}\overrightarrow{CE}$,

pelo que P é o ponto médio do segmento de reta $[CE]$ e consequentemente é o centro da face $[CFED]$ do cubo.

b. Como o ponto F pertence à reta AF, as suas coordenadas são da forma $(-6-5k\ ,\ -11-7k\ ,\ 2+k)$, $k \in \mathbb{R}$.

Por outro lado, o ponto F pertence ao plano que contém o ponto C, paralelo ao plano yOz, pelo que os dois pontos têm a mesma abcissa.

Assim, $-6-5k=-1 \Leftrightarrow k=-1$ e $F(-1\ ,\ -4\ ,\ 1)$.

O ponto G pertence ao plano que contém o ponto C, paralelo ao plano yOz, pelo que os dois pontos têm a mesma abcissa.

Assim, tem-se $G(-1\ ,\ y\ ,\ z)$.

Considerando o triângulo retângulo $[CFG]$, tem-se:

$\overline{CF} = \overline{FG}$:

$$\sqrt{(-1+1)^2+(y+4)^2+(z-1)^2} = 5 \Leftrightarrow (y+4)^2+(z-1)^2 = 25$$

$\overline{CG}^2 = \overline{CF}^2 + \overline{FG}^2$:

$$\sqrt{(-1+1)^2+(y-0)^2+(z-4)^2}^{\,2} = \sqrt{(-1+1)^2+(-4-0)^2+(1-4)^2}^{\,2} + \sqrt{(-1+1)^2+(y+4)^2+(z-1)^2}^{\,2} \Leftrightarrow$$

$$\Leftrightarrow y^2+(z-4)^2 = (-4)^2+(-3)^2+(y+4)^2+(z-1)^2 \Leftrightarrow$$

$$\Leftrightarrow -8z+16 = 16+9+8y+16-2z+1 \Leftrightarrow -8y = 6z+26 \Leftrightarrow y = -\frac{3}{4}z - \frac{13}{4}$$

Substituindo y por $-\frac{3}{4}z - \frac{13}{4}$, vem

$$\left(-\frac{3}{4}z - \frac{13}{4} + 4\right)^2 + (z-1)^2 = 25 \Leftrightarrow \left(-\frac{3}{4}z + \frac{3}{4}\right)^2 + (z-1)^2 = 25 \Leftrightarrow$$

$$\Leftrightarrow \frac{9}{16}z^2 - \frac{18}{16}z + \frac{9}{16} + z^2 - 2z + 1 - 25 = 0 \Leftrightarrow 25z^2 - 50z - 375 = 0 \Leftrightarrow$$

$$\Leftrightarrow z^2 - 2z - 15 = 0 \Leftrightarrow z = \frac{2 \pm \sqrt{(-2)^2 - 4(-15)}}{2} \Leftrightarrow z = \frac{2 \pm 8}{2} \Leftrightarrow z = -3 \vee z = 5$$

Como o ponto G tem cota menor do que a do ponto C, conclui-se que $G(-1\ ,\ y\ ,\ -3)$.

Assim, $y = -\frac{3}{4}(-3) - \frac{13}{4} \Leftrightarrow y = -1$ e $G(-1\ ,\ -1\ ,\ -3)$.

O ponto E pertence ao plano ADE, pelo que tem abcissa 4 .

Por outro lado, o ponto E pertence à reta EF, que é perpendicular ao plano yOz, pelo que tem equação $y = -4 \wedge z = 1$.

Assim, $E(4\ ,\ -4\ ,\ 1)$.

De $P=C+\frac{1}{2}\overrightarrow{CE}$, vem $P=(-1\,,\,0\,,\,4)+\frac{1}{2}(4+1\,,\,-4-0\,,\,1-4) \Leftrightarrow P\left(\frac{3}{2}\,,\,-2\,,\,\frac{5}{2}\right)$.

$\overrightarrow{GP}\left(\frac{3}{2}+1\,,\,-2+1\,,\,\frac{5}{2}+3\right) \Leftrightarrow \overrightarrow{GP}\left(\frac{5}{2}\,,\,-1\,,\,\frac{11}{2}\right)$

$(x\,,\,y\,,\,z)=(-1\,,\,-1\,,\,-3)+\lambda\,(5\,,\,-2\,,\,11)\,,\ \lambda\in\mathbb{R}$

c. $\overrightarrow{EV}+\overrightarrow{EP}=\left(\frac{3}{2}-4\,,\,-\frac{7}{2}+4\,,\,\frac{9}{2}-1\right)+\left(\frac{3}{2}-4\,,\,-2+4\,,\,\frac{5}{2}-1\right)=$

$=\left(-\frac{5}{2}\,,\,\frac{1}{2}\,,\,\frac{7}{2}\right)+\left(-\frac{5}{2}\,,\,2\,,\,\frac{3}{2}\right)=\left(-5\,,\,\frac{5}{2}\,,\,5\right)$

$\lambda\left(-5\,,\,\frac{5}{2}\,,\,5\right)=\left(-5\lambda\,,\,\frac{5}{2}\lambda\,,\,5\lambda\right)\,,\ \lambda\in\mathbb{R}$

$\sqrt{(-5\lambda)^2+\left(\frac{5}{2}\lambda\right)^2+(5\lambda)^2}=\sqrt{75} \Leftrightarrow \frac{225}{4}\lambda^2=75 \Leftrightarrow \lambda^2=\frac{300}{225} \Leftrightarrow$

$\Leftrightarrow \lambda^2=\frac{4}{3} \Leftrightarrow \lambda=-\frac{2\sqrt{3}}{3} \vee \lambda=\frac{2\sqrt{3}}{3}$

Se $\lambda=-\frac{2\sqrt{3}}{3}$, $-\frac{2\sqrt{3}}{3}\left(-5\,,\,\frac{5}{2}\,,\,5\right)=\left(\frac{10\sqrt{3}}{3}\,,\,-\frac{5\sqrt{3}}{3}\,,\,-\frac{10\sqrt{3}}{3}\right)$;

Se $\lambda=\frac{2\sqrt{3}}{3}$, $\frac{2\sqrt{3}}{3}\left(-5\,,\,\frac{5}{2}\,,\,5\right)=\left(-\frac{10\sqrt{3}}{3}\,,\,\frac{5\sqrt{3}}{3}\,,\,\frac{10\sqrt{3}}{3}\right)$.

Como o sentido é oposto, $\left(\frac{10\sqrt{3}}{3}\,,\,-\frac{5\sqrt{3}}{3}\,,\,-\frac{10\sqrt{3}}{3}\right)$.

29.5 Centro: $M_{[AF]}=\left(\frac{4-1}{2}\,,\,\frac{3-4}{2}\,,\,\frac{0+1}{2}\right)=\left(\frac{3}{2}\,,\,-\frac{1}{2}\,,\,\frac{1}{2}\right)$

$\sqrt{5^2+5^2+5^2}=\sqrt{75}$ medida da diagonal espacial do cubo

Raio $=\frac{\sqrt{75}}{2}$

$\left(x-\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2+\left(z-\frac{1}{2}\right)^2=\frac{75}{4}$

29.6 A área total do sólido obtém-se adicionando a área de cinco faces do cubo com a área das quatro faces laterais da pirâmide.

Consideremos, por exemplo, a face $[VEF]$. Tem-se:

$\overline{VE}=\sqrt{\left(\frac{3}{2}-4\right)^2+\left(-\frac{7}{2}+4\right)^2+\left(\frac{9}{2}-1\right)^2}=\sqrt{\frac{75}{4}}=\frac{5\sqrt{3}}{2}$

$\overline{VF}=\sqrt{\left(\frac{3}{2}+1\right)^2+\left(-\frac{7}{2}+4\right)^2+\left(\frac{9}{2}-1\right)^2}=\sqrt{\frac{75}{4}}=\frac{5\sqrt{3}}{2}$

A altura do triângulo $[VEF]$ é $\sqrt{\left(\sqrt{\frac{75}{4}}\right)^2-\left(\frac{5}{2}\right)^2}=\sqrt{\frac{50}{4}}=\frac{5\sqrt{2}}{2}$

e a área desse triângulo é $\frac{5\times\frac{5\sqrt{2}}{2}}{2}=\frac{25\sqrt{2}}{4}$.

Logo, a área total do sólido é $5\times 5^2+4\times\frac{25\sqrt{2}}{4}=125+25\sqrt{2}$.