MÁXIMO

11.º ano

Matemática A

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Trigonometria

Porto Editora

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Tarefa de revisão

1. $\cos(36°) = \dfrac{\overline{AB}}{\overline{BC}} \Leftrightarrow \overline{BC} = \dfrac{\overline{AB}}{\cos(36°)}$

$\Leftrightarrow \overline{BC} = \dfrac{5}{\cos(36°)}$

$\overline{BC} \approx 6{,}1803$

$\tan(36°) = \dfrac{\overline{AC}}{\overline{AB}} \Leftrightarrow \overline{AC} = \tan(36°) \times \overline{AB}$

$\Leftrightarrow \overline{AC} = \tan(36°) \times 5$

$\overline{AC} \approx 3{,}6327$

$P = \overline{AB} + \overline{AC} + \overline{BC}$

$P \approx 5 + 3{,}6327 + 6{,}1803 \approx 14{,}813$

O perímetro do triângulo é, aproximadamente, 14,8 cm.

2. $\tan\left(B\hat{A}C\right) = \dfrac{100}{60} \Leftrightarrow \tan\left(B\hat{A}C\right) = \dfrac{5}{3}$

$B\hat{A}C = \tan^{-1}\left(\dfrac{5}{3}\right)$

$C\hat{A}D = 90° - \tan^{-1}\left(\dfrac{5}{3}\right) \approx 31{,}0°$

$C\hat{A}D \approx 31{,}0°$.

3. $\tan\alpha = \dfrac{\overline{DE}}{\overline{DC}} \Leftrightarrow \tan\alpha = \dfrac{2}{3}$

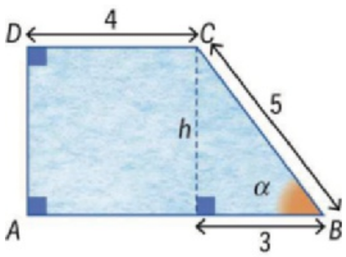
$\alpha = \tan^{-1}\left(\dfrac{2}{3}\right)$

$\alpha \approx 33{,}69°$

$\beta = 45° - \alpha \approx 45° - 33{,}69° = 11{,}31°$

$\alpha \approx 33{,}69°$ e $\beta \approx 11{,}31°$

4.1.



$\cos\alpha = \dfrac{3}{5}$

$\alpha \approx 53{,}13°$

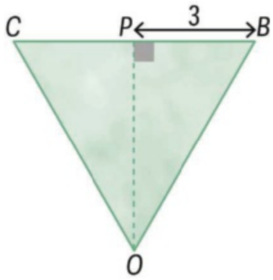
$C\hat{B}A \approx 53{,}13°$

4.2. $5^2 = 3^2 + h^2 \Leftrightarrow h^2 = 25 - 9 \Leftrightarrow h^2 = 16 \underset{(h>0)}{\Leftrightarrow} h = 4$

$A_{[ABCD]} = \dfrac{\overline{AB} + \overline{DC}}{2} \times \overline{AD} = \dfrac{7+4}{2} \times 4 = 22 \text{ cm}^2$

5. $P = 36$ m

$l = \dfrac{36}{6} = 6$ m



$\tan(30°) = \dfrac{\overline{PB}}{\overline{PO}} \Leftrightarrow \overline{PO} = \dfrac{\overline{PB}}{\tan(30°)} \Leftrightarrow$

$\Leftrightarrow \overline{PO} = \dfrac{3}{\frac{\sqrt{3}}{3}} \Leftrightarrow \overline{PO} = 3\sqrt{3}$

$A_{[OCB]} = \dfrac{6 \times 3\sqrt{3}}{2} = 9\sqrt{3}$

$A_{[ABCDEF]} = 6 \times A_{[OCB]} = 6 \times 9\sqrt{3} \approx 93{,}5 \text{ m}^2$

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Tarefa 1

1.1. Como o triângulo $[ABC]$ é equilátero, $\overline{AB} = k$.

M é o ponto médio de $[AB]$, logo

$\overline{AM} = \dfrac{1}{2}\overline{AB} = \dfrac{1}{2}k = \dfrac{k}{2}$.

1.2. Como o triângulo $[AMC]$ é retângulo, pode-se aplicar o Teorema de Pitágoras.

$\overline{AC}^2 = \overline{AM}^2 + \overline{MC}^2 \Leftrightarrow k^2 = \left(\dfrac{k}{2}\right)^2 + \overline{MC}^2$

$\Leftrightarrow k^2 = \dfrac{k^2}{4} + \overline{MC}^2 \Leftrightarrow \overline{MC}^2 = k^2 - \dfrac{k^2}{4}$

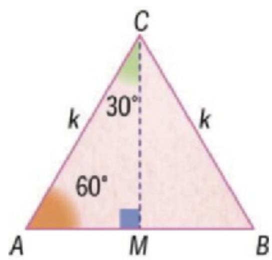
$\Leftrightarrow \overline{MC}^2 = \dfrac{3k^2}{4} \underset{(\overline{MC}>0)}{\Leftrightarrow} \overline{MC} = \sqrt{\dfrac{3k^2}{4}} \underset{(k>0)}{\Leftrightarrow} \overline{MC} = \dfrac{\sqrt{3}k}{2}$

1.3. Como o triângulo $[ABC]$ é equiátero, os seus ângulos interno têm amplitude 60º.

Assim, $B\hat{A}C = 60°$.

$A\hat{C}B = 60°$

$A\hat{C}M = \dfrac{1}{2} \times 60° = 30°$



1.4. $\sin(30°) = \dfrac{\overline{AM}}{\overline{AC}} = \dfrac{\frac{k}{2}}{k} = \dfrac{k}{2k} = \dfrac{1}{2}$

$\cos(30°) = \dfrac{\overline{MC}}{\overline{AC}} = \dfrac{\frac{\sqrt{3}k}{2}}{k} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

$$\tan(30^\circ) = \frac{\overline{AM}}{\overline{MC}} = \frac{\frac{k}{2}}{\frac{\sqrt{3}k}{2}} = \frac{2k}{2\sqrt{3}k} = \frac{1}{\sqrt{3}} =$$

$$= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$$

1.5. $\sin(60^\circ) = \frac{\overline{MC}}{\overline{AC}} = \frac{\frac{\sqrt{3}k}{2}}{k} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$$\cos(60^\circ) = \frac{\overline{AM}}{\overline{AC}} = \frac{\frac{k}{2}}{k} = \frac{k}{2k} = \frac{1}{2}$$

$$\tan(60^\circ) = \frac{\overline{MC}}{\overline{AM}} = \frac{\frac{\sqrt{3}k}{2}}{\frac{k}{2}} = \frac{2\sqrt{3}k}{2k} = \sqrt{3}$$

2.1. Aplicando o Teorema de Pitágoras ao triângulo $[DEF]$ vem:

$$\overline{FE}^2 = k^2 + k^2 \Leftrightarrow \overline{FE}^2 = 2k^2 \underset{(\overline{FE}>0)}{\Leftrightarrow} \overline{FE} = \sqrt{2k^2}$$

$$\underset{(k>0)}{\Leftrightarrow} \overline{FE} = \sqrt{2}k$$

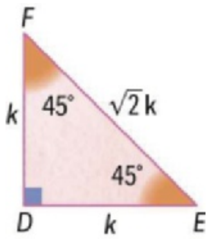
2.2. Como o triângulo $[DEF]$ é isósceles, tem dois lados iguais, logo tem dois ângulos internos iguais. Como o triângulo $[DEF]$ é retângulo, tem um ângulo interno com amplitude 90º.

$180^\circ - 90^\circ = 90^\circ$

$90^\circ : 2 = 45^\circ$

$F\hat{E}D = D\hat{F}E = 45^\circ$

2.3.



$$\sin(45^\circ) = \frac{\overline{DF}}{\overline{FE}} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos(45^\circ) = \frac{\overline{DE}}{\overline{FE}} = \frac{k}{\sqrt{2}k} = \frac{\sqrt{2}}{2}$$

$$\tan(45^\circ) = \frac{\overline{DF}}{\overline{DE}} = \frac{k}{k} = 1$$

3.1. $\sin(60^\circ) - 2\tan(45^\circ) + \cos(30^\circ) = \frac{\sqrt{3}}{2} - 2 \times 1 + \frac{\sqrt{3}}{2}$

$$= \frac{2\sqrt{3}}{2} - 2 = \sqrt{3} - 2$$

3.2. $\sin^2(45^\circ) + \cos^2(45^\circ) = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4}$

$$= \frac{4}{4} = 1$$

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1.1. A razão trigonométrica que relaciona os dois catetos é a tangente.

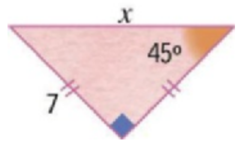
$$\tan(30^\circ) = \frac{x}{\sqrt{3}} \Leftrightarrow \frac{\sqrt{3}}{3} = \frac{x}{\sqrt{3}} \Leftrightarrow x = \frac{\sqrt{3} \times \sqrt{3}}{3} \Leftrightarrow x = 1$$

1.2. A razão trigonométrica que relaciona o cateto adjacente e a hipotenusa é o cosseno.

$$\cos x = \frac{2}{4} \Leftrightarrow \cos x = \frac{1}{2}$$

$$x = \cos^{-1}\left(\frac{1}{2}\right) \Leftrightarrow x = 60^\circ$$

1.3.



$$\sin(45^\circ) = \frac{7}{x} \Leftrightarrow \frac{\sqrt{2}}{2} = \frac{7}{x} \Leftrightarrow \sqrt{2}x = 14 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{14}{\sqrt{2}} \Leftrightarrow x = \frac{14\sqrt{2}}{2} \Leftrightarrow x = 7\sqrt{2}$$

Outro processo:

Como o triângulo é retângulo e são conhecidos os dois catetos, para determinar a hipotenusa pode-se aplicar o Teorema de Pitágoras.

$$x^2 = 7^2 + 7^2 \Leftrightarrow x^2 = 98 \underset{(x>0)}{\Leftrightarrow} x = \sqrt{98} \Leftrightarrow x = 7\sqrt{2}$$

Cálculos auxiliares

98	2
49	7
7	7
1	

$98 = 2 \times 7^2$

$\sqrt{98} = \sqrt{2 \times 7^2} = 7\sqrt{2}$

2. A razão trigonométrica que relaciona os dois catetos é a tangente.

$$\tan(60^\circ) = \frac{\overline{AB}}{50\sqrt{3}} \Leftrightarrow \sqrt{3} = \frac{\overline{AB}}{50\sqrt{3}} \Leftrightarrow \overline{AB} = \sqrt{3} \times 50\sqrt{3}$$

$$\Leftrightarrow \overline{AB} = 150 \text{ m}$$

O comprimento do lago é 150 m.

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3.1. $\alpha = 60^\circ$ e $\beta = 45^\circ$

$$\cos\alpha + \cos^2\beta + 2\tan^2\beta = \frac{1}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 + 2 \times 1^2$$

$$= \frac{1}{2} + \frac{2}{4} + 2 = 3$$

3.2. $P = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$ $\qquad \overline{AB} = \overline{BC} = \overline{DA}$

$P = \overline{AB} + \overline{BC} + x + \overline{DA}$

$\cos(45°) = \dfrac{\overline{BC}}{\overline{CD}} \Leftrightarrow \overline{BC} = \dfrac{\sqrt{2}}{2}x$

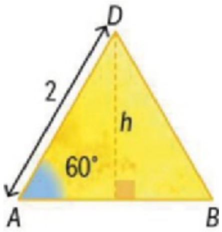
$P = 3 \times \dfrac{\sqrt{2}}{2}x + x \Leftrightarrow P = \dfrac{3\sqrt{2}}{2}x + x$

3.3. $A_{[BCD]} = \dfrac{\overline{BC} \times \overline{BD}}{2} = \dfrac{2 \times 2}{2} = 2 \text{ m}^2$

$\sin(60°) = \dfrac{h}{2}$

$\Leftrightarrow h = 2 \times \dfrac{\sqrt{3}}{2}$

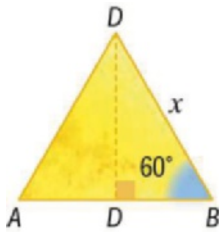
$\Leftrightarrow h = \sqrt{3}$



$A_{[ABD]} = \dfrac{\overline{AB} \times h}{2} = \dfrac{2 \times \sqrt{3}}{2} = \sqrt{3} \text{ m}^2$

$A_{[ABCD]} = \left(2 + \sqrt{3}\right) \text{ m}^2$

4. Como o triângulo $[ABC]$ é equilátero, $A\hat{B}C = 60°$. $[CD]$ é a altura do triângulo $[ABC]$.



$\sin(60°) = \dfrac{\overline{CD}}{x} \Leftrightarrow \overline{CD} = \dfrac{\sqrt{3}}{2}x$

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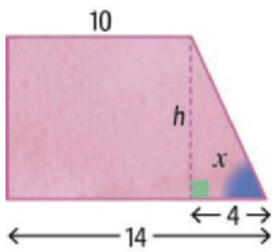
5.



5.1. **a)** $x = 60°$

$\tan(60°) = \dfrac{h}{4} \Leftrightarrow h = 4\tan(60°) \Leftrightarrow h = 4\sqrt{3}$

$A = \dfrac{14 + 10}{2} \times 4\sqrt{3} = 48\sqrt{3}$ u.a.

b) $x = 45°$

$\tan(45°) = \dfrac{h}{4} \Leftrightarrow h = 4\tan(45°) \Leftrightarrow h = 4$

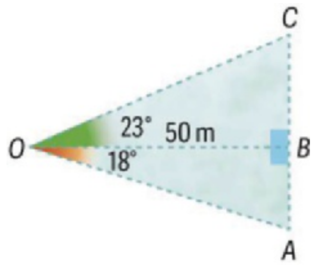
$A = \dfrac{14 + 10}{2} \times 4 = 48$ u.a.

5.2. Seja h a altura do trapézio.

$\text{Área} = \dfrac{14 + 10}{2} \times h$	$\tan x = \dfrac{h}{4}$
$= 12h = 12 \times 4\tan x$	$\Leftrightarrow h = 4\tan x$
$= 48\tan x$	

$A_{[ABCD]} = 48\tan x$ u. a.

6.



$\tan(23°) = \dfrac{\overline{BC}}{50} \Leftrightarrow \overline{BC} = 50 \times \tan(23°)$

$\tan(18°) = \dfrac{\overline{AB}}{50} \Leftrightarrow \overline{AB} = 50 \times \tan(18°)$

$\overline{AC} = \overline{AB} + \overline{BC} = 50\tan(18°) + 50\tan(23°) \approx 37{,}5 \text{ m}$

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7.

7.1. **a)** $\tan(70°) = \dfrac{20}{\overline{AB}} \Leftrightarrow \overline{AB} = \dfrac{20}{\tan(70°)}$

$\text{Diâmetro} = 2 \times \overline{AB} = 2 \times \dfrac{20}{\tan(70°)} \approx 14{,}6 \text{ cm}$

O diâmetro da base do chapéu é, aproximadamente, 14,6 cm.

b) $V = \dfrac{1}{3}A_b \times h = \dfrac{1}{3}\pi \times \dfrac{20}{\tan(70°)} \times 20 \approx 1110 \text{ cm}^3$

O volume do chapéu é, aproximadamente, 1110 cm^3.

7.2. Quanto menor é o raio da base, mais x se aproxima de 90°, mas nunca pode ser 90°. Quanto maior é o raio da base, mais x se aproxima de 0°, mas nunca pode ser 0°. Por isso, $x \in]0°, 90°[$.

7.3.

$$V_{cone} = \frac{1}{3} A_b \times h$$
$$= \frac{1}{3}\pi r^2 \times h = \frac{1}{3}\pi \frac{20^2}{\tan^2 x} \times 20$$
$$= \frac{20^3 \pi}{3\tan^2 x} = \frac{8000\pi}{3\tan^2 x}$$

Cálculos auxiliares

Seja r o raio da base do cone

$$\tan x = \frac{20}{r}$$
$$\Leftrightarrow r = \frac{20}{\tan x}$$

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8.1. $\tan(37°) = \frac{\overline{AD}}{\overline{AB}} \Leftrightarrow \overline{AD} = \tan(37°) \times 20$

$\overline{AD} \approx 15{,}071 \approx 15{,}1$ m

8.2. $37° + 3{,}4° = 40{,}4°$

$\tan(40{,}4°) = \frac{\overline{AC}}{\overline{AB}} \Leftrightarrow \overline{AC} = \tan(40{,}4°) \times 20$

$\overline{AC} \approx 17{,}021 \approx 17{,}0$ m

8.3. $\overline{CD} = \overline{AC} - \overline{AD} =$

$= \tan(40{,}4°) \times 20 - \tan(37°) \times 20 \approx 2{,}0$ m

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9.1.

$$\begin{cases} \tan(33°) = \frac{h}{x} \\ \tan(30°) = \frac{h}{x+5} \end{cases} \Leftrightarrow \begin{cases} h = x\tan(33°) \\ (x+5)\tan(30°) = h \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_ \\ x\tan(30°) + 5\tan(30°) = x\tan(33°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_ \\ x(\tan(33°) - \tan(30°)) = 5\tan(30°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_ \\ x = \frac{5\tan(30°)}{\tan(33°) - \tan(30°)} \end{cases}$$

$$\begin{cases} \_\_\_\_\_\_\_\_ \\ x \approx 40{,}062 \end{cases}$$

$\overline{BC} = x \approx 40{,}1$ m

9.2.

$h = x \times \tan(33°) \approx 40{,}062 \times \tan(33°)$

$h \approx 26{,}017$

$h \approx 26{,}0$ m

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10.1. $25° + 5° = 30°$

$$\begin{cases} \tan(25°) = \frac{h}{x} \\ \tan(30°) = \frac{20+h}{x} \end{cases} \Leftrightarrow \begin{cases} h = x\tan(25°) \\ x\tan(30°) = 20 + h \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_ \\ x\tan(30°) = 20 + x\tan(25°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_ \\ x(\tan(30°) - \tan(25°)) = 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_ \\ x = \frac{20}{\tan(30°) - \tan(25°)} \end{cases}$$

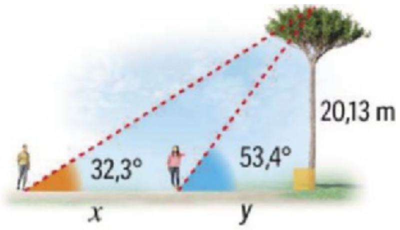
$$\begin{cases} \_\_\_\_\_\_\_\_ \\ x \approx 180{,}111 \end{cases}$$

$\overline{AB} = x \approx 180{,}1$ m

10.2. $h = x\tan(25°) \approx 180{,}111 \times \tan(25°) \approx 83{,}987$

$h \approx 84{,}0$ m

11.



$$\begin{cases} \tan(32{,}3°) = \frac{20{,}13}{x+y} \\ \tan(53{,}4°) = \frac{20{,}13}{y} \end{cases}$$

$$\begin{cases} 0{,}632 = \frac{20{,}13}{x+y} \\ 1{,}347 = \frac{20{,}13}{y} \end{cases} \Leftrightarrow \begin{cases} 0{,}632(x+y) = 20{,}13 \\ y = \frac{20{,}13}{1{,}347} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 0{,}632x + 0{,}632y = 20{,}13 \\ y \approx 14{,}944 \end{cases}$$

$$\Leftrightarrow \begin{cases} 0{,}632x + 0{,}632 \times 14{,}944 = 20{,}13 \\ \_\_\_\_\_\_\_\_ \end{cases}$$

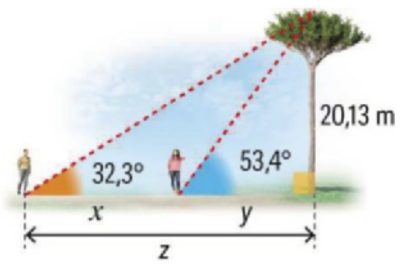
$$\Leftrightarrow \begin{cases} 0{,}632x + 9{,}445 = 20{,}13 \\ \_\_\_\_\_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} 0{,}632x = 10{,}685 \\ \_\_\_\_\_\_\_\_ \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{10{,}685}{0{,}632} \\ \_\_\_\_\_\_\_\_ \end{cases}$$

$$\begin{cases} x \approx 17 \\ y \approx 14{,}944 \end{cases}$$

A distância entre as duas amigas é, aproximadamente, 17 m.

Outro modo:



$\tan(53,4°) = \dfrac{20,13}{y} \Leftrightarrow y = \dfrac{20,13}{\tan(53,4°)}$

$\tan(32,3°) = \dfrac{20,13}{z} \Leftrightarrow z = \dfrac{20,13}{\tan(32,3°)}$

$x + y = z \Leftrightarrow x = z - y \Leftrightarrow x = \dfrac{20,13}{\tan(32,3°)} - \dfrac{20,13}{\tan(53,4°)}$

$x \approx 17$ m

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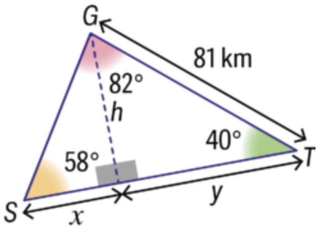
12.1.



$180° - 82° - 58° = 40°$

$\sin(40°) = \dfrac{h}{81} \Leftrightarrow h = 81\sin(40°)$

$h \approx 52,066$

$\sin(58°) = \dfrac{h}{\overline{SG}} \Leftrightarrow \overline{SG} = \dfrac{h}{\sin(58°)}$

$\overline{SG} \approx \dfrac{52,066}{\sin(58°)} \approx 61,395$

$\overline{SG} \approx 61$ km

12.2. $\tan(58°) = \dfrac{h}{x} \Leftrightarrow x = \dfrac{h}{\tan(58°)}$

$x \approx \dfrac{52,066}{\tan(58°)}$

$\Leftrightarrow x \approx 32,534$

$\cos(40°) = \dfrac{y}{81} \Leftrightarrow y = 81 \times \cos(40°)$

$y \approx 62,050$

$\overline{ST} \approx 32,534 + 62,050 \approx 94,584$

$\overline{ST} \approx 95$ km

13.1. $180° - 98° - 60° = 22°$

$\sin(60°) = \dfrac{h}{40} \Leftrightarrow h = 40\sin(60°) \Leftrightarrow h = 40 \times \dfrac{\sqrt{3}}{2}$

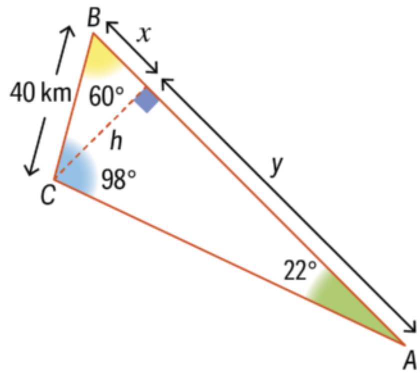
$h \approx 34,641$

$\sin(22°) = \dfrac{h}{\overline{AC}}$

$\sin(22°) \approx \dfrac{34,641}{\overline{AC}} \Leftrightarrow \overline{AC} \approx \dfrac{34,641}{\sin(22°)} \Leftrightarrow \overline{AC} \approx 92,473$

$\overline{AC} \approx 92,5$ km

13.2.



$\cos(60°) = \dfrac{x}{40} \Leftrightarrow x = 40\cos(60°) \Leftrightarrow x = 40 \times \dfrac{1}{2} = 20$

$\tan(22°) = \dfrac{h}{y} \Leftrightarrow y = \dfrac{h}{\tan(22°)}$

$y \approx \dfrac{34,641}{\tan(22°)} \approx 85,739$

$\overline{AB} \approx 20 + 85,739 = 105,739$

$\overline{AB} \approx 105,7$ km

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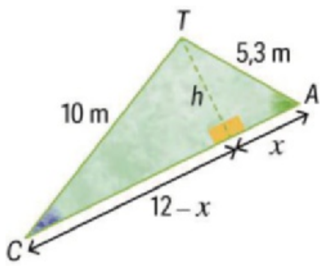
14.



$5,3^2 = h^2 + x^2 \Leftrightarrow h^2 = 5,3^2 - x^2$

$10^2 = h^2 + (12 - x)^2 \Leftrightarrow h^2 = 10^2 - (12 - x)^2$

$5,3^2 - x^2 = 10^2 - (12 - x)^2$

$\Leftrightarrow 28,09 - x^2 = 100 - 144 + 24x - x^2 \Leftrightarrow 24x = 72,09$

$\Leftrightarrow x = 3,00375$

Vamos representar por *C*, *A* e *T* as amplitudes dos ângulos de vértices *C*, *A* e *T*, respetivamente.

$\cos A = \dfrac{x}{5,3} \Leftrightarrow \cos A = \dfrac{3,00375}{5,3} \Leftrightarrow$

$\Leftrightarrow A = \cos^{-1}\left(\dfrac{3,00375}{5,3}\right)$

$A \approx 55,5°$

$12 - x = 12 - 3,00375 = 8,99625$

$\cos C = \dfrac{12 - x}{10} \Leftrightarrow \cos C = \dfrac{8,99625}{10} \Leftrightarrow$

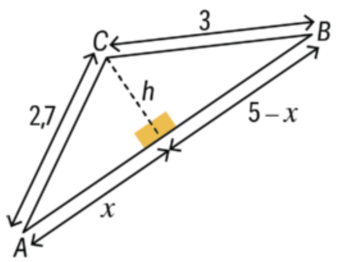
$\Leftrightarrow C = \cos^{-1}\left(\dfrac{8,99625}{10}\right)$

$C \approx 25,9°$

$T \approx 180° - 55,5° - 25,9° \approx 98,6°$

$T\hat{A}C \approx 55,5°$, $A\hat{C}T \approx 25,9°$ e $C\hat{T}A \approx 98,6°$

15.



$2,7^2 = h^2 + x^2 \Leftrightarrow h^2 = 2,7^2 - x^2$

$3^2 = h^2 + (5-x)^2 \Leftrightarrow h^2 = 3^2 - (5-x)^2$

$2,7^2 - x^2 = 3^2 - (5-x)^2 \Leftrightarrow 7,29 \cancel{-x^2} = 9 - 25 + 10x \cancel{-x^2}$

$\Leftrightarrow 10x = 23,29 \Leftrightarrow x = 2,329$

Vamos representar por A, B e C as amplitudes dos ângulos de vértices A, B e C, respetivamente.

$\cos A = \frac{x}{2,7} \Leftrightarrow \cos A = \frac{2,329}{2,7} \Leftrightarrow A = \cos^{-1}\left(\frac{2,329}{2,7}\right)$

$A \approx 30,4°$

$5 - x = 5 - 2,329 = 2,671$

$\cos B = \frac{5-x}{3} \Leftrightarrow \cos B = \frac{2,671}{3} \Leftrightarrow B = \cos^{-1}\left(\frac{2,671}{3}\right)$

$B \approx 27,1°$

$C \approx 180° - 30,4° - 27,1° \approx 122,5°$

$B\hat{A}C \approx 30,4°$, $C\hat{B}A \approx 27,1°$ e $A\hat{C}B \approx 122,5°$

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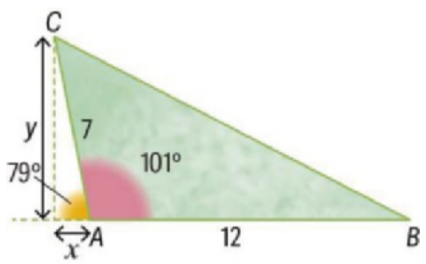
16.1.



$180° - 101° = 79°$

$\cos(79°) = \frac{x}{7} \Leftrightarrow x = 7\cos(79°)$

$x \approx 1,336$

$x + 12 = 13,336$

$\sin(79°) = \frac{y}{7} \Leftrightarrow y = 7\sin(79°)$

$y \approx 6,871$

$\overline{CB}^2 = (x+12)^2 + y^2 \underset{(\overline{CB}>0)}{\Leftrightarrow} \overline{CB} = \sqrt{(x+12)^2 + y^2}$

$\overline{CB} = \sqrt{13,336^2 + 6,871^2} \Leftrightarrow \overline{CB} \approx 15,002$

$\overline{CB} \approx 15,0$ km

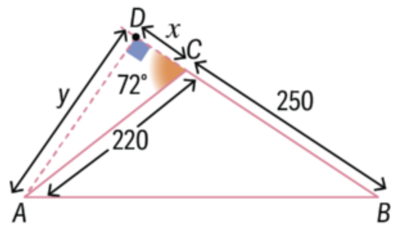
16.2. $\tan C\hat{B}A = \frac{y}{x+12}$

$\tan C\hat{B}A \approx \frac{6,871}{13,336}$

$\Leftrightarrow C\hat{B}A \approx \tan^{-1}\left(\frac{6,871}{13,336}\right) \Leftrightarrow C\hat{B}A \approx 27,259°$

$C\hat{B}A \approx 27,3°$

17.



$180° - 72° = 108°$

$\cos(72°) = \frac{x}{220} \Leftrightarrow x = 220\cos(72°)$

$x \approx 67,984$

$\sin(72°) = \frac{y}{220} \Leftrightarrow y = 220\sin(72°)$

$y \approx 209,232$

$\overline{BD} = x + 250 \approx 67,984 + 250 \approx 317,984$

$\overline{AB}^2 = \overline{AD}^2 + \overline{BD}^2 \Leftrightarrow \overline{AB}^2 = y^2 + (x+250)^2$

$\overline{AB} \approx \sqrt{209,232^2 + 317,984^2} \Leftrightarrow \overline{AB} \approx 380,647$

$\overline{AB} \approx 381$ m

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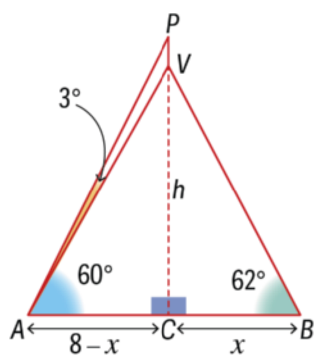
18.



$\tan(62°) = \frac{h}{x} \Leftrightarrow h = x\tan(62°)$

$\tan(60°) = \frac{h}{8-x} \Leftrightarrow h = (8-x)\tan(60°)$

$x\tan(62°) = 8\tan(60°) - x\tan(60°)$

$\Leftrightarrow x\left(\tan(62°) + \tan(60°)\right) = 8\tan(60°)$

$\Leftrightarrow x = \frac{8\tan(60°)}{\tan(62°) + \tan(60°)}$

$x \approx 3,835$

$\tan(63°) = \frac{\overline{PC}}{8-x} \Leftrightarrow \overline{PC} = (8-x)\tan(63°)$

$\overline{PC} \approx (8-3{,}835)\tan(63°) \Leftrightarrow \overline{PC} \approx 8{,}174$

$h = x\tan(62°)$; $h \approx 3{,}835\tan(62°) \approx 7{,}2126$

$\overline{PV} = \overline{PC} - \overline{VC} = \overline{PC} - h \approx 8{,}175 - 7{,}2126 \approx 0{,}9624$

$\overline{PV} \approx 0{,}96$ m

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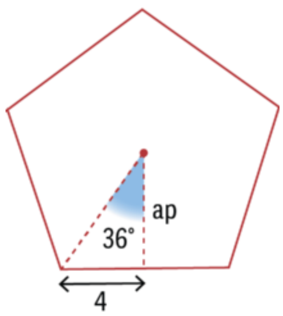
Tarefas de consolidação 1

1.1. Não é necessariamente verdadeira, porque o comprimento do cateto oposto pode ser 6 e o comprimento da hipotenusa pode ser 10, por exemplo.

1.2. É falsa, porque num triângulo retângulo, o comprimento do cateto adjacente não pode ser maior que o comprimento da hipotenusa, daí que $\cos\alpha < 1$ e $\frac{4}{3} > 1$.

2.



$P = 40$ cm ; $l = \frac{40}{5} = 8$ cm

$360° : 5 = 72°$ $72° : 2 = 36°$

Designemos por *ap* o apótema do pentágono.

$\tan(36°) = \frac{4}{ap} \Leftrightarrow ap = \frac{4}{\tan(36°)}$; $ap \approx 5{,}51$ cm

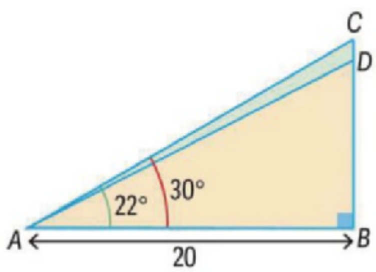
$V_{prisma} = A_{base} \times altura = \frac{5 \times l \times ap}{2} \times altura$

$V_{prisma} \approx \frac{5 \times 8 \times 5{,}51}{2} \times 16 = 1763{,}2 \text{ cm}^3$

$1{,}5 \text{ L} = 1{,}5 \text{ dm}^3 = 1500 \text{ cm}^3$

Uma garrafa de 1,5 L não é suficiente para encher completamente a jarra, porque a sua capacidade é, aproximadamente, $1763{,}2 \text{ cm}^3$.

3.



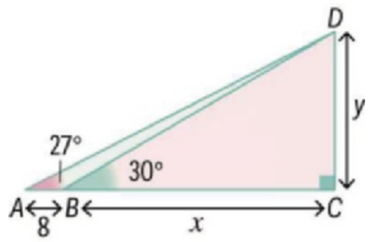
$\tan(27°) = \frac{\overline{BD}}{20} \Leftrightarrow \overline{BD} = 20\tan(27°)$

$\tan(30°) = \frac{\overline{BC}}{20} \Leftrightarrow \overline{BC} = 20\tan(30°) \Leftrightarrow$

$\Leftrightarrow \overline{BC} = 20 \times \frac{\sqrt{3}}{3}$

$\overline{CD} = \overline{BC} - \overline{BD} = 20 \times \frac{\sqrt{3}}{3} - 20\tan(27°) \approx 1{,}4$ m

4.



$$\begin{cases} \tan(30°) = \frac{y}{x} \\ \tan(27°) = \frac{y}{x+8} \end{cases} \Leftrightarrow \begin{cases} y = x\tan(30°) \\ y = (x+8)\tan(27°) \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \frac{\sqrt{3}}{3}x \\ \frac{\sqrt{3}}{3}x = (x+8)\tan(27°) \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ \frac{\sqrt{3}x}{3\tan(27°)} = x+8 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{———} \\ \frac{\sqrt{3}}{3\tan(27°)}x - x = 8 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ \left(\frac{\sqrt{3}}{3\tan(27°)} - 1\right)x = 8 \end{cases}$$

$$\begin{cases} y \approx \frac{\sqrt{3}}{3} \times 60{,}099 \\ x \approx 60{,}099 \end{cases} \Leftrightarrow \begin{cases} y \approx 34{,}698 \\ x \approx 60{,}099 \end{cases}$$

$\overline{CD} \approx 34{,}7$ m

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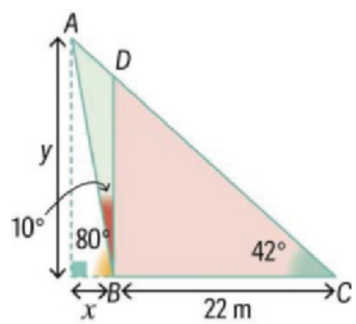
5.



$$\begin{cases} \tan(80°) = \frac{y}{x} \\ \tan(42°) = \frac{y}{x+22} \end{cases} \Leftrightarrow \begin{cases} y = x\tan(80°) \\ y = (x+22)\tan(42°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{———} \\ x\tan(80°) = (x+22)\tan(42°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_\_\_ \\ \frac{\tan(80°)}{\tan(42°)}x = x + 22 \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_\_\_ \\ \frac{\tan(80°)}{\tan(42°)}x - x = 22 \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_\_\_ \\ \left(\frac{\tan(80°)}{\tan(42°)} - 1\right)x = 22 \end{cases}$$

$$\begin{cases} y \approx 4{,}1520 \times \tan(80°) \\ x \approx 4{,}1520 \end{cases}$$

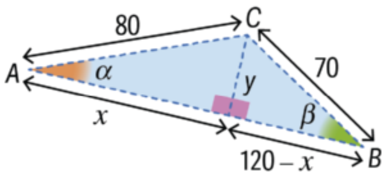
$$\begin{cases} y \approx 23{,}5472 \\ x \approx 4{,}1520 \end{cases}$$

$$\sin(80°) \approx \frac{23{,}5472}{\overline{BA}} \Leftrightarrow \overline{BA} \approx \frac{23{,}5472}{\sin(80°)} \Leftrightarrow$$

$$\Leftrightarrow \overline{BA} \approx 23{,}9105$$

$\overline{BA} \approx 23{,}9$ m

6.1.



$$70^2 = y^2 + (120 - x)^2 \Leftrightarrow y^2 = 70^2 - (120 - x)^2$$

$$80^2 = y^2 + x^2 \Leftrightarrow y^2 = 80^2 - x^2$$

$$70^2 - (120 - x)^2 = 80^2 - x^2$$

$$\Leftrightarrow 4900 - (14\ 400 - 240x + x^2) = 6400 - x^2$$

$$\Leftrightarrow 4900 - 14\ 400 + 240x - \cancel{x^2} = 6400 - \cancel{x^2}$$

$$\Leftrightarrow 240x = 15\ 900 \Leftrightarrow x = 66{,}25$$

$$\cos\alpha = \frac{66{,}25}{80} \Leftrightarrow \alpha = \cos^{-1}\left(\frac{66{,}25}{80}\right)$$

$\alpha \approx 34{,}1°$

$B\hat{A}C \approx 34{,}1°$

6.2. $\cos\beta = \frac{120 - x}{70} \Leftrightarrow \cos\beta = \frac{120 - 66{,}25}{70}$

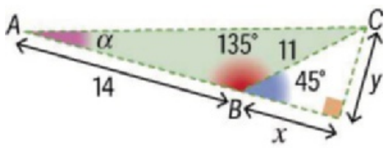
$$\Leftrightarrow \cos\beta = \frac{53{,}75}{70} \Leftrightarrow \beta = \cos^{-1}\left(\frac{53{,}75}{70}\right)$$

$\beta \approx 39{,}8$

$A\hat{B}C = 39{,}8°$

$A\hat{C}B \approx 180° - 34{,}1° - 39{,}8° = 106{,}1°$

7.1.



$180° - 135° = 45°$

$$\cos(45°) = \frac{x}{11} \Leftrightarrow x = 11\cos(45°) \Leftrightarrow x = 11\frac{\sqrt{2}}{2}$$

$$\sin(45°) = \frac{y}{11} \Leftrightarrow y = 11\sin(45°) \Leftrightarrow y = 11\frac{\sqrt{2}}{2}$$

$$\overline{AC}^2 = (14 + x)^2 + y^2 \underset{(\overline{AC} > 0)}{\Leftrightarrow} \overline{AC} = \sqrt{(14 + x)^2 + y^2}$$

$$\Leftrightarrow \overline{AC} = \sqrt{\left(14 + \frac{11\sqrt{2}}{2}\right)^2 + \left(\frac{11\sqrt{2}}{2}\right)^2}$$

$\overline{AC} \approx 23{,}1255$

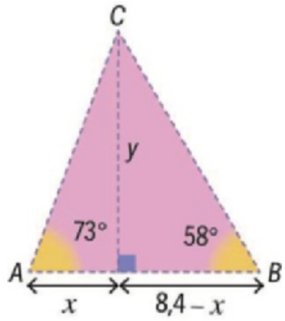
A distância de A a C é, aproximadamente, $23{,}13$ m.

7.2. $\tan(B\hat{A}C) = \frac{y}{14 + x} \Leftrightarrow \tan(B\hat{A}C) = \frac{\frac{11\sqrt{2}}{2}}{14 + \frac{11\sqrt{2}}{2}}$

$$\Leftrightarrow B\hat{A}C = \tan^{-1}\left(\frac{\frac{11\sqrt{2}}{2}}{14 + \frac{11\sqrt{2}}{2}}\right); \quad B\hat{A}C \approx 19{,}6544°$$

A amplitude do ângulo BAC é, aproximadamente, 19,65º.

8.1.



$$\begin{cases} \tan(73°) = \frac{y}{x} \\ \tan(58°) = \frac{y}{8{,}4 - x} \end{cases} \Leftrightarrow \begin{cases} y = x\tan(73°) \\ y = (8{,}4 - x)\tan(58°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_\_\_ \\ x\tan(73°) = 8{,}4\tan(58°) - x\tan(58°) \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_\_\_ \\ x(\tan(73°) + \tan(58°)) = 8{,}4\tan(58°) \end{cases}$$

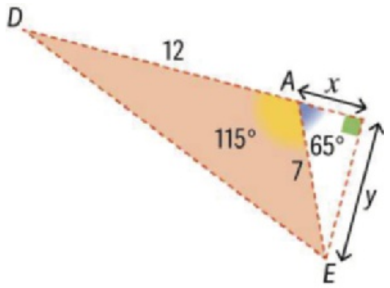
$$\Leftrightarrow \begin{cases} \_\_\_\_\_\_\_\_\_\_ \\ x = \frac{8{,}4\tan(58°)}{\tan(73°) + \tan(58°)} \end{cases}$$

$$\begin{cases} y \approx 2{,}760\tan(73°) \\ x \approx 2{,}760 \end{cases} \Leftrightarrow \begin{cases} y \approx 9{,}028 \\ x \approx 2{,}760 \end{cases}$$

$$A_{vela} = \frac{\overline{AB} \times y}{2} \approx \frac{8{,}4 \times 9{,}028}{2} \approx 37{,}9176$$

A área da vela é, aproximadamente, $37{,}9$ m^2.

8.2.



$180° - 115° = 65°$

$\sin(65°) = \frac{y}{7} \Leftrightarrow y = 7\sin(65°)$

$y \approx 6{,}344$

$\cos(65°) = \frac{x}{7} \Leftrightarrow x = 7\cos(65°)$

$x \approx 2{,}958$

$\overline{DE}^2 = (12+x)^2 + y^2 \underset{(\overline{DE}>0)}{\Leftrightarrow} \overline{DE} = \sqrt{(12+x)^2 + y^2}$

$\overline{DE} \approx \sqrt{(12+2{,}958)^2 + 6{,}344^2} \Leftrightarrow \overline{DE} \approx 16{,}248$

A distância de D a E é, aproximadamente, 16,2 m.

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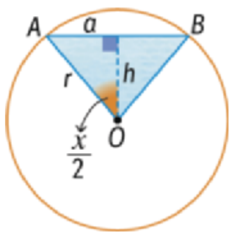
Avaliação formativa 1

1.1.



$\cos\left(\frac{x}{2}\right) = \frac{h}{r} \Leftrightarrow h = r\cos\left(\frac{x}{2}\right).$

$\sin\left(\frac{x}{2}\right) = \frac{a}{r} \Leftrightarrow a = r\sin\left(\frac{x}{2}\right)$

$\overline{AB} = 2a = 2r\sin\left(\frac{x}{2}\right)$

$Área = \frac{2r\sin\left(\frac{x}{2}\right) \times r\cos\left(\frac{x}{2}\right)}{2} =$

$= r^2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$ c.q.m

1.2. $x = 180° - O\hat{A}B - A\hat{B}O = 180° - 60° - 60° = 60°$

$A_{[ABC]} = r^2 \times \sin(30°) \times \cos(30°) =$

$= r^2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{r^2\sqrt{3}}{4}$ u.a

2. $\cos\alpha = \frac{\overline{OC}}{4} \Leftrightarrow \overline{OC} = 4\cos\alpha$

$\overline{BC} = \sqrt{4^2 - (4\cos\alpha)^2} = \sqrt{16 - 16\cos^2\alpha}$

$d^2 = \overline{BC}^2 + \overline{AC}^2 = \left(\sqrt{16 - 16\cos^2\alpha}\right)^2 + (4 + 4\cos\alpha)^2$

$\Leftrightarrow d^2 = 16 - 16\cos^2\alpha + 16 + 32\cos\alpha + 16\cos^2\alpha$

$\Leftrightarrow d^2 = 32 + 32\cos\alpha$

$\underset{(d>0)}{\Leftrightarrow} d = \sqrt{32 + 32\cos\alpha} \Leftrightarrow d = \sqrt{32(1+\cos\alpha)}$

$\Leftrightarrow d = \sqrt{2^2 \times 2^2 \times 2(1+\cos\alpha)} \Leftrightarrow$

$\Leftrightarrow d = \sqrt{16}\sqrt{12(1+\cos\alpha)}$

$\Leftrightarrow d = 4\sqrt{2(1+\cos\alpha)} \Leftrightarrow d = 4\sqrt{2 + 2\cos\alpha}$ c.q.m.

Cálculos auxiliares

32	2
16	2
8	2
4	2
2	2
1	

$32 = 2^2 \times 2^2 \times 2$

$\sqrt{32} = \sqrt{2^2 \times 2^2 \times 2}$

3.1. $\sin(60°) = \frac{h}{50} \Leftrightarrow h = 50\sin(60°) \Leftrightarrow h = 50 \times \frac{\sqrt{3}}{2}$

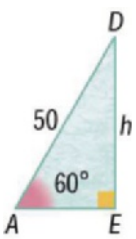
$\Leftrightarrow h = 25\sqrt{3}$

$A_{terreno} = base \times altura = 80 \times 25\sqrt{3} = 2000\sqrt{3}$

$\approx 3464{,}1016$

A área do terreno é, aproximadamente, 3464 m^2.

3.2.



Seja E a projeção ortogonal de D sobre a reta AB.

$\cos(60°) = \frac{\overline{AE}}{50} \Leftrightarrow \overline{AE} = 50\cos(60°)$

$\Leftrightarrow \overline{AE} = 50 \times \frac{1}{2} = 25$ m

$\overline{EB} = 80 - 25 = 55$ m

$\overline{DB}^2 = \overline{DE}^2 + \overline{EB}^2 \underset{(\overline{DB}>0)}{\Leftrightarrow} \overline{DB} = \sqrt{\overline{DE}^2 + \overline{EB}^2}$

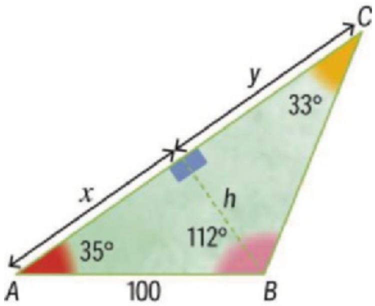
$\Leftrightarrow \overline{DB} = \sqrt{\left(25\sqrt{3}\right)^2 + 55^2} \Leftrightarrow \overline{DB} = \sqrt{1875 + 3025}$

$\Leftrightarrow \overline{DB} = \sqrt{4900} \Leftrightarrow \overline{DB} = 70$

$\overline{AD} + \overline{AB} + \overline{DB} = 50 + 80 + 70 = 200$

A Helena precisa de comprar 200 metros de rede.

4.



$\sin(35^\circ) = \frac{h}{100} \Leftrightarrow h = 100\sin(35^\circ)$

$h \approx 57,3576$ m

I – a)

$A\hat{C}B = 180^\circ - 35^\circ - 112^\circ = 33^\circ$

$\sin(33^\circ) = \frac{h}{\overline{BC}} \Leftrightarrow \overline{BC} = \frac{h}{\sin(33^\circ)}$

$\overline{BC} \approx \frac{57,3576}{\sin(33^\circ)} \Leftrightarrow \overline{BC} \approx 105,3131$

II – c)

$\cos(35^\circ) = \frac{x}{100} \Leftrightarrow x = 100\cos(35^\circ)$

$x \approx 81,9152$

$\cos(33^\circ) = \frac{y}{\overline{BC}} \Leftrightarrow y = \overline{BC} \times \cos(33^\circ)$

$y \approx 105,3131 \times \cos(33^\circ) \Leftrightarrow y \approx 88,3230$

$\overline{AC} = x + y \approx 81,9152 + 88,3230 \approx 170,2382$

$P = \overline{AB} + \overline{BC} + \overline{AC} \approx 100 + 105,3131 + 170,2382$

$\approx 375,5513$ m^2

III – c)

I $\to$ a ; II $\to$ c ; III $\to$ c

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Tarefa 2

1.1. Figura 1: $n = 5$; Figura 2: $n = 12$

1.2. Figura 1: $\frac{360^\circ}{5} = 72^\circ$

$72^\circ - 360^\circ = -288^\circ$, por exemplo

Figura 2: $\frac{360^\circ}{12} = 30^\circ$

$30^\circ - 360^\circ = -330^\circ$, por exemplo

2.1. $\frac{360^\circ}{180^\circ} = 2$

A rosácea seria composta por 2 triângulos.

2.2. $540^\circ = 360^\circ + 180^\circ$

A rosácea seria composta por 2 triângulos.

2.3. $990^\circ = 3 \times 360^\circ - 90^\circ$

$\frac{360^\circ}{90^\circ} = 4$

A rosácea seria composta por 4 triângulos.

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19.1. $A\hat{O}B = \frac{360^\circ}{6^\circ} = 60^\circ$

19.2. a) $300^\circ = 5 \times 60^\circ$

F é a imagem do ponto *A*.

b) $-240^\circ = 4 \times (-60^\circ)$

C é a imagem do ponto *A*.

19.3. Rotação de centro *O* e amplitude 60º.

Rotação de centro *O* e amplitude -300° .

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20.1. a) $45^\circ + 3 \times 360^\circ = 1125^\circ$

b) $-120^\circ - 2 \times 360^\circ = -840^\circ$

20.2. a) $3750^\circ = 150^\circ + 10 \times 360^\circ$; $(150^\circ, 10)$

b) $-2000^\circ = -200^\circ - 5 \times 360^\circ$; $(-200^\circ, -5)$

21. $360^\circ : 8 = 45^\circ$

21.1. a) $225^\circ = 5 \times 45^\circ$

$A\hat{O}F = 225^\circ$

O lado extremidade é $\dot{O}F$.

b) $-135^\circ = -3 \times 45^\circ$

$A\hat{O}F = -135^\circ$

O lado extremidade é $\dot{O}F$.

21.2. a) $1575^\circ = 135^\circ + 4 \times 360^\circ$;

$(135^\circ, 4)$

$A\hat{O}D = 3 \times 45^\circ = 135^\circ$

O lado extremidade é $\dot{O}D$.

b) $-1170^\circ = -90^\circ - 3 \times 360^\circ$

$(-90^\circ, -3)$

$A\hat{O}G = -2 \times 45^\circ = -90^\circ$

O lado extremidade é $\dot{O}G$.

21.3. a) $A\hat{O}C = 2 \times 45^\circ = 90^\circ$

$\alpha = 90^\circ + k \times 360^\circ,\ k \in \mathbb{Z}$

b) $A\hat{O}A = 0^\circ$

$\alpha = 0^\circ + k \times 360^\circ = k \times 360^\circ,\ k \in \mathbb{Z}$

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22.1. a) $A\hat{O}B = B\hat{O}D = D\hat{O}F = F\hat{O}A = \frac{360^\circ}{4} = 90^\circ$

B é o transformado de *A* pela rotação de centro *O* e ângulo definido por $(90^\circ, 3)$

b) $900^\circ = 180^\circ + 2 \times 360^\circ$

$A\hat{O}D = 2 \times 90^\circ = 180^\circ$

D é o transformado de *A* pela rotação de centro *O* e amplitude 900º.

c) $A\hat{O}C = C\hat{O}E = E\hat{O}A = \frac{360^\circ}{3} = 120^\circ$

$A\hat{O}E = 2 \times 120^\circ = 240^\circ$

E é o transformado de *A* pela rotação de centro *O* e ângulo definido por $(240°, 1)$.

d) $-1530° = -90° - 4 \times 360°$

$A\hat{O}F = -90°$

F é o transformado de *A* pela rotação de centro *O* e ângulo de amplitude $-1530°$.

e) $A\hat{O}E = -120°$

E é o transformado de *A* pela rotação de centro *O* e ângulo definido por $(-120°, 0)$.

f) $-1080° = 0° - 3 \times 360°$

A é o transformado de *A* pela rotação de centro *O* e ângulo de amplitude $-1080°$.

22.2. $\alpha = -90°$ ou $\alpha = 3 \times 90° = 270°$, por exemplo.

22.3. a) O lado extremidade é $\dot{O}C$.

b) $-180° = -2 \times 90°$

O lado extremidade é $\dot{O}D$.

22.4. a) $1920° = 120° + 5 \times 360°$

$E\hat{O}A = 120°$

O lado origem é $\dot{O}E$.

b) $-990° = -270° - 2 \times 360°$

O lado origem é $\dot{O}F$.

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23. Equipa A

$360°$ —— 250 m

$72\,000°$ —— d_A

$$d_A = \frac{72\,000° \times 250}{360°} = 50\,000 \text{ m}$$

Equipa B

$270° + 200 \times 360° = 72\,270°$

$360°$ ___ 250 m

$72\,270°$ ___ d_B

$$d_B = \frac{72\,270° \times 250}{360°} = 50\,187{,}5 \text{ m}$$

$d_B - d_A = 50\,187{,}5 - 50\,000 = 187{,}5$ m

A equipa vencedora foi a equipa B. Percorreu mais 187,5 m do que a equipa A.

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24.1. $\alpha = 269°$

$\alpha = 3 \times 90° = 270°$

$269° \in]180°, 270°[$

Então, α pertence ao 3.º Quadrante.

24.2. $\alpha = 361° = 360° + 1°$

$1° \in]0°, 90°[$

Então, α pertence ao 1.º Quadrante.

24.3. $\alpha = -361° = -360° - 1°$

$-1° \in]-90°, 0°[$

Então, α pertence ao 4.º Quadrante.

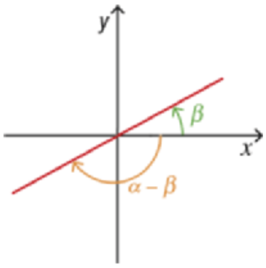
24.4. $\alpha = 820° = 100° + 2 \times 360°$

$100° \in]90°, 180°[$

Então, α pertence ao 2.º Quadrante.

24.5. $\alpha = \beta - 180°$ e $\beta \in 1.° \text{ Q}$

α pertence ao 3.º Quadrante.



24.6. $\beta \in 3.° \text{ Q}$

Seja θ o ângulo generalizado com lado extremidade coincidente com o de β.

$\theta \in]180°, 270°[$, logo,

$2\theta \in]360°, 540°[\quad (540° = 360° + 180°)$

Ou seja, $2\theta \in]0°, 180°[$.

Da mesma forma, $2\beta \in]0°, 180°[$.

Como somando um múltiplo de uma volta completa, $5 \times 360°$, não altera o lado extremidade do ângulo, tem-se que

$\alpha = 2\beta + 5 \times 360° \in]0°, 180°[$

α pertence ao 1.º ou ao 2.º Quadrante ou ao semieixo positivo *Oy*.

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Tarefa 3

1.1. A abcissa de *P* é o $\cos\alpha$ e a ordenada de *P* é o $\sin\alpha$.

1.2. a) comprimento da hipotenusa

b) $\overline{OP}$ **c)** 1

d) comprimento do cateto adjacente

e) $\overline{OQ}$ **f)** x

g) $\cos\alpha$ **h)** $\sin\alpha$

3. $P(r\cos\alpha, r\sin\alpha)$

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25.1. $180° < 218° < 270°$, logo:

- $\alpha \in 3.° \text{ Q}$
- $\cos\alpha < 0$
- $\sin\alpha < 0$

25.2. $270° < 272° < 360°$, logo:

- $\alpha \in 4.° \text{ Q}$
- $\cos\alpha > 0$
- $\sin\alpha < 0$

25.3. $90° < 107° < 180°$, logo:

- $\alpha \in 2.° \text{ Q}$
- $\cos\alpha < 0$
- $\sin\alpha > 0$

25.4. $-270° < -235° < -180°$, logo:

- $\alpha \in 2.° \text{ Q}$
- $\cos\alpha < 0$
- $\sin\alpha > 0$

26.1. $\sin(0°) + 2\cos(-180°) = 0 + 2(-1) = -2$

26.2. $\cos^3(-90°) - \sin(-270°)\cos(180°) = 0^3 - 1\times(-1)$

$= 0 + 1 = 1$

26.3. $\dfrac{\cos(270°)}{\sin(20°)} = \dfrac{0}{\sin(20°)} = 0$

27. $\left(\cos(30°), \sin(30°)\right) = \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

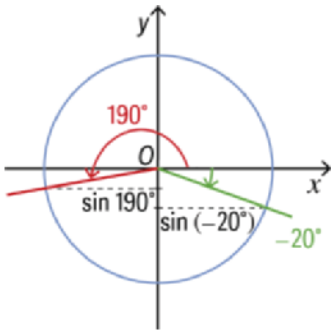
28.1. $180° < 190° < 270°$

$190° \in 3.° \text{ Q}$. Então $\sin(190°) < 0$.

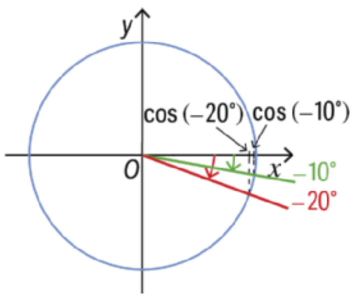
$35° \in 1.° \text{ Q}$. Então $\sin(35°) > 0$.

$\sin(190°) < \sin(35°)$

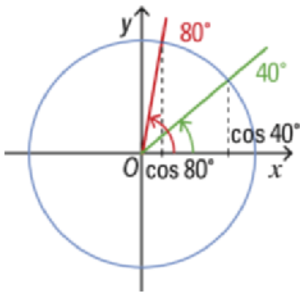
28.2. $\sin(190°) > \sin(-20°)$



28.3. $\cos(-10°) > \cos(-20°)$



28.4. $\cos(80°) < \cos(40°)$



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29.

$A_{[OAB]} = \dfrac{\overline{AB} \times x_B}{2}$ $\qquad B(\cos\alpha, \sin\alpha)$

$\overline{AB} = 2y_B = 2\sin\alpha$ e $x_B = \cos\alpha$

$A_{[OAB]} = \dfrac{2\sin\alpha\cos\alpha}{2} = \sin\alpha\cos\alpha$

30.

$C(\cos\alpha, \sin\alpha)$; $\cos\alpha < 0$ e $\sin\alpha > 0$

$-\cos\alpha > 0$

$\overline{BC} = 2(-\cos\alpha) = -2\cos\alpha$

$\overline{AB} = y_C = \sin\alpha$

$\overline{OA} = -x_C = -\cos\alpha$

$A_{[OABC]} = \dfrac{\overline{BC} + \overline{OA}}{2} \times \overline{AB} = \dfrac{-2\cos\alpha + (-\cos\alpha)}{2} \times \sin\alpha$

$= \dfrac{-3\cos\alpha\sin\alpha}{2} = -\dfrac{3}{2}\sin\alpha\cos\alpha$

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31.1. $\alpha = 361° = 360° + 1°$

$\sin\alpha = \sin(420°)$

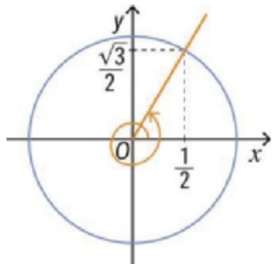
$= \sin(60° + 1\times 360°)$

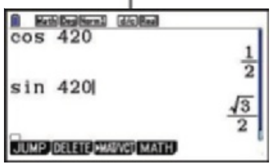
$= \sin(60°) = \dfrac{\sqrt{3}}{2}$

$\cos\alpha = \cos(420°)$

$= \cos(60° + 1\times 360°)$

$= \cos(60°) = \dfrac{1}{2}$





31.2. $\alpha = 1260° = 180° + 3\times 360° = (180°, 3)$

$\sin\alpha = \sin(1260°)$

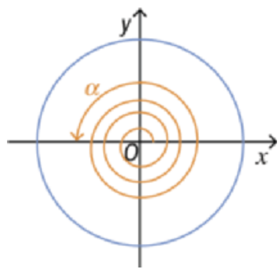
$= \sin(180° + 3\times 360°)$

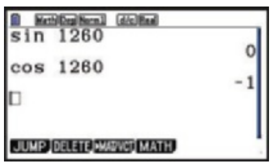
$= \sin(180°) = 0$

$\cos\alpha = \cos(1260°)$

$= \cos(180° + 3\times 360°)$

$= \cos(180°) = -1$





31.3. $\alpha = -1800° = 0° - 5 \times 360° = (0°, -5)$

$\sin\alpha = \sin(-1800°)$

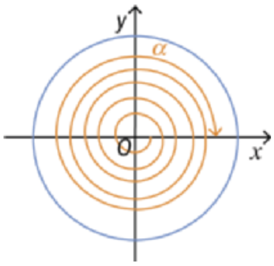
$= \sin(0° - 5 \times 360°)$

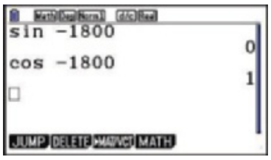
$= \sin(0°) = 0$

$\cos\alpha = \cos(-1800°)$

$= \cos(0° - 5 \times 360°)$

$= \cos(0°) = 1$





31.4. $\alpha = -315° = 45° - 360°$

$\sin\alpha = \sin(-315°)$

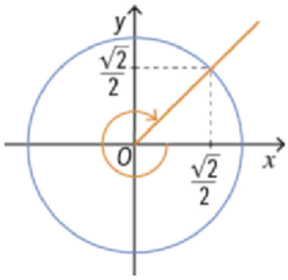
$= \sin(45° - 360°)$

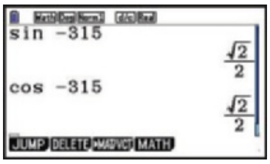
$= \sin(45°) = \frac{\sqrt{2}}{2}$

$\cos\alpha = \cos(-315°)$

$= \cos(45° - 360°)$

$= \cos(45°) = \frac{\sqrt{2}}{2}$





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32.1. Seja *R* o ponto de interseção de *OP* com a circunferência que pertence ao 1.º quadrante.
Q é o transformado de *R* pela rotação de meia-volta em torno de *O*.
$240° = 60° + 180°$

$R(\cos(60°), \sin(60°)) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Então, $Q\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$P(1, \tan(240°)) = (1, \tan(60°)) = (1, \sqrt{3})$

32.2. $Q(\cos(240°), \sin(240°)) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\cos(240°) = -\frac{1}{2}$; $\sin(240°) = -\frac{\sqrt{3}}{2}$

33.1. $\sin\alpha < 0$. Então $\alpha \in 3.° \text{ Q}$ ou $\alpha \in 4.° \text{ Q}$

$\tan\alpha > 0$. Então $\alpha \in 1.° \text{ Q}$ ou $\alpha \in 3.° \text{ Q}$

$\sin\alpha < 0 \vee \tan\alpha > 0$

Logo, α pertence ao 1.º, 3.º ou 4.º.

33.2. $\tan(-\alpha) > 0$. Então $-\alpha \in 1.° \text{ Q}$ ou $-\alpha \in 3.° \text{ Q}$

Se $-\alpha \in 1.° \text{ Q}$ então $\alpha \in 4.° \text{ Q}$.

Se $-\alpha \in 3.° \text{ Q}$ então $\alpha \in 2.° \text{ Q}$

α pertence ao 2.º ou ao 4.º Q.

33.3. $P = \sin\alpha \times \cos\alpha \times \tan\alpha$

α	1.º Q	2.º Q	3.º Q	4.º Q
$\sin\alpha$	+	+	–	–
$\cos\alpha$	+	–	–	+
$\tan\alpha$	+	–	+	–
P	+	+	+	+

α pertence ao 1.º, 2.º, 3.º ou 4.º Q.

33.4. $Q = \frac{\cos\alpha}{\sin\alpha}$

α	1.º Q	2.º Q	3.º Q	4.º Q
$\cos\alpha$	+	–	–	+
$\sin\alpha$	+	+	–	–
Q	+	–	+	–

α pertence ao 2.º ou 4.º Q.

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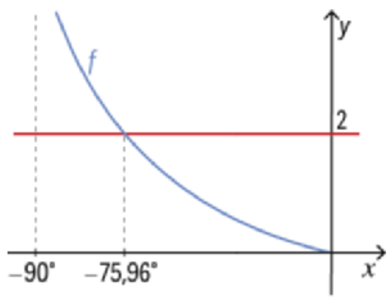
34.1. $\tan\alpha < 0$. Então $-\tan\alpha > 0$.

$A_{[OQR]} = \frac{\overline{OR} \times \overline{QR}}{2} = \frac{1 \times (-\tan\alpha)}{2} = -\frac{\tan\alpha}{2}$

34.2. $A_{[OQR]} = 2 \Leftrightarrow -\frac{\tan\alpha}{2} = 2$

$f(x) = -\frac{\tan x}{2}$

A área do triângulo $[OQR]$ é igual a 2 para $\alpha \approx -76°$.



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35.1. $780° = 60° + 2 \times 360° = (60°, 2)$

$\sin(780°) = \sin(60° + 2 \times 360°) = \sin(60°) = \frac{\sqrt{3}}{2}$

$\cos(780°) = \cos(60° + 2 \times 360°) = \cos(60°) = \frac{1}{2}$

$\tan(780°) = \tan(60° + 2 \times 360°) = \tan(60°) = \sqrt{3}$

35.2. $765° = 45° + 2 \times 360° = (45°,\ 2)$

$\sin(765°) = \sin(45° + 2 \times 360°) = \sin(45°) = \frac{\sqrt{2}}{2}$

$\cos(765°) = \cos(45° + 2 \times 360°) = \cos(45°) = \frac{\sqrt{2}}{2}$

$\tan(765°) = \tan(45° + 2 \times 360°) = \tan(45°) = 1$

35.3. $1470° = 30° + 4 \times 360° = (30°,\ 4)$

$\sin(1470°) = \sin(30° + 4 \times 360°) = \sin(30°) = \frac{1}{2}$

$\cos(1470°) = \cos(30° + 4 \times 360°) = \cos(30°) = \frac{\sqrt{3}}{2}$

$\tan(1470°) = \tan(30° + 4 \times 360°) = \tan(30°) = \frac{\sqrt{3}}{3}$

35.4. $-810° = -90° - 2 \times 360° = (-90°,\ -2)$

$\sin(-810°) = \sin(-90° - 2 \times 360°) = \sin(-90°) = -1$

$\cos(-810°) = \cos(-90° - 2 \times 360°) = \cos(-90°) = 0$

Não existe $\tan(-90°)$. Logo, não existe $\tan(-810°)$.

36.1. $\theta = 60°$
ou $\theta = 60° - 360° = -300°$
ou $\theta = 60° + 360° = 420°$
Por exemplo, 60º, –300º e 420º.

36.2. $\tan\theta = \sqrt{3}$. Então θ pertence ao 1.º quadrante ou ao 3.º quadrante.
Se $\theta \in 1.º$ Q:

$\sin\theta = \sin(60°) = \frac{\sqrt{3}}{2}$ e $\cos\theta = \cos(60°) = \frac{1}{2}$

Se $\theta \in 3.º$ Q:

$\sin\theta = -\sin(60°) = -\frac{\sqrt{3}}{2}$

e $\cos\theta = -\cos(60°) = -\frac{1}{2}$

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Tarefas de consolidação 2

1.1. $\alpha = -96°$ ou $\alpha = 360° - 96° = 264°$

1.2. **a)** $\beta = -2 \times 96° = -192°$ ou $\beta = 360° - 192° = 168°$

b) $[ABC]$ é um triângulo isósceles $(\overline{AB} = \overline{AC})$

$C\hat{B}A = A\hat{C}B = \frac{180° - 96°}{2} = 42°$

$C\hat{O}A = 2C\hat{B}A = 2 \times 42° = 84°$

$\beta = 84°$ ou $\beta = 84° - 360° = -276°$

c) $\beta = -84°$ ou $\beta = 276°$

2. $180° < 200° < 270°$
Logo, os lados extremidade pertencem ao 1.º ou ao 3.º quadrantes.

3. Equação reduzida da circunferência trigonométrica: $x^2 + y^2 = 1$

$A\left(-\frac{3}{5},\ y\right)$ é um ponto da circunferência trigonométrica.

Então, $\left(-\frac{3}{5}\right)^2 + y^2 = 1 \Leftrightarrow \frac{9}{25} + y^2 = 1$

$\Leftrightarrow y^2 = 1 - \frac{9}{25} \Leftrightarrow y^2 = \frac{16}{25} \Leftrightarrow y = \pm\sqrt{\frac{16}{25}} \Leftrightarrow y = \pm\frac{4}{5}$

$y = -\frac{4}{5}$ ou $y = \frac{4}{5}$. Existem dois pontos da circunferência trigonométrica com abcissa igual a $-\frac{3}{5}$.

4. I: Verdadeira
$180° < 225° < 270°$. Logo, $225° \in 3.º$ Q
$225° = 180° + 45°$. Então, 225º é uma rotação de meia-volta do ângulo de amplitude 45º.

$\sin(225°) = -\sin(45°) = -\frac{\sqrt{2}}{2}$

$\cos(225°) = -\cos(45°) = -\frac{\sqrt{2}}{2}$

Logo, $\sin(225°) = \cos(225°)$.

II: Falsa
$-1 \leq \cos\alpha \leq 1$, qualquer que seja o valor de α e $\frac{10}{9} > 1$

III: Falsa
Nos 3.º e 4.º quadrantes, $\sin\alpha < 0$.
Então, $-\sin\alpha > 0$.

5.1. $-1 \leq \cos\beta \leq 1$, qualquer que seja o valor de β.
$\downarrow \times 5$
$\Leftrightarrow -5 \leq 5\cos\beta \leq 5$
Máximo: 5; Mínimo: –5

5.2. $-1 \leq \sin\beta \leq 1$, qualquer que seja β.
$\downarrow \times 2$
$\Leftrightarrow -2 \leq 2\sin\beta \leq 2$
$\downarrow -3$
$\Leftrightarrow -5 \leq 2\sin\beta - 3 \leq -1$
Máximo: –1; Mínimo: –5

5.3. $-1 \leq \sin\beta \leq 1$, qualquer que seja β.
$\downarrow \times(-1)$
$\Leftrightarrow 1 \geq -\sin\beta \geq -1$
$\Leftrightarrow -1 \leq -\sin\beta \leq 1$
$\downarrow +3$
$\Leftrightarrow 2 \leq 3 - \sin\beta \leq 4$
Máximo: 4; Mínimo: 2

5.4. $-1 \leq \sin\beta \leq 1$, qualquer que seja β.
Então, $0 \leq \sin^2\beta \leq 1$.
Máximo: 1; Mínimo: 0

5.5. $\tan\beta \in]-\infty, +\infty[$, qualquer que seja β .

Então, $\tan^2\beta \geq 0$

$\downarrow +3$

$\Leftrightarrow \tan^2\beta + 3 \geq 3$

Máximo: não tem; Mínimo: 3

5.6. $-1 \leq \cos\beta \leq 1$, qualquer que seja β .

Então, $0 \leq \cos^2\beta \leq 1$

$\downarrow \times(-1)$

$\Leftrightarrow 0 \geq -\cos^2\beta \geq -1$

$\Leftrightarrow -1 \leq -\cos^2\beta \leq 0$

$\downarrow +1$

$\Leftrightarrow 0 \leq -\cos^2\beta \leq 1$

Máximo: 1; Mínimo: 0

6.1. Se α é um ângulo do 2.º quadrante, então

$0 < \sin\alpha < 1 \Leftrightarrow 0 < 1-2k < 1 \Leftrightarrow 0 < 1-2k \wedge 1-2k < 1$

$\Leftrightarrow 2k < 1 \wedge -2k < 0 \Leftrightarrow k < \frac{1}{2} \wedge k > 0$

$k \in \left]0, \frac{1}{2}\right[$

6.2. $\alpha \in]-180°, 0°[$. Então, α pertence ao 3.º Q, ao 4.º Q ou ao semieixo negativo Oy .

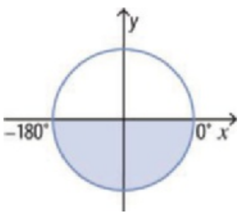
$-1 \leq \sin\alpha < 0 \Leftrightarrow -1 \leq 1-2k < 0$

$\Leftrightarrow -1 \leq 1-2k \wedge 1-2k < 0$

$\Leftrightarrow 2k \leq 2 \wedge -2k < -1$

$\Leftrightarrow k \leq 1 \wedge k > \frac{1}{2}$

$k \in \left]\frac{1}{2}, 1\right]$



7. $-270° < -200° < -180°$. Então, $\alpha \in$ 2.º Q

$\sin\alpha > 0$, $\cos\alpha < 0$, $\tan\alpha < 0$

$180° < \beta < 270°$. Então $\beta \in$ 3.º Q

$\sin\beta < 0$, $\cos\beta < 0$, $\tan\beta > 0$

7.1. $\underbrace{\sin\alpha}_{>0} - \underbrace{\cos\beta}_{<0} > 0$

$\sin\alpha - \cos\beta$ é positivo.

7.2. $\underbrace{\tan\beta}_{>0} \times \underbrace{\cos\alpha}_{<0} < 0$

Então, $-2\tan\beta \times \cos\alpha > 0$.

$-2\tan\beta \times \cos\alpha$ é positivo.

7.3. $\tan\alpha < 0$. Então $2\tan\alpha < 0$.

$\underbrace{\sin\beta}_{<0} + \underbrace{2\tan\alpha}_{<0} < 0$.

$\sin\beta + 2\tan\alpha$ é negativo.

7.4. $180° < \beta < 270°$

$\Leftrightarrow 180° - 200° < -200° + \beta < -200° + 270°$

$\Leftrightarrow -20° < \alpha + \beta < 70°$

$\alpha + \beta$ pertence ao 1.º, ao 4.º quadrante ou ao semieixo positivo Ox . Então, $\cos(\alpha + \beta) > 0$.

$\cos(\alpha + \beta)$ é positivo.

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8.1. $P(\cos(30°), \sin(30°))$

Então, $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$Q(2\cos(60°), 2\sin(60°))$

Então, $Q\left(2 \times \frac{1}{2}, 2 \times \frac{\sqrt{3}}{2}\right) = \left(1, \sqrt{3}\right)$.

8.2. R é um ponto da circunferência de raio 1 e centro na origem do referencial, ou seja é um ponto da circunferência trigonométrica.

Então, $R(\cos\alpha, \sin\alpha)$ e $R\left(-\frac{4}{5}, y_R\right)$.

Logo, $\cos\alpha = -\frac{4}{5}$.

Equação reduzida da circunferência trigonométrica: $x^2 + y^2 = 1$

$\left(-\frac{4}{5}\right)^2 + y_R^2 = 1 \Leftrightarrow \frac{16}{25} + y_R^2 = 1 \Leftrightarrow y_R^2 = 1 - \frac{16}{25}$

$\Leftrightarrow y_R^2 = \sqrt{\frac{9}{25}} \Leftrightarrow y_R = \pm\sqrt{\frac{9}{25}} \Leftrightarrow y_R = \pm\frac{3}{5}$

Como R é um ponto do 3.º quadrante, tem-se que $y_R = -\frac{3}{5}$.

Então, $\cos\alpha = -\frac{4}{5}$ e $\sin\alpha = -\frac{3}{5}$.

8.3. S é um ponto de uma circunferência de raio 2 centrada na origem.

$S(2\cos\beta, 2\sin\beta)$ e $S(1, y_S)$.

Então, $2\cos\beta = 1 \Leftrightarrow \cos\beta = \frac{1}{2}$.

Equação da circunferência: $x^2 + y^2 = 4$

$1^2 + y_S^2 = 4 \Leftrightarrow y_S^2 = 3 \Leftrightarrow y_S = \pm\sqrt{3}$

Como S é um ponto do 4.º quadrante, tem-se que $y_S = -\sqrt{3}$. Então, $2\sin\beta = -\sqrt{3} \Leftrightarrow \sin\beta = -\frac{\sqrt{3}}{2}$.

$\cos\beta = \frac{1}{2}$ e $\sin\beta = -\frac{\sqrt{3}}{2}$

9.1. $\alpha \in$ 1.º Q . Então, $\alpha + 180° \in$ 3.º Q

$\cos(\alpha + 180°) < 0$

$\underbrace{\sin\alpha}_{>0} \times \underbrace{\cos(\alpha + 180°)}_{<0} < 0$

$\sin\alpha \times \cos(\alpha + 180°)$ é negativo.

9.2. $\alpha \in$ 1.º Q . Então, $\alpha + 90° \in$ 2.º Q .

Logo, $\cos(\alpha + 90°) < 0$.

$\alpha \in$ 1.º Q . Então $\alpha - 90° \in$ 4.º Q .

Logo, $\tan(\alpha - 90°) < 0$.

$\underbrace{\cos(\alpha + 90°)}_{<0} \times \underbrace{\tan(\alpha - 90°)}_{<0} > 0$

$\cos(\alpha + 90°) \times \tan(\alpha - 90°)$ é positivo.

9.3. $\alpha - 90° \in 4.°$ Q e $\alpha + 90° \in 2.°$ Q (visto na alínea anterior)

$\cos(\alpha - 90°) > 0$ e $\sin(\alpha + 90°) > 0$

$\underbrace{\cos(\alpha - 90°)}_{>0} + \underbrace{\sin(\alpha + 90°)}_{>0} > 0$

$\cos(\alpha - 90°) + \sin(\alpha + 90°)$ é positivo.

9.4. $\alpha + 180° \in 3.°$ Q e $\alpha - 90° \in 4.°$ Q (visto nas alíneas anteriores)

$\tan(\alpha + 180°) > 0$ e $\sin(\alpha - 90°) < 0$

$\underbrace{\tan(\alpha + 180°)}_{>0} - \underbrace{\sin(\alpha - 90°)}_{<0} > 0$

$\tan(\alpha + 180°) - \sin(\alpha - 90°)$ é positivo.

10. $A(\cos\alpha, \sin\alpha)$

$\cos\alpha < 0 \Leftrightarrow -\cos\alpha > 0$ e $\sin\alpha > 0$

$A_{[ABCD]} = \overline{AD} \times \overline{AB}$

$\overline{AD} = 2 \times (-\cos\alpha) = -2\cos\alpha$

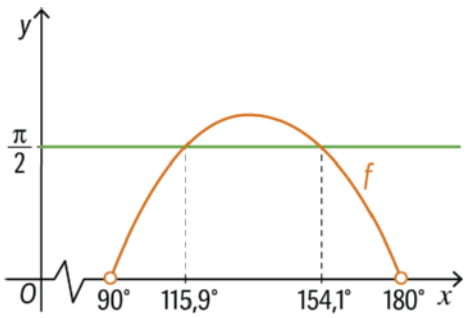
$\overline{AB} = 2\sin\alpha$

$A_{[ABCD]} = -2\cos\alpha \times 2\sin\alpha = -4\sin\alpha\cos\alpha$

$A_{círculo} = \pi \times 1^2 = \pi$

Pretende-se os valores de α para os quais

$A_{[ABCD]} > \frac{1}{2} A_{círculo} \Leftrightarrow -4\sin\alpha\cos\alpha > \frac{\pi}{2}$



$f(x) = -4\sin x \cos x$

A área do retângulo é superior a metade da área do círculo trigonométrico, para $\alpha \in [116°, 154°]$.

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Avaliação formativa 2

1. $\alpha = 60°$ ou $\alpha = 60° - 360° = -300°$, por exemplo.

2. $P(2\cos(45°), 2\sin(45°)) = \left(2 \times \frac{\sqrt{2}}{2}, 2 \times \frac{\sqrt{2}}{2}\right)$

$= (\sqrt{2}, \sqrt{2})$

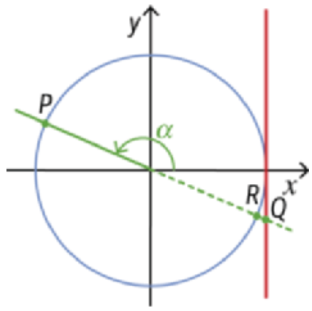
Opção (D)

3. I – c)

II – $\cos\alpha < 0$. Então, $-\cos\alpha > 0$. b)

III – $P(\cos\alpha, \sin\alpha)$; $Q(1, \tan\alpha)$

$\overline{PQ} = \sqrt{(1 - \cos\alpha)^2 + (\tan\alpha - \sin\alpha)^2}$



Seja R o ponto de interseção da reta PQ com a circunferência.

$R(-\cos\alpha, -\sin\alpha)$

$\overline{PR} = 2r = 2 \times 1 = 2$

$\overline{RQ} = \sqrt{(1 + \cos\alpha)^2 + (\tan\alpha + \sin\alpha)^2}$

$\overline{PQ} = \overline{PR} + \overline{RQ} = 2 + \sqrt{(1 + \cos\alpha)^2 + (\tan\alpha + \sin\alpha)^2}$

Então, III – c)

I $\rightarrow$ c) ; II $\rightarrow$ b) ; III $\rightarrow$ c)

4.1. $\overline{AB} = 2x_A = 2\cos\alpha$

$h_{[OAB]} = y_A = \sin\alpha$

$A_{[OAB]} = \frac{\overline{AB} \times h_{[OAB]}}{2} = \frac{2\cos\alpha \times \sin\alpha}{2} = \sin\alpha \times \cos\alpha$

4.2. $A(30°) = \sin(30°) \times \cos(30°) = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$A(60°) = \sin(60°) \times \cos(60°) = \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$

$A(30°) = A(60°)$

Significa que para $\alpha = 30°$ e para $\alpha = 60°$, obtém-se triângulos com a mesma área.

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1. 57,3º

2. A amplitude mantém-se aproximadamente igual a 57,3º para todo o valor do raio experimentado.

3. **a)** $2\pi r$ **b)** r

c) $r \times 360°$ **d)** $2\pi r$

4. π rad

5. $A\hat{O}B = 50°$ e $A\hat{O}C = 1$ rad

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37.1.

graus	radianos
180	π
135	x

$x = \frac{135\pi}{180} = \frac{3\pi}{4}$

$135° = \frac{3\pi}{4}$ rad

37.2.

graus radianos

$180 \_\_\_ \pi$

$270 \_\_\_ x$

$x = \frac{270 \times \pi}{180} = \frac{3\pi}{2}$

$270^{\circ} = \frac{3\pi}{2}$ rad

37.3.

graus radianos

$180 \_\_\_ \pi$

$150 \_\_\_ x$

$x = \frac{150 \times \pi}{180} = \frac{5\pi}{6}$

$150^{\circ} = \frac{5\pi}{6}$ rad

37.4.

graus radianos

$180 \_\_\_ \pi$

$330 \_\_\_ x$

$x = \frac{330 \times \pi}{180} = \frac{11\pi}{6}$

$330^{\circ} = \frac{11\pi}{6}$ rad

37.5.

graus radianos

$180 \_\_\_ \pi$

$450 \_\_\_ x$

$x = \frac{450 \times \pi}{180} = \frac{5\pi}{2}$

$450^{\circ} = \frac{5\pi}{2}$ rad

37.6.

graus radianos

$180 \_\_\_ \pi$

$252 \_\_\_ x$

$x = \frac{252 \times \pi}{180} = \frac{7\pi}{5}$

$252^{\circ} = \frac{7\pi}{5}$ rad

38.1.

graus radianos

$180 \_\_\_ \pi$

$x \_\_\_ \frac{2\pi}{3}$

$x = \frac{\frac{2\pi}{3} \times 180}{\pi} = 120$

$\frac{2\pi}{3}$ rad $= 120^{\circ}$

38.2.

graus radianos

$180 \_\_\_ \pi$

$x \_\_\_ \frac{5\pi}{6}$

$x = \frac{\frac{5\pi}{6} \times 180}{\pi} = 150$

$\frac{5\pi}{6}$ rad $= 150^{\circ}$

38.3.

graus radianos

$180 \_\_\_ \pi$

$x \_\_\_ \frac{5\pi}{4}$

$x = \frac{\frac{5\pi}{4} \times 180}{\pi} = 225$

$\frac{5\pi}{4}$ rad $= 225^{\circ}$

38.4.

graus radianos

$180 \_\_\_ \pi$

$x \_\_\_ \frac{5\pi}{8}$

$x = \frac{\frac{5\pi}{8} \times 180}{\pi} = 112,5$

$\frac{5\pi}{8}$ rad $= 112,5^{\circ}$

39.

graus radianos

$180 \_\_\_ \pi$

$120 \_\_\_ x$

$x = \frac{120 \times \pi}{180} = \frac{2\pi}{3}$

$D\hat{C}B = D\hat{A}B = \frac{2\pi}{3}$ rad

$C\hat{B}A = C\hat{D}A = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$ rad

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40.1. $\pi < \frac{8}{7}\pi < \frac{3}{2}\pi$; α pertence ao 3.º quadrante

40.2. $-\frac{\pi}{2} < -\frac{\pi}{5} < 0$; α pertence ao 4.º quadrante

40.3. $\frac{3\pi}{2} < \frac{13\pi}{7} < 2\pi$; α pertence ao 4.º quadrante

40.4. Adicionando uma volta completa a α , tem-se

$\beta = \alpha + 2\pi = -\frac{7\pi}{2} + 2\pi = -\frac{3\pi}{2}$

α e β pertencem ao semieixo positivo *Oy*.

40.5. Retirando 600 voltas completas a α , tem-se

$\beta = 1201\pi - 600 \times 2\pi = \pi$

α e β pertencem ao semieixo negativo Ox .

41.1. $\alpha = -30^{\circ} - 3 \times 360^{\circ} = -1110^{\circ}$

graus radianos

$180 \_\_\_ \pi$

$-1110 \_\_\_ x$

$x = \frac{-1110 \times \pi}{180} = -\frac{37\pi}{6}$

$\alpha = -\frac{37\pi}{6}$ rad

41.2. $\frac{23\pi}{2} - 5 \times 2\pi = \frac{3\pi}{2}$; $\frac{\theta}{2} = \frac{3\pi}{2} \Leftrightarrow \theta = 3\pi$;

$n - 1 = 5 \Leftrightarrow n = 6$; $\theta = 3\pi$ e $n = 6$

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Tarefa 5

1.1. $P_{\circ} = 2\pi r = 2\pi \times 3 = 6\pi$ cm

2π rad ___ 6π cm
2 rad ___ x
$x = \frac{2 \times 6\pi}{2\pi} = 6$ cm

O comprimento do arco de circunferência é 6 cm.

1.2. $A_{\circ} = \pi r^2 = \pi \times 3^2 = 9\pi$ cm²

2π rad ___ 9π cm²
2 rad ___ x
$x = \frac{2 \times 9\pi}{2\pi} = 9$ cm²

A área do setor circular é 9 cm².

2.1. $P_{\circ} = 2\pi r$

2π rad ___ $2\pi r$
α rad ___ x
$x = \frac{\alpha \times 2\pi r}{2\pi} = \alpha r$

O comprimento do arco de circunferência definido por α é αr.

2.2. $A_{\circ} = \pi r^2$

2π rad ___ πr^2
α rad ___ x
$x = \frac{\alpha \times \pi r^2}{2\pi} = \frac{\alpha r^2}{2}$

A área do setor circular definido por α é $\frac{\alpha r^2}{2}$.

42. $\alpha = ?$; $r = 10$ cm;

$\alpha r = 14\pi \Leftrightarrow 10\alpha = 14\pi \Leftrightarrow \alpha = \frac{14\pi}{10} \Leftrightarrow \alpha = \frac{7\pi}{5}$ rad

graus	radianos
180	π
x	$\frac{7\pi}{5}$

$x = \frac{\frac{7\pi}{5} \times 180}{\pi} = 252$

$\alpha = \frac{7\pi}{5}$ rad $= 252$º

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43. $A\hat{O}B = 45º = \frac{\pi}{2}$ rad

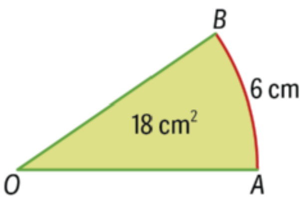
$$A = \frac{\alpha r^2}{2} = \frac{\frac{\pi}{2} \times 10^2}{2} = 25\pi \text{ cm}^2$$

A área do setor circular é 25π cm².

Alternativa:

$$A = \frac{\pi r^2}{4} = \frac{\pi \times 10^2}{4} = 25\pi \text{ cm}^2$$

44.



$$\begin{cases} \alpha r = 6 \\ \frac{\alpha r^2}{2} = 18 \end{cases} \Leftrightarrow \begin{cases} \alpha r = 6 \\ \alpha r^2 = 36 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \alpha = \frac{6}{r} \\ \frac{6}{r} \times r^2 = 36 \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_ \\ 6r = 36 \end{cases} \Leftrightarrow \begin{cases} \alpha = \frac{6}{6} = 1 \\ r = 6 \end{cases}$$

Raio 6 cm e amplitude do ângulo ao centro 1 rad.

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Tarefa 6

1. $\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$ rad.

Ponto C

2. $\frac{2\pi}{12}$ rad $= \frac{\pi}{6}$ rad;

A: $\alpha = 0$ rad; B: $\alpha = \frac{\pi}{6}$ rad;

C: $\alpha = \frac{2\pi}{6} = \frac{\pi}{3}$ rad; D: $\alpha = \frac{3\pi}{6} = \frac{\pi}{2}$ rad;

E: $\alpha = \frac{4\pi}{6} = \frac{2\pi}{3}$ rad; F: $\alpha = \frac{5\pi}{6}$ rad;

G: $\alpha = \frac{6\pi}{6} = \pi$ rad; H: $\alpha = \frac{7\pi}{6}$ rad;

I: $\alpha = \frac{8\pi}{6} = \frac{4\pi}{3}$ rad; J: $\alpha = \frac{9\pi}{6} = \frac{3\pi}{2}$ rad;

K: $\alpha = \frac{10\pi}{6} = \frac{5\pi}{3}$ rad; L: $\alpha = \frac{11\pi}{6}$ rad;

$\frac{2\pi}{8} = \frac{\pi}{4}$ rad;

M: $\alpha = 0$ rad; N: $\alpha = \frac{\pi}{4}$ rad;

P: $\alpha = \frac{2\pi}{4} = \frac{\pi}{2}$ rad; Q: $\alpha = \frac{3\pi}{4}$ rad;

R: $\alpha = \frac{4\pi}{4} = \pi$ rad; S: $\alpha = \frac{5\pi}{4}$ rad;

T: $\alpha = \frac{6\pi}{4} = \frac{3\pi}{2}$ rad; U: $\alpha = \frac{7\pi}{4}$ rad

3. $C\left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right)$, $C\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $N\left(2\cos\frac{\pi}{4}, 2\sin\frac{\pi}{4}\right)$;

$N\left(2 \times \frac{\sqrt{2}}{2}, 2 \times \frac{\sqrt{2}}{2}\right)$, $N\left(\sqrt{2}, \sqrt{2}\right)$

4.1. $\sin\left(\frac{5\pi}{3}\right) < 0$; $\cos\left(\frac{5\pi}{3}\right) > 0$; $\tan\left(\frac{5\pi}{3}\right) < 0$

4.2. $\sin\left(\frac{5\pi}{4}\right) < 0$; $\cos\left(\frac{5\pi}{4}\right) < 0$; $\tan\left(\frac{5\pi}{4}\right) > 0$

45. $\sin(C\hat{B}A)=\frac{2}{4} \Leftrightarrow \sin(C\hat{B}A)=\frac{1}{2} \Leftrightarrow C\hat{B}A=\frac{\pi}{6}$ rad

$C\hat{B}A=\pi-\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{2}$ rad ;

$\sin\left(\frac{\pi}{3}\right)=\frac{2}{\overline{AC}} \Leftrightarrow \frac{\sqrt{3}}{2}=\frac{2}{\overline{AC}} \Leftrightarrow$

$\Leftrightarrow \overline{AC}=\frac{2\times 2}{\sqrt{3}}_{(\times\sqrt{3})} \Leftrightarrow \overline{AC}=\frac{4\sqrt{3}}{3}$;

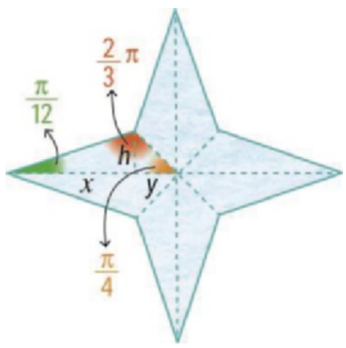
$\sin\left(\frac{\pi}{3}\right)=\frac{\overline{CB}}{\overline{AB}} \Leftrightarrow \frac{\sqrt{3}}{2}=\frac{4}{\overline{AB}} \Leftrightarrow \overline{AB}=\frac{4\times 2}{\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow \overline{AB}=\frac{8}{\sqrt{3}}_{(\times\sqrt{3})} \Leftrightarrow \overline{AB}=\frac{8\sqrt{3}}{3}$;

$C\hat{B}A=\frac{\pi}{2}$ rad; $C\hat{B}A=\frac{\pi}{6}$ rad ; $\overline{AC}=\frac{4\sqrt{3}}{3}$;

$\overline{AB}=\frac{8\sqrt{3}}{3}$

46. $\frac{2\pi}{8}=\frac{\pi}{4}$



$\pi-\frac{2\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$

$\sin\frac{\pi}{12}=\frac{h}{5} \Leftrightarrow$

$\Leftrightarrow h=5\sin\frac{\pi}{12}$

$h\approx 1{,}2941$

$\cos\frac{\pi}{12}=\frac{x}{5} \Leftrightarrow x=5\cos\frac{\pi}{12}$;

$x\approx 4{,}8296$

$\tan\frac{\pi}{4}=\frac{h}{y} \Leftrightarrow y=\frac{h}{\tan\frac{\pi}{4}} \Leftrightarrow y=h$

$y\approx 1{,}2941$

$A=8\times\frac{(x+y)\times h}{2}=$

$\approx 8\times\frac{(4{,}8296+1{,}2941)\times 1{,}2941}{2}\approx 31{,}6987$

A área do polígono é 31,7 cm^2.

47.1. $\sin\frac{\pi}{2}\left(\tan\frac{\pi}{4}-\cos\pi\right)=1\times(1-(-1))=1\times 2=2$

47.2. $\cos\left(\frac{7\pi}{3}\right)-\sin\left(-\frac{11\pi}{6}\right)=$

$=\cos\left(2\pi+\frac{\pi}{3}\right)-\sin\left(-\frac{12\pi}{6}+\frac{\pi}{6}\right)=$

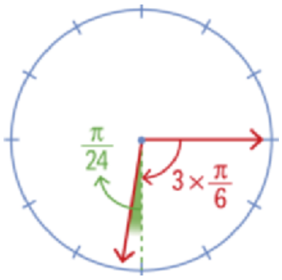
$=\cos\frac{\pi}{3}-\sin\frac{\pi}{6}=\frac{1}{2}-\frac{1}{2}=0$

Tarefas de consolidação 3

1.1. $-\frac{9}{2}\pi=-\frac{4}{2}\pi-\frac{4}{2}\pi-\frac{\pi}{2}=-2\pi-2\pi-\frac{\pi}{2}$

O ponteiro dos minutos deu duas voltas mais $\frac{1}{4}$ de volta, ou seja, passaram duas horas e 15 minutos das 4 horas. O relógio agora marca 6 h 15 min .

1.2.



$2\pi : 12=\frac{\pi}{6}$ rad

60 min______ $\frac{\pi}{6}$ rad

15 min______ x

$x=\frac{15\times\frac{\pi}{6}}{60}=\frac{\pi}{24}$ rad ; $\frac{\pi}{24}+3\times\frac{\pi}{6}=\frac{13\pi}{24}$

A amplitude do ângulo convexo formado pelos ponteiros do relógio às 18 h 15 min é $\frac{13\pi}{24}$ rad.

2. $\alpha=\frac{3}{2}\beta$; $d=30$ cm; $r=15$ cm

2.1. $\alpha+\beta=2\pi \Leftrightarrow \frac{3}{2}\beta+\beta=2\pi \Leftrightarrow \frac{5}{2}\beta=2\pi \Leftrightarrow$

$\Leftrightarrow 5\beta=4\pi \Leftrightarrow \beta=\frac{4\pi}{5}$; $\beta\times r=\frac{4\pi}{5}\times 15=12\pi$

O comprimento da borda da piza que a Vitória comeu é 12π cm.

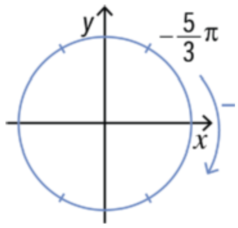
2.2. $\alpha=\frac{3}{2}\beta$; $\alpha=\frac{3}{2}\times\frac{4\pi}{5}=\frac{12\pi}{10}=\frac{6\pi}{5}$;

$\frac{\alpha r^2}{2}$; $\frac{\frac{6\pi}{5}\times 15^2}{2}\approx 424$ cm^2

A área da parte da piza que o Salvador comeu é, aproximadamente 424 cm^2.

3. I. $\alpha=-\frac{59\pi}{3}=\underbrace{-9\times 2\pi}_{\text{9 voltas completas}}-\frac{5}{3}\pi$

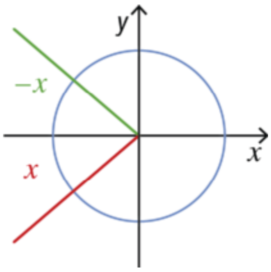
$-2\pi<-\frac{5}{3}\pi<-\frac{3}{2}\pi$



O ângulo $-\frac{59\pi}{3}$ descreve nove voltas no sentido negativo e o ângulo $-\frac{5\pi}{3}$.

que se situa no 1º quadrante. Logo, o ângulo α de amplitude $-\frac{59\pi}{3}$ pertence ao 1º quadrante.

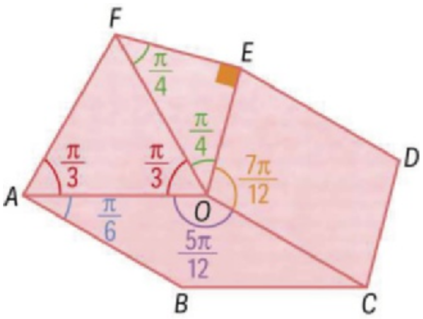
II. $x \in \left]\pi, \frac{3}{2}\pi\right[$, $x \in 3.^{\circ}Q$, $-x \in 2.^{\circ}Q$



$\underbrace{\sin x}_{-} \times \underbrace{\tan(-x)}_{-} > 0$

$x \in 3.^{\circ}$ quadrante e no $3.^{\circ}$ quadrante, o seno é negativo. $-x \in 2.^{\circ}$ quadrante e no $2.^{\circ}$ quadrante, a tangente é negativa. Assim, $\sin x \times \tan(-x)$ designa um número real positivo.

4.



$F\hat{A}O = A\hat{O}F = A\hat{F}O = \frac{\pi}{3}$

$O\hat{F}E = F\hat{O}E = \frac{\pi}{4}$

$B\hat{A}O = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$A\hat{O}C = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$; $C\hat{O}E = 2\pi - \frac{\pi}{4} - \frac{\pi}{3} - \frac{5\pi}{6} = \frac{7\pi}{12}$;

$D\hat{C}O = \pi - \frac{7\pi}{12} = \frac{5\pi}{12}$

A amplitude do ângulo DCO é $\frac{5\pi}{12}$ rad.

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5.1. $2\pi : 8 = \frac{\pi}{4}$ rad

a) $\dot{O}G$ **b)** $\dot{O}E$

c) $\dot{O}A$ **d)** $-\frac{\pi}{2} = -\frac{2\pi}{4}$; $\dot{O}B$

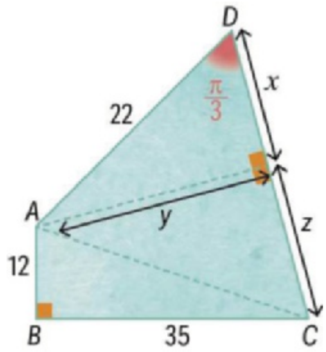
5.2. **a)** $\frac{5}{2}\pi = \frac{4}{2}\pi + \frac{\pi}{2} = 2\pi + \frac{\pi}{2} = 2\pi + \frac{2\pi}{4}$; $\dot{O}F$

b) $-\frac{15\pi}{4} = -\frac{16\pi}{4} + \frac{\pi}{4} = -4\pi + \frac{\pi}{4}$; $\dot{O}E$

c) -4π ; $\dot{O}D$

d) $-\frac{11\pi}{2} = -\frac{12\pi}{2} + \frac{\pi}{2} = -6\pi + \frac{2\pi}{4}$; $\dot{O}F$

6.



$\cos\frac{\pi}{3} = \frac{x}{22} \Leftrightarrow$

$\Leftrightarrow x = 22\cos\frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = 22 \times \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow x = 11$

$\sin\frac{\pi}{3} = \frac{y}{22} \Leftrightarrow y = 22\sin\frac{\pi}{3} \Leftrightarrow y = 22 \times \frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow y = 11\sqrt{3}$

$\overline{AC}^2 = 12^2 + 35^2 \Leftrightarrow \overline{AC}^2 = 1369 \Leftrightarrow$

$\underset{(\overline{AC}>0)}{\Leftrightarrow} \overline{AC} = \sqrt{1369}$;

$\overline{AC}^2 = y^2 + z^2 \Leftrightarrow$

$\Leftrightarrow z^2 = \overline{AC}^2 - y^2 \Leftrightarrow z^2 = 1369 - \left(11\sqrt{3}\right)^2 \Leftrightarrow$

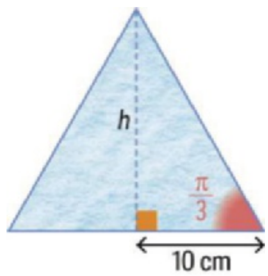
$\Leftrightarrow z^2 = 1369 - 121 \times 3 \Leftrightarrow z^2 = 1006 \Leftrightarrow z = \sqrt{1006}$;

$A_{[ABCD]} = A_{[ABC]} + A_{[ACD]} =$

$= \frac{12 \times 35}{2} + \frac{\left(11 + \sqrt{1006}\right) \times 11\sqrt{3}}{2} \approx 616{,}93887$

A área do quadrilátero [ABCD] é, aproximadamente, 616,9 cm^2.

7.



$\tan\frac{\pi}{3} = \frac{h}{10} \Leftrightarrow \sqrt{3} = \frac{h}{10} \Leftrightarrow$

$\Leftrightarrow h = 10\sqrt{3}$ cm

$V = \frac{1}{3} A_b \times h$

$V = \frac{1}{3}\pi \times 10^2 \times 10\sqrt{3} \approx 1813{,}8$ cm^3

O volume do cone é, aproximadamente, 1813,8 cm^3.

8. $A_{\substack{\text{coroa}\\\text{circular}}} = A_{\substack{\text{setor}\\\text{maior}}} - A_{\substack{\text{setor}\\\text{menor}}} = \frac{\alpha R^2}{2} - \frac{\alpha r^2}{2} =$

$= \frac{\alpha (2r)^2 - \alpha r^2}{2} = \frac{4\alpha r^2 - \alpha r^2}{2} = \frac{3\alpha r^2}{2} = \frac{3\alpha}{2} r^2$

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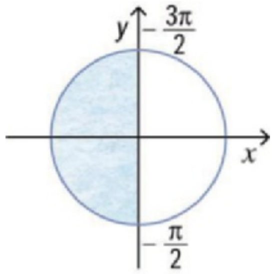
Avaliação formativa 3

1. $a = \pi - \frac{4\pi}{5} = \frac{\pi}{5}$; $a + c = \frac{\pi}{5} + \frac{\pi}{5} = \frac{2\pi}{5}$ (V)

Opção (A)

2. $x \in \left]-\frac{3}{2}\pi, -\frac{\pi}{2}\right[$

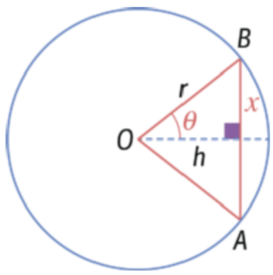


No 2.º ou 3.º quadrantes, e em $-\pi$, o $\cos x < 0$.
Logo, $2\cos x < 0$. Opção (B)

3. $\frac{46\pi}{9} = \underbrace{4\pi}_{(1)} + \frac{10\pi}{9}$, em que (1) representa duas voltas completas; $\frac{10\pi}{9} \in$ 3.º quadrante

I - b); II - c); III - a)

4.1.



$\overline{AB} = 2x$;

$\sin\theta = \frac{x}{r} \Leftrightarrow x = r\sin\theta$

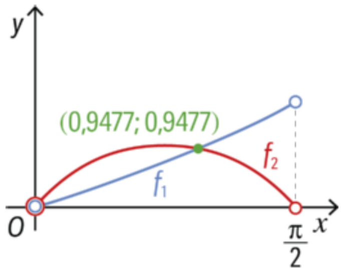
$\cos\theta = \frac{h}{r} \Leftrightarrow h = r\cos\theta$

$\overline{AB} = 2r\sin\theta$;

$A_{[AOB]} = \frac{2r\sin\theta \times r\cos\theta}{2} = r^2 \sin\theta\cos\theta$ c.q.m.

4.2. $\frac{\alpha r^2}{2}$; $\alpha = 2\theta$; $S(\theta) = \frac{2\theta r^2}{2} = \theta r^2$

- $S(\theta) = 2A(\theta)$; $\theta r^2 = 2r^2 \sin\theta\cos\theta \Leftrightarrow$
  $\Leftrightarrow \theta = 2\sin\theta\cos\theta$
- $f_1(x) = x$; $f_2(x) = 2\cos x \sin x$



$\theta \approx 0{,}95$ rad

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Tarefa 7

A circunferência trigonométrica é uma circunferência centrada na origem do referencial e raio igual a 1. Então, pode ser definida pela equação $x^2 + y^2 = 1$. Qualquer ponto P da circunferência tem coordenadas $(\cos\alpha, \sin\alpha)$, sendo α o ângulo definido pelo semieixo positivo Ox e pela semirreta $\dot{O}P$. Assim, atendendo à equação da circunferência, temos que:

$\cos^2\alpha + \sin^2\alpha = 1 \Leftrightarrow \sin^2\alpha + \cos^2\alpha = 1$

48. $\sin^2\alpha + \cos^2\alpha = 1$

Então, $\sin^2\alpha + \left(-\frac{1}{3}\right)^2 = 1 \Leftrightarrow \sin^2\alpha + \frac{1}{9} = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \sin^2\alpha = \frac{8}{9}$

Como $\alpha \in 3.^{\circ}$Q, $\sin\alpha < 0$, pelo que

$\sin\alpha = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$.

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49. C pertence à circunferência trigonométrica. Então, $C(\cos\alpha, \sin\alpha)$. Logo, $\sin\alpha = \frac{2}{3}$.

$\overline{AC} = 2\sin\alpha = 2 \times \frac{2}{3} = \frac{4}{3}$

$\sin^2\alpha + \cos^2\alpha = 1$

Então, $\left(\frac{2}{3}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow \frac{4}{9} + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = 1 - \frac{4}{9} \Leftrightarrow \cos^2\alpha = \frac{5}{9}$

Como $\alpha \in 2.^{\circ}$Q, $\cos\alpha < 0$, pelo que

$\cos\alpha = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$.

$\overline{AB} = 2 \times \left|-\frac{\sqrt{5}}{3}\right| = \frac{2\sqrt{5}}{3}$;

$A_{[ABC]} = \frac{\overline{AB} \times \overline{AC}}{2} = \frac{\frac{2\sqrt{5}}{3} \times \frac{4}{3}}{2} = \frac{4\sqrt{5}}{9}$ u. a.

50.1. $\sin^2 x - \sin^4 x = \sin^2 x(1 - \sin^2 x) =$

$= (1 - \cos^2 x) \times \cos^2 x = \cos^2 x - \cos^4 x$

C.A.: $\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \cos^2 x = 1 - \sin^2 x \Leftrightarrow$

$\Leftrightarrow \sin^2 x = 1 - \cos^2 x$

50.2. $\frac{1 - \cos x}{\underset{\times(1-\cos x)}{\sin x}} + \frac{\sin x}{\underset{\times(\sin x)}{1 - \cos x}} = \frac{(1 - \cos x)^2 + \sin^2 x}{\sin x(1 - \cos x)} =$

$= \frac{1 - 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1 - \cos x)} = \frac{1 - 2\cos x + 1}{\sin x(1 - \cos x)} =$

$= \frac{2 - 2\cos x}{\sin x(1 - \cos x)} = \frac{2\cancel{(1 - \cos x)}}{\sin x\cancel{(1 - \cos x)}} = \frac{2}{\sin x}$

50.3. $\frac{\sin x}{1 - \cos x} = \frac{\sin x(1 - \cos x)}{(1 - \cos x)(1 - \cos x)} = \frac{\sin x(1 - \cos x)}{1 - \cos^2 x} =$

$\underset{(1)}{=} \frac{\sin x(1 - \cos x)}{\sin^2 x} = \frac{1 - \cos x}{\sin x}$

(1) $\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \sin^2 x = 1 - \cos^2 x$

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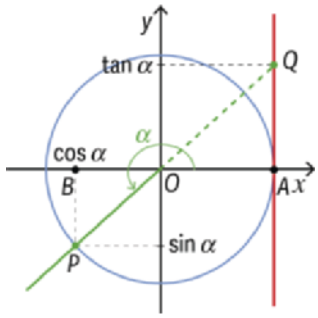
Tarefa 8

1. $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

2.1. $\tan\alpha = 0$; $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{0}{-1} = 0$

Logo, $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

2.2. Seja α um ângulo do 3.º Q. $P(\cos\alpha, \sin\alpha)$, $Q(1, \tan\alpha)$. Como os triângulos [*OAQ*] e [*OBP*] são semelhantes,

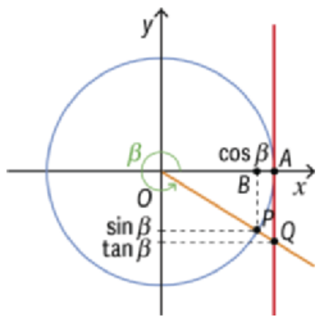


$\dfrac{\overline{AQ}}{\overline{BP}} = \dfrac{\overline{OA}}{\overline{OB}} \Leftrightarrow$

$\Leftrightarrow \dfrac{\tan\alpha}{-\sin\alpha} = \dfrac{1}{-\cos\alpha} \Leftrightarrow$

$\Leftrightarrow \tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

Seja β um ângulo do 4ºQ. $P(\cos\beta, \sin\beta)$, $Q(1, \tan\beta)$. Como os triângulos [*OAQ*] e [*OBP*] são semelhantes,



$\dfrac{\overline{AQ}}{\overline{BP}} = \dfrac{\overline{OA}}{\overline{OB}} \Leftrightarrow$

$\Leftrightarrow \dfrac{-\tan\beta}{-\sin\beta} = \dfrac{1}{\cos\beta} \Leftrightarrow \tan\beta = \dfrac{\sin\beta}{\cos\beta}$

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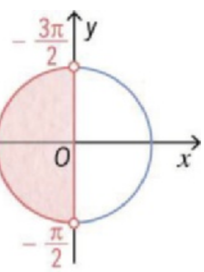
51. $-\sin\alpha = -\dfrac{3}{4} \Leftrightarrow \sin\alpha = \dfrac{3}{4}$;

$\sin^2\alpha + \cos^2\alpha = 1$

Então, $\left(\dfrac{3}{4}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow \dfrac{9}{16} + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = 1 - \dfrac{9}{16} \Leftrightarrow \cos^2\alpha = \dfrac{7}{16}$

$\alpha \in \left]-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}\right[$, pelo que $\cos\alpha = -\dfrac{\sqrt{7}}{4}$.



$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = -\dfrac{3}{\sqrt{7}} = -\dfrac{3\sqrt{7}}{7}$

$\dfrac{1}{\cos\alpha} - \tan\alpha = \dfrac{1}{-\frac{\sqrt{7}}{4}} - \left(-\dfrac{3\sqrt{7}}{7}\right) = -\dfrac{4}{\sqrt{7}} + \dfrac{3\sqrt{7}}{7} =$

$= \dfrac{-4\sqrt{7} + 3\sqrt{7}}{7} = -\dfrac{\sqrt{7}}{7}$

52.1. $\underset{\times(1+\sin x)}{\dfrac{1}{1-\sin x}} - \underset{\times(1-\sin x)}{\dfrac{1}{1+\sin x}} = \dfrac{1+\sin x - (1-\sin x)}{(1+\sin x)(1-\sin x)} =$

$= \dfrac{1+\sin x - 1 + \sin x}{1-\sin^2 x} \underset{(1)}{=} \dfrac{2\sin x}{\cos^2 x} = 2\dfrac{\sin x}{\cos x} \times \dfrac{1}{\cos x} =$

$= 2\tan x \times \dfrac{1}{\cos x} = \dfrac{2\tan x}{\cos x}$

sendo (1) $\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \cos^2 x = 1 - \sin^2 x$

52.2. $\dfrac{\tan^3 x}{\cos^2 x} - \tan^3 x = \tan^3 x\left(\dfrac{1}{\cos^2 x} - 1\right) =$

$= \tan^3 x\left(\dfrac{1-\cos^2 x}{\cos^2 x}\right) \underset{(1)}{=} \tan^3 x \times \dfrac{\sin^2 x}{\cos^2 x} =$

$= \tan^3 x \times \left(\dfrac{\sin x}{\cos x}\right)^2 = \tan^3 x \times \tan^2 x = \tan^5 x$

sendo (1) $\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \sin^2 x = 1 - \cos^2 x$

52.3. $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \dfrac{1-\left(\frac{\sin x}{\cos x}\right)^2}{1+\left(\frac{\sin x}{\cos x}\right)^2} = \dfrac{1-\frac{\sin^2 x}{\cos^2 x}}{1+\frac{\sin^2 x}{\cos^2 x}} =$

$= \dfrac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} = \dfrac{\cancel{\cos^2 x}\left(\cos^2 x - \sin^2 x\right)}{\cancel{\cos^2 x} \times 1} =$

$= \cos^2 x - \sin^2 x$

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53. $1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha}$

Então, $1 + 2{,}4^2 = \dfrac{1}{\cos^2\alpha} \Leftrightarrow 1 + 5{,}76 = \dfrac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow 6{,}76 = \dfrac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \dfrac{1}{6{,}76} \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = \dfrac{25}{169}$

Como $\alpha \in 3.^{\text{o}}\,Q$, $\cos\alpha < 0$, pelo que $\cos\alpha = -\sqrt{\dfrac{25}{169}} = -\dfrac{5}{13}$.

54.1. $P(1, \tan\beta)$ Então, $\tan\beta = 3$

$\dfrac{6}{\sqrt{3} - \tan\beta} = \dfrac{6}{\sqrt{3} - 3} = \dfrac{6\left(\sqrt{3}+3\right)}{\left(\sqrt{3}-3\right)\left(\sqrt{3}+3\right)} =$

$$= \frac{6\left(\sqrt{3}+3\right)}{\sqrt{3}^2-3^2} = \frac{6\left(\sqrt{3}+3\right)}{3-9} = \frac{6\left(\sqrt{3}+3\right)}{-6} = -\sqrt{3}-3$$

54.2. $1+\tan^2\beta = \dfrac{1}{\cos^2\beta}$

Então, $1+3^2 = \dfrac{1}{\cos^2\beta} \Leftrightarrow 1+9 = \dfrac{1}{\cos^2\beta} \Leftrightarrow$

$\Leftrightarrow 10 = \dfrac{1}{\cos^2\beta} \Leftrightarrow \cos^2\beta = \dfrac{1}{10}$

Como $\beta \in 3^{\text{o}}\,\text{Q}$, $\cos\beta < 0$, pelo que

$\cos\beta = -\sqrt{\dfrac{1}{10}} = -\dfrac{1}{\sqrt{10}} = -\dfrac{\sqrt{10}}{10}$

$\tan\beta = \dfrac{\sin\beta}{\cos\beta} \Leftrightarrow 3 = \dfrac{\sin\beta}{-\dfrac{\sqrt{10}}{10}} \Leftrightarrow \sin\beta = -\dfrac{3\sqrt{10}}{10}$

$\cos\beta - 2\sin\beta = -\dfrac{\sqrt{10}}{10} - 2\left(-\dfrac{3\sqrt{10}}{10}\right) =$

$= \dfrac{-\sqrt{10}+6\sqrt{10}}{10} = \dfrac{5\sqrt{10}}{10} = \dfrac{\sqrt{10}}{2}$

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55.1. $P(\cos\alpha, \sin\alpha)$; $B(1, \tan\alpha)$; $\overline{OC} = 1$; $\overline{AB} = \tan\alpha$;

$\overline{OA} = 1$;

$A(\alpha) = \dfrac{\overline{OC}+\overline{AB}}{2} \times \overline{OA} = \dfrac{1+\tan\alpha}{2} \times 1 =$

$= \dfrac{1+\tan\alpha}{2}$

55.2. $P(\cos\alpha, \sin\alpha)$

Seja x_p a abcissa do ponto *P*.

$x_p = \dfrac{\sqrt{10}}{10} \Leftrightarrow \cos\alpha = \dfrac{\sqrt{10}}{10}$

$\tan^2\alpha + 1 = \dfrac{1}{\left(\dfrac{\sqrt{10}}{10}\right)^2} \Leftrightarrow$

$\Leftrightarrow \tan^2\alpha = \dfrac{1}{\dfrac{10}{100}} - 1 \Leftrightarrow \tan^2\alpha = 10-1 \Leftrightarrow \tan^2\alpha = 9$

Como $\alpha \in 1^{\text{o}}\,\text{Q}$, $\tan\alpha > 0$, logo, $\tan\alpha = \sqrt{9} = 3$.

$A(\alpha) = \dfrac{1+\tan\alpha}{2} = \dfrac{1+3}{2} = 2$ u. a.

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Tarefas de consolidação 1

1. 1.º erro: O André usou valores arredondados e, na pergunta, não pede valores arredondados.

2.º erro: Quando calcula, na máquina, $\cos^{-1}\left(-\dfrac{1}{3}\right)$, a calculadora devolve-lhe o valor arredondado de um ângulo do segundo quadrante cujo cosseno é $-\dfrac{1}{3}$ e não do 3.º quadrante.

Resposta certa: Usando a fórmula fundamental da trigonometria, temos:

$\sin^2\alpha + \left(-\dfrac{1}{3}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1-\dfrac{1}{9} \Leftrightarrow \sin^2\alpha = \dfrac{8}{9}$

Como $\alpha \in 3.^{\text{o}}\,\text{Q}$, $\sin\alpha < 0$, logo,

$\sin\alpha = -\sqrt{\dfrac{8}{9}} = -\dfrac{2\sqrt{2}}{3}$.

2.1. $A\hat{C}B = \dfrac{\pi}{2} - \alpha$; $\sin\left(\dfrac{\pi}{2}-\alpha\right) = \sin A\hat{C}B = \dfrac{\overline{AB}}{\overline{BC}} = \cos\alpha$;

$\cos\left(\dfrac{\pi}{2}-\alpha\right) = \cos A\hat{C}B = \dfrac{\overline{AC}}{\overline{BC}} = \sin\alpha$

2.2. $\sin^2\left(\dfrac{\pi}{2}-\alpha\right) - 2\cos^2\left(\dfrac{\pi}{2}-\alpha\right) = \cos^2\alpha - 2\sin^2\alpha =$

$\underset{(1)}{=} 1-\sin^2\alpha - 2\sin^2\alpha = 1-3\sin^2\alpha$ com

(1) $\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = 1-\sin^2\alpha$

3. $P(\cos\alpha, \sin\alpha)$ e $P\left(-\dfrac{4}{5}, \sin\alpha\right)$

Então, $\cos\alpha = -\dfrac{4}{5}$.

$\sin^2\alpha + \cos^2\alpha = 1$

Então, $\sin^2\alpha + \left(-\dfrac{4}{5}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1-\dfrac{16}{25} \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = \dfrac{9}{25}$. Como $\alpha \in 3.^{\text{o}}\,\text{Q}$, $\sin\alpha < 0$, pelo que, $\sin\alpha = -\sqrt{\dfrac{9}{25}} = -\dfrac{3}{5}$.

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{-\dfrac{3}{5}}{-\dfrac{4}{5}} = \dfrac{3}{4}$

4. $\cos\alpha + \dfrac{1}{4\cos\alpha} - 1 = 0 \Leftrightarrow$

$\Leftrightarrow \dfrac{4\cos^2\alpha + 1 - 4\cos\alpha}{4\cos\alpha} = 0 \Leftrightarrow \dfrac{(2\cos\alpha - 1)^2}{4\cos\alpha} = 0 \Leftrightarrow$

$\Leftrightarrow (2\cos\alpha - 1)^2 = 0 \ \wedge \ 4\cos\alpha \neq 0 \Leftrightarrow$

$\Leftrightarrow 2\cos\alpha - 1 = 0 \ \wedge \ \cos\alpha \neq 0 \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \dfrac{1}{2} \ \wedge \ \cos\alpha \neq 0$;

$\sin^2\alpha + \cos^2\alpha = 1$

Então, $\sin^2\alpha + \left(\dfrac{1}{2}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1-\dfrac{1}{4} \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = \dfrac{3}{4}$.

$\sin\alpha \times \tan\alpha = \sin\alpha \times \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sin^2\alpha}{\cos\alpha} = \dfrac{\dfrac{3}{4}}{\dfrac{1}{2}} = \dfrac{6}{4} = \dfrac{3}{2}$

5. $\sin^2\alpha+\cos^2\alpha=1$

Então, $\left(\dfrac{\sqrt{k-2}}{k}\right)^2+\left(\dfrac{2}{k}\right)^2=1\Leftrightarrow\dfrac{k-2}{k^2}+\dfrac{4}{k^2}=1\Leftrightarrow$

$\Leftrightarrow\dfrac{k+2}{k^2}-1=0\Leftrightarrow\dfrac{k+2-k^2}{k^2}=0\Leftrightarrow$

$\Leftrightarrow -k^2+k+2=0\ \wedge\ k^2\neq0\Leftrightarrow$

$\Leftrightarrow k=\dfrac{-1\pm\sqrt{1^2-4(-1)\times2}}{2(-1)}\ \wedge\ k\neq0\Leftrightarrow$

$\Leftrightarrow k=\dfrac{-1\pm\sqrt{9}}{-2}\ \wedge\ k\neq0\Leftrightarrow$

$\Leftrightarrow\left(k=\dfrac{-1+3}{-2}\ \vee\ k=\dfrac{-1-3}{-2}\right)\wedge\ k\neq0\Leftrightarrow$

$\Leftrightarrow(k=-1\ \vee\ k=2)\ \wedge\ k\neq0$. Como

$\sin\alpha=\dfrac{\sqrt{k-2}}{k}$, tem-se que, $k-2\geq0\Leftrightarrow k\geq2$.

Logo, $k=2$.

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6. $P(\cos\alpha,\sin\alpha)$ e $T(1,\tan\alpha)$. Q resulta de uma rotação de meia-volta do ponto P e centro O.
Então, $Q(-\cos\alpha,-\sin\alpha)$.

7.1. $\dfrac{\cos^3\alpha-2\cos\alpha+\dfrac{1}{\cos\alpha}}{\cos\alpha\sin^2\alpha}=\dfrac{\dfrac{\cos^4\alpha-2\cos^2\alpha+1}{\cos\alpha}}{\cos\alpha\sin^2\alpha}=$

$=\dfrac{(\cos^2\alpha-1)^2}{\cos^2\alpha\sin^2\alpha}=\dfrac{(1-\cos^2\alpha)^2}{\cos^2\alpha\sin^2\alpha}\underset{(1)}{=}\dfrac{(\sin^2\alpha)^2}{\cos^2\alpha\sin^2\alpha}=$

$=\dfrac{\sin^2\alpha}{\cos^2\alpha}=\left(\dfrac{\sin\alpha}{\cos\alpha}\right)^2=\tan^2\alpha$

Em que (1) $\sin^2\alpha+\cos^2\alpha=1\Leftrightarrow\sin^2\alpha=1-\cos^2\alpha$

7.2. $\dfrac{\sin\alpha\cos\alpha}{\sin\alpha-\cos\alpha}+\dfrac{\cos\alpha}{1-\tan\alpha}=$

$=\dfrac{\sin\alpha\cos\alpha}{\sin\alpha-\cos\alpha}+\dfrac{\cos\alpha}{1-\dfrac{\sin\alpha}{\cos\alpha}}=$

$=\dfrac{\sin\alpha\cos\alpha}{\sin\alpha-\cos\alpha}+\dfrac{\cos\alpha}{\dfrac{\cos\alpha-\sin\alpha}{\cos\alpha}}=$

$=\dfrac{\sin\alpha\cos\alpha}{\sin\alpha-\cos\alpha}+\dfrac{\cos^2\alpha}{\cos\alpha-\sin\alpha}=$

$=\dfrac{\sin\alpha\cos\alpha}{\sin\alpha-\cos\alpha}-\dfrac{\cos^2\alpha}{\sin\alpha-\cos\alpha}=$

$=\dfrac{\sin\alpha\cos\alpha-\cos^2\alpha}{\sin\alpha-\cos\alpha}=\dfrac{\cos\alpha\,\cancel{(\sin\alpha-\cos\alpha)}}{\cancel{\sin\alpha-\cos\alpha}}=$

$=\cos\alpha$

8.1. $B(2\cos\alpha,2\sin\alpha)$; $\overline{CB}=2\times2\cos\alpha=4\cos\alpha$;

$\overline{CD}=2\sin\alpha$; $\overline{OD}=2\cos\alpha$.

Seja c o comprimento do arco $\overset{\frown}{AB}$.

$c=\alpha\times2=2\alpha$

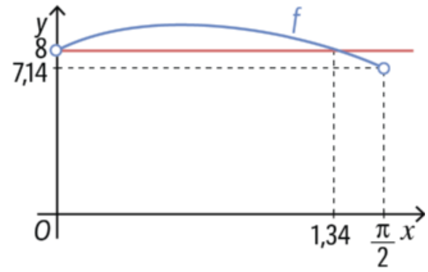
Perímetro $=\overline{OA}+c+\overline{BC}+\overline{CD}+\overline{DO}=$

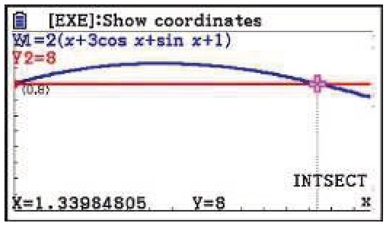
$=2+2\alpha+4\cos\alpha+2\sin\alpha+2\cos\alpha=$

$=2\alpha+6\cos\alpha+2\sin\alpha+2=$

$=2(\alpha+3\cos\alpha+\sin\alpha+1)$

8.2. Diâmetro da circunferência $=2\times2=4$
Seja f a função que, a cada valor de α faz corresponder o perímetro da região sombreada.
Pretende-se resolver graficamente a condição $f(\alpha)<2\times4\Leftrightarrow f(\alpha)<8$.





$1{,}34<\alpha<\dfrac{\pi}{2}$

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Avaliação formativa 4

1. $3\cos\theta=-\dfrac{7}{4}\Leftrightarrow\cos\theta=-\dfrac{7}{12}$; I → b)

$\sin^2\theta+\cos^2\theta=1$

Então, $\sin^2\theta+\left(-\dfrac{7}{12}\right)^2=1\Leftrightarrow\sin^2\theta=1-\dfrac{49}{144}\Leftrightarrow$

$\Leftrightarrow\sin^2\theta=\dfrac{95}{144}$. Como $\theta\in3.º\,Q$, $\sin\theta<0$, pelo que, $\sin\theta=-\sqrt{\dfrac{95}{144}}=-\dfrac{\sqrt{95}}{12}$. II → a)

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{-\dfrac{\sqrt{95}}{12}}{-\dfrac{7}{12}}=\dfrac{\sqrt{95}}{7}$; III → b)

I → b); II → a); III → b)

2. $\sin^2\left(\dfrac{\alpha}{2}\right)+\cos^2\left(\dfrac{\alpha}{2}\right)=1$

$-0{,}25=-\dfrac{1}{4}$

Então, $\left(-\dfrac{1}{4}\right)^2+\cos^2\left(\dfrac{\alpha}{2}\right)=1\Leftrightarrow$

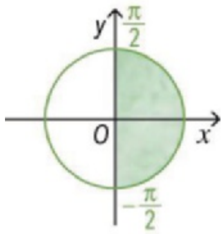
$\Leftrightarrow\cos^2\left(\dfrac{\alpha}{2}\right)=1-\dfrac{1}{16}\Leftrightarrow\cos^2\left(\dfrac{\alpha}{2}\right)=\dfrac{15}{16}$

Como $\cos\left(\frac{\alpha}{2}\right) > 0$, tem-se que $\cos\left(\frac{\alpha}{2}\right) = \frac{\sqrt{15}}{4}$.

C.A.:

$-\pi < \alpha < \pi \xrightarrow{:2} -\frac{\pi}{2} < \frac{\alpha}{2} < \frac{\pi}{2}$

$\cos\left(\frac{\alpha}{2}\right) > 0$



$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{-\frac{1}{4}}{\frac{\sqrt{15}}{4}} = -\frac{1}{\sqrt{15}} = -\frac{\sqrt{15}}{15}$

$4\cos\left(\frac{\alpha}{2}\right) + 15\tan\left(\frac{\alpha}{2}\right) =$

$= 4 \times \left(\frac{\sqrt{15}}{4}\right) + 15 \times \left(-\frac{\sqrt{15}}{15}\right) = \sqrt{15} - \sqrt{15} = 0$

3. $\tan^2 x - \sin^2 x = \tan^2 x\left(1 - \frac{\sin^2 x}{\tan^2 x}\right) =$

$= \tan^2 x\left(1 - \frac{\sin^2 x}{\left(\frac{\sin x}{\cos x}\right)^2}\right) = \tan^2 x\left(1 - \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}}\right) =$

$= \tan^2 x\left(1 - \frac{\cancel{\sin^2 x}\cos^2 x}{\cancel{\sin^2 x}}\right) = \tan^2 x\left(1 - \cos^2 x\right) \underset{(1)}{=}$

$= \tan^2 x \times \sin^2 x$

em que (1) $\sin^2 x + \cos^2 x = 1 \Leftrightarrow \sin^2 x = 1 - \cos^2 x$

4. $P(\cos\alpha, \sin\alpha)$ Então, $\overline{OQ} = \cos\alpha$ e $\overline{PQ} = \sin\alpha$.

$\overline{QA} = \overline{PQ} = \sin\alpha$; $\overline{OA} = \overline{OQ} + \overline{QA} = \cos\alpha + \sin\alpha$.

$[OAB]$ e $[OPQ]$ são triângulos semelhantes.

Então, $\frac{\overline{AB}}{\overline{OA}} = \frac{\overline{PQ}}{\overline{OQ}} \Leftrightarrow \frac{\overline{AB}}{\cos\alpha + \sin\alpha} = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow$

$\Leftrightarrow \frac{\overline{AB}}{\cos\alpha + \sin\alpha} = \tan\alpha \Leftrightarrow$

$\Leftrightarrow \overline{AB} = \tan\alpha(\cos\alpha + \sin\alpha)$

$A_{[OABC]} = \overline{OA} \times \overline{AB} =$

$= (\cos\alpha + \sin\alpha) \times \tan\alpha(\cos\alpha + \sin\alpha) =$

$= \tan\alpha(\cos\alpha + \sin\alpha)^2$

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Tarefa 9

1. $P(a\ ,\ b)$

$a = \cos\ \alpha$, $b = \sin\ \alpha$

$A(b\ ,\ a)$ $B(-b\ ,\ a)$ $C(-a\ ,\ b)$

$D(-a\ ,\ -b)$ $E(-b\ ,\ -a)$ $F(b\ ,\ -a)$

$G(a\ ,\ -b)$

2. $\pi - \alpha \to C$; $-\alpha \to G$

$\frac{\pi}{2} - \alpha \to A$; $\pi + \alpha \to D$

$\frac{3}{2}\pi + \alpha \to F$; $\frac{\pi}{2} + \alpha \to B$

$\frac{3}{2}\pi - \alpha \to E$

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Tarefa 10

1. 4.º Quadrante. O seno e a tangente são negativos e o cosseno é positivo.

2.1. $\sin(-\alpha) = -\sin\ \alpha$

2.2. $\cos(-\alpha) = \cos\ \alpha$

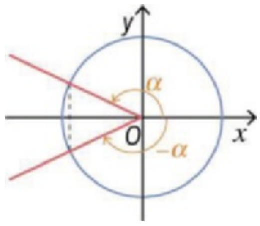
2.3. $\tan(-\alpha) = -\tan\ \alpha$

3. Se $\alpha \in 2.^{\text{o}}Q$, $-\alpha \in 3.^{\text{o}}Q$

$\sin(-\alpha) = -\sin\ \alpha$

$\cos(-\alpha) = \cos\ \alpha$

$\tan(-\alpha) = -\tan\ \alpha$

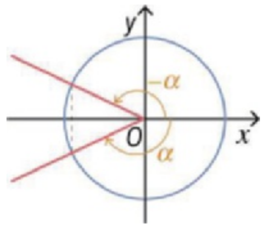


Se $\alpha \in 3.^{\text{o}}Q$, $-\alpha \in 2.^{\text{o}}Q$

$\sin(-\alpha) = -\sin\ \alpha$

$\cos(-\alpha) = \cos\ \alpha$

$\tan(-\alpha) = -\tan\ \alpha$

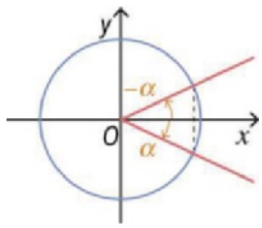


Se $\alpha \in 4.^{\text{o}}Q$, $-\alpha \in 1.^{\text{o}}Q$

$\sin(-\alpha) = -\sin\ \alpha$

$\cos(-\alpha) = \cos\ \alpha$

$\tan(-\alpha) = -\tan\ \alpha$



Também se α pertence a um semieixo coordenado, se mantém as relações estabelecidas, quando definidas.

56.1. $\sin\left(-\frac{\pi}{4} + 2\pi\right) = \sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

56.2. $\cos\left(\frac{12\pi}{6} - \frac{\pi}{6}\right) = \cos\left(2\pi - \frac{\pi}{6}\right)$

$= \cos\left(-\frac{\pi}{6}\right) = \cos\ \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

56.3. $\tan\left(\frac{15\pi}{4}\right) = \tan\left(\frac{16\pi}{4} - \frac{\pi}{4}\right) = \tan\left(4\pi - \frac{\pi}{4}\right)$

$= \tan\left(-\frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$

56.4. $\cos\left(\frac{5\pi}{3}\right)-\sin\left(-\frac{\pi}{6}\right)=\cos\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)-\left(-\sin\frac{\pi}{6}\right)$

$=\cos\left(2\pi-\frac{\pi}{3}\right)+\sin\frac{\pi}{6}=\cos\left(-\frac{\pi}{3}\right)+\sin\frac{\pi}{6}$

$=\cos\frac{\pi}{3}+\sin\frac{\pi}{6}=\frac{1}{2}+\frac{1}{2}=1$

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Tarefa 11

1. $\pi-\alpha\in 2.^{\circ}$ Quadrante
O seno é positivo e o cosseno e a tangente são negativos.

2.1. $\sin(\pi-\alpha)=\sin\ \alpha$

2.2. $\cos(\pi-\alpha)=-\cos\ \alpha$

2.3. $\tan(\pi-\alpha)=-\tan\ \alpha$

3. $\pi+\alpha\in 3.^{\circ}$ Quadrante
O seno e o cosseno são negativos e a tangente é positiva.

4.1. $\sin(\pi+\alpha)=-\sin\alpha$

4.2. $\cos(\pi+\alpha)=-\cos\alpha$

4.3. $\tan(\pi+\alpha)=\tan\ \alpha$

5. Se $\alpha\in 2.^{o}$ Quadrante , $\pi+\alpha\in 4.^{o}$ Quadrante e $\pi-\alpha\in 1.^{o}$ Quadrante .

$\sin(\pi+\alpha)=-\sin\ \alpha$

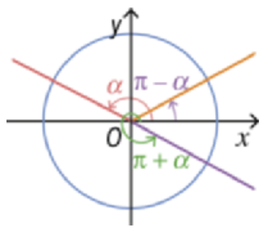
$\cos(\pi+\alpha)=-\cos\ \alpha$

$\tan(\pi+\alpha)=\tan\ \alpha$

$\sin(\pi-\alpha)=\sin\ \alpha$

$\cos(\pi-\alpha)=-\cos\ \alpha$

$\tan(\pi-\alpha)=-\tan\ \alpha$



Se $\alpha\in 3.^{o}$ Quadrante , $\pi+\alpha\in 1.^{o}$ Quadrante e $\pi-\alpha\in 4.^{o}$ Quadrante

$\sin(\pi+\alpha)=-\sin\ \alpha$

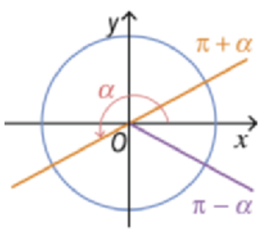
$\cos(\pi+\alpha)=-\cos\ \alpha$

$\tan(\pi+\alpha)=\tan\ \alpha$

$\sin(\pi-\alpha)=\sin\ \alpha$

$\cos(\pi-\alpha)=-\cos\ \alpha$

$\tan(\pi-\alpha)=-\tan\ \alpha$



Se $\alpha\in 4.^{o}$ Quadrante , $\pi+\alpha\in 2.^{o}$ Quadrante e $\pi-\alpha\in 3.^{o}$ Quadrante

$\sin(\pi+\alpha)=-\sin\ \alpha$

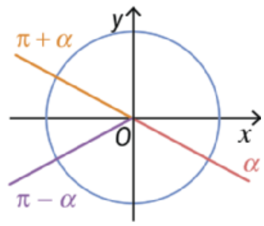
$\cos(\pi+\alpha)=-\cos\ \alpha$

$\tan(\pi+\alpha)=\tan\ \alpha$

$\sin(\pi-\alpha)=\sin\ \alpha$

$\cos(\pi-\alpha)=-\cos\ \alpha$

$\tan(\pi-\alpha)=-\tan\ \alpha$



Também se α pertence a um semieixo coordenado, se mantém as relações estabelecidas, quando definidas.

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57.1. $\cos\frac{3\pi}{4}=\cos\left(\pi-\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

57.2. $\tan\frac{4\pi}{3}=\tan\left(\pi+\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

57.3. $\sin\left(-\frac{5\pi}{6}\right)=-\sin\frac{5\pi}{6}=-\sin\left(\pi-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

57.4. $\cos\frac{29\pi}{4}-\sin\frac{13\pi}{4}=\cos\left(\frac{28\pi}{4}+\frac{\pi}{4}\right)-\sin\left(\frac{12\pi}{4}+\frac{\pi}{4}\right)$

$=\cos\left(7\pi-\frac{\pi}{4}\right)-\sin\left(3\pi+\frac{\pi}{4}\right)$

$=\cos\left(6\pi+\pi+\frac{\pi}{4}\right)-\sin\left(2\pi+\pi+\frac{\pi}{4}\right)$

$=\cos\left(\pi+\frac{\pi}{4}\right)-\sin\left(\pi+\frac{\pi}{4}\right)$

$=-\cos\frac{\pi}{4}-\left(-\sin\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=0$

57.5. $\tan\frac{7\pi}{6}-\frac{1}{\cos\frac{2\pi}{3}}=\tan\left(\pi+\frac{\pi}{6}\right)-\frac{1}{\cos\left(\pi-\frac{\pi}{3}\right)}$

$=\tan\frac{\pi}{6}-\frac{1}{-\cos\frac{\pi}{3}}=\frac{\sqrt{3}}{3}+\frac{1}{\frac{1}{2}}=\frac{\sqrt{3}}{3}+2=\frac{6+\sqrt{3}}{3}$

58.1. $\cos(-\alpha)-\cos(\pi-\alpha)=\cos\alpha-(-\cos\alpha)$

$=\cos\alpha+\cos\alpha\ =2\cos\alpha$

58.2. $\frac{\sin(-\alpha)}{\tan(\pi+\alpha)}-\cos(\pi+\alpha)=\frac{-\sin\alpha}{\tan\alpha}-(-\cos\alpha)$

$=\frac{-\sin\alpha}{\frac{\sin\alpha}{\cos\alpha}}+\cos\alpha=\frac{-\sin\alpha\cos\alpha}{\sin\alpha}+\cos\alpha$

$=-\cos\alpha+\cos\alpha=0$

58.3. $\sin(\pi-\alpha)\sin\alpha+\cos^2(\pi+\alpha)$

$=\sin\alpha\times\sin\alpha+\left[\cos(\pi+\alpha)\right]^2$

$=\sin^2\alpha+(-\cos\alpha)^2$

$=\sin^2\alpha+\cos^2\alpha$

$=1$

58.4. $\sin(-\alpha)-2\sin(3\pi+\alpha)+\tan(100\pi)$

$=-\sin\alpha-2\sin(\pi+\alpha)+\tan 0$

$=-\sin\alpha+2\sin\alpha$

$=\sin\alpha$

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Tarefa 12

1. $\frac{\pi}{2}-\alpha\in 1.^{\text{o}}$ Quadrante

O seno e o cosseno são positivos.

2.1. $\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

2.2. $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

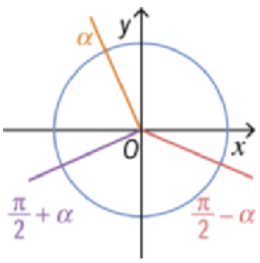
3. $\frac{\pi}{2}+\alpha\in 2.^{\text{o}}$ Quadrante

O seno é positivo e o cosseno é negativo.

4.1. $\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$

4.2. $\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$

5. Se $\alpha\in 2.^{\text{o}}$ Quadrante, $\frac{\pi}{2}-\alpha\in 4.^{\text{o}}$ Quadrante e $\frac{\pi}{2}+\alpha\in 3.^{\text{o}}$ Quadrante.



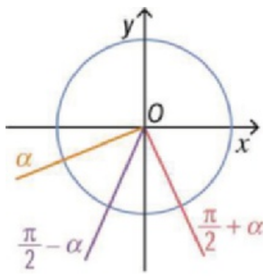
$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$

Se $\alpha\in 3.^{\text{o}}$ Quadrante, $\frac{\pi}{2}-\alpha\in 3.^{\text{o}}$ Quadrante e $\frac{\pi}{2}+\alpha\in 4.^{\text{o}}$ Quadrante.



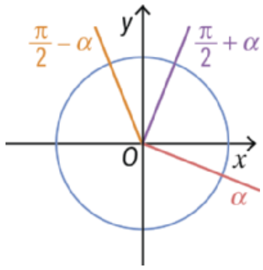
$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$

Se $\alpha\in 4.^{\text{o}}$ Quadrante, $\frac{\pi}{2}-\alpha\in 2.^{\text{o}}$ Quadrante e $\frac{\pi}{2}+\alpha\in 1.^{\text{o}}$ Quadrante.



$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$

Também se α pertence a um semieixo coordenado, se mantém as relações estabelecidas.

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59.1. $\cos\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\times\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)=\sin\frac{\pi}{6}\times\cos\frac{\pi}{6}$

$=\frac{1}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}$

59.2. $\sin\left(\frac{11\pi}{6}\right)-\tan\left(\frac{5\pi}{4}\right)=\sin\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)-\tan\left(\pi+\frac{\pi}{4}\right)$

$=\sin\left(2\pi-\frac{\pi}{6}\right)-\tan\frac{\pi}{4}=\sin\left(-\frac{\pi}{6}\right)-1$

$=-\sin\frac{\pi}{6}-1=-\frac{1}{2}-1=-\frac{3}{2}$

59.3. $\cos\left(\frac{\pi}{2}+\frac{5\pi}{2}\right)-2\sin\frac{5\pi}{6}=-\sin\frac{5\pi}{2}-2\sin\left(\pi-\frac{\pi}{6}\right)$

$=-\sin\left(\frac{4\pi}{2}+\frac{\pi}{2}\right)-2\sin\frac{\pi}{6}=-\sin\left(2\pi+\frac{\pi}{2}\right)-2\times\frac{1}{2}$

$=-\sin\frac{\pi}{2}-1=-1-1=-2$

59.4. $\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\times\tan\left(-\frac{3}{4}\pi\right)$

$=-\sin\frac{\pi}{6}\times\left[-\tan\left(\frac{3}{4}\pi\right)\right]=\sin\frac{\pi}{6}\times\tan\left(\frac{4\pi}{4}-\frac{\pi}{4}\right)=$

$=\frac{1}{2}\times\tan\left(\pi-\frac{\pi}{4}\right)=\frac{1}{2}\times\left(-\tan\frac{\pi}{4}\right)=$

$=\frac{1}{2}\times(-1)=-\frac{1}{2}$

60. $\cos\left(\frac{\pi}{2}+\alpha\right)=-\frac{3}{4}\Leftrightarrow-\sin\alpha=-\frac{3}{4}$

$\Leftrightarrow\sin\alpha=\frac{3}{4}$

$\alpha\in\left]\frac{\pi}{2},\pi\right[\Leftrightarrow\alpha\in 2.^{\text{o}}$ Quadrante

60.1. $\sin^2\alpha+\cos^2\alpha=1 \Leftrightarrow \cos^2\alpha=1-\left(\frac{3}{4}\right)^2$

$\Leftrightarrow \cos^2\alpha=1-\frac{9}{16}$

$\Leftrightarrow \cos^2\alpha=\frac{7}{16}$

60.2. $\tan(\pi-\alpha)=-\tan\alpha=-\frac{\sin\alpha}{\cos\alpha}=-\frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}}$

$=\frac{3}{\sqrt{7}}=\frac{3\sqrt{7}}{7}$

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Tarefa 13

1. $\frac{3\pi}{2}-\alpha\in 3.^{\text{o}}$ Quadrante

O seno e o cosseno são negativos.

2.1. $\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$

2.2. $\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$

3. $\frac{3\pi}{2}+\alpha\in 4.^{\text{o}}$ Quadrante .

O seno é negativo e o cosseno positivo.

4.1. $\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$

4.2. $\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha$

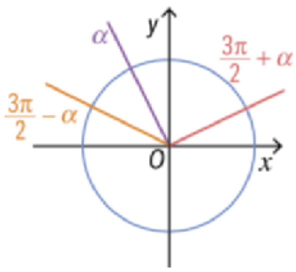
5. Se $\alpha\in 2.^{\text{o}}$ Quadrante , $\frac{3\pi}{2}-\alpha\in 2.^{\text{o}}$ Quadrante e $\frac{3\pi}{2}+\alpha\in 1.^{\text{o}}$ Quadrante .

$\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$

$\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha$



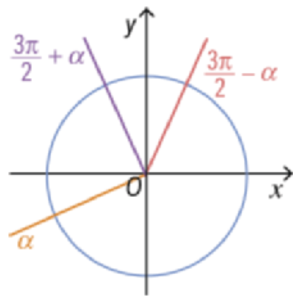
Se $\alpha\in 3.^{\text{o}}$ Quadrante , $\frac{3\pi}{2}-\alpha\in 1.^{\text{o}}$ Quadrante e $\frac{3\pi}{2}+\alpha\in 2.^{\text{o}}$ Quadrante .

$\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$

$\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha$



Se $\alpha\in 4.^{\text{o}}$ Quadrante , $\frac{3\pi}{2}-\alpha\in 4.^{\text{o}}$ Quadrante e $\frac{3\pi}{2}+\alpha\in 3.^{\text{o}}$ Quadrante

Cálculos auxiliares:

$\cos^2\alpha=\frac{7}{16}$

$\Leftrightarrow \cos\alpha=\pm\frac{\sqrt{7}}{4}$

Como $\alpha\in 2.^{\text{o}}$ Quadrante ,

$\cos\alpha=-\frac{\sqrt{7}}{4}$

$\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$

$\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$

$\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha$

Também se α pertence a um semieixo coordenado, se mantém as relações estabelecidas.

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61.1. $\cos\left(\frac{3\pi}{2}+\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

61.2. $\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)-\cos\left(\frac{3\pi}{2}-\frac{\pi}{6}\right)$

$=\cos\frac{\pi}{3}-\left(-\sin\frac{\pi}{6}\right)=\frac{1}{2}+\frac{1}{2}=1$

61.3. $\cos\left(-\frac{\pi}{2}+\frac{\pi}{3}\right)-\sin\left(\frac{7\pi}{3}\right)=$

$=\cos\left(\frac{3\pi}{2}+\frac{\pi}{3}\right)+\sin\left(\frac{6\pi}{3}+\frac{\pi}{3}\right)$

$=\sin\frac{\pi}{3}+\sin\left(2\pi+\frac{\pi}{3}\right)=\sin\frac{\pi}{3}+\sin\frac{\pi}{3}=2\times\frac{\sqrt{3}}{2}$

$=\sqrt{3}$

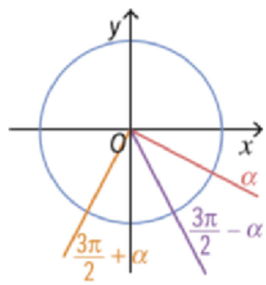
61.4. $\tan\left(-\frac{\pi}{3}\right)+2\sin\left(\frac{2\pi}{3}\right)=$

$=-\tan\frac{\pi}{3}+2\sin\left(\pi-\frac{\pi}{3}\right)$

$=-\sqrt{3}+2\sin\frac{\pi}{3}=$

$=-\sqrt{3}+2\frac{\sqrt{3}}{2}=$

$=-\sqrt{3}+\sqrt{3}=0$



62. $A\left(\cos\left(\frac{3}{2}\pi-\alpha\right),\ \sin\left(\frac{3}{2}\pi-\alpha\right)\right)=\left(-\sin\alpha\,,\,-\cos\alpha\right)$

$D\left(-\sin\alpha\ ,\ 0\right)$

$A_{[ABCD]}=\overline{AB}\times\overline{AD}=2\left|-\sin\alpha\right|\times\left|-\cos\alpha\right|$

$=2\cos\alpha\sin\alpha$

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Tarefas de consolidação 5

1. I – Qualquer que seja $\alpha\in\mathbb{R}$, $\cos(2\alpha)=2\cos\alpha$

Seja $\alpha=\pi$, $\cos(2\times\pi)=1$

$2\cos\pi=2\times(-1)=-2$

Logo $\cos(2\pi)\neq2\cos\pi$.

I – Falsa

II – Qualquer que seja $\alpha\in\mathbb{R}$,
$\sin(\pi+\alpha)=\sin\pi+\sin\alpha$

Seja $\alpha=\frac{\pi}{2}$

$\sin\left(\pi+\frac{\pi}{2}\right)=\sin\left(\frac{3\pi}{2}\right)=-1$

$\sin\pi+\sin\frac{\pi}{2}=0+1=1$

Logo, $\sin\left(\pi+\frac{\pi}{2}\right)\neq\sin\pi+\sin\frac{\pi}{2}$

II – Falsa

2.1. $\cos\frac{5\pi}{4}-\sin\left(-\frac{3}{4}\pi\right)-2\tan\left(\frac{4\pi}{3}\right)=$

$=\cos\left(\pi+\frac{\pi}{4}\right)+\sin\frac{3}{4}\pi-2\tan\left(\pi+\frac{\pi}{3}\right)$

$=-\cos\frac{\pi}{4}+\sin\left(\pi-\frac{\pi}{4}\right)-2\tan\frac{\pi}{3}$

$=-\frac{\sqrt{2}}{2}+\sin\frac{\pi}{4}-2\times\sqrt{3}$

$=-\frac{\sqrt{2}}{2}+-\frac{\sqrt{2}}{2}-2\sqrt{3}=-2\sqrt{3}$

2.2. $\dfrac{\cos\frac{7\pi}{6}-\sin\left(-\frac{5\pi}{3}\right)}{\tan\frac{13\pi}{6}}=\dfrac{\cos\frac{7\pi}{6}+\sin\frac{5\pi}{3}}{\tan\frac{13\pi}{6}}$

$=\dfrac{\cos\left(\pi+\frac{\pi}{6}\right)+\sin\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)}{\tan\left(\frac{12\pi}{6}+\frac{\pi}{6}\right)}=$

$=\dfrac{-\cos\frac{\pi}{6}+\sin\left(2\pi-\frac{\pi}{3}\right)}{\tan\left(2\pi+\frac{\pi}{6}\right)}=\dfrac{-\frac{\sqrt{3}}{2}-\sin\frac{\pi}{3}}{\tan\frac{\pi}{6}}$

$=\dfrac{-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{3}}=\dfrac{-\sqrt{3}}{\frac{\sqrt{3}}{3}}=-3$

2.3. $\cos\left(\frac{11\pi}{6}\right)+\sin\left(-\frac{5\pi}{3}\right)=\cos\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)-\sin\left(\frac{5\pi}{3}\right)$

$=\cos\left(2\pi-\frac{\pi}{6}\right)-\sin\left(2\pi-\frac{\pi}{3}\right)=\cos\left(-\frac{\pi}{6}\right)-\sin\left(-\frac{\pi}{3}\right)$

$=\cos\frac{\pi}{6}+\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\frac{2\sqrt{3}}{2}=\sqrt{3}$

2.4. $\sqrt{2}\sin\frac{7\pi}{4}+\sqrt{3}\tan\frac{4\pi}{3}$

$=\sqrt{2}\sin\left(\frac{8\pi}{4}-\frac{\pi}{4}\right)+\sqrt{3}\tan\left(\pi+\frac{\pi}{3}\right)$

$=\sqrt{2}\sin\left(2\pi-\frac{\pi}{4}\right)+\sqrt{3}\tan\frac{\pi}{3}$

$=-\sqrt{2}\sin\frac{\pi}{4}+\sqrt{3}\times\sqrt{3}$

$=-\sqrt{2}\times\frac{\sqrt{2}}{2}+3=-1+3=2$

3.1. $P\left(\cos\ 120°,\ \sin\ 120°\right)$

$\cos\ 120°=\cos\left(180°-60°\right)=-\cos\ 60°=-\frac{1}{2}$

$\sin\ 120°=\sin\left(180°-60°\right)=\sin\ 60°=\frac{\sqrt{3}}{2}$

$P\left(-\frac{1}{2}\ ,\ \frac{\sqrt{3}}{2}\right)$

3.2. $\sin\ 300°=\sin\left(360°-60°\right)=\sin\left(-60°\right)$

$=-\sin\ 60°=-\frac{\sqrt{3}}{2}$

$\cos\ 300°=\cos\left(360°-60°\right)=\cos\left(-60°\right)$

$=\cos\ 60°=\frac{1}{2}$

Alternativa:
Como $120°+180°=300°$,
$Q\left(\cos\ 300°\ ,\ \sin\ 300°\right)$ e Q é o simétrico de P relativamente a O, tem-se que

$\cos\ 300°=-\cos\ 120°=\frac{1}{2}$

$\sin\ 300°=-\sin\ 120°=-\frac{\sqrt{3}}{2}$

3.3. $T\left(1\ ,\ \tan\ 300^\circ\right)=\left(1\ ,\ \tan\ 120^\circ\right)$

$\tan\ 120^\circ=\tan\left(180^\circ-60^\circ\right)=-\tan\ 60^\circ=-\sqrt{3}$

$T\left(1\ ,\ -\sqrt{3}\right)$

4.1. $\tan\left(\frac{\pi}{2}-\alpha\right)\tan\alpha=\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)}\times\frac{\sin\alpha}{\cos\alpha}$

$=\frac{\cos\alpha}{\sin\alpha}\times\frac{\sin\alpha}{\cos\alpha}=1$

4.2. $\frac{\tan\left(\pi-\alpha\right)\sin\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{3\pi}{2}-\alpha\right)}=\frac{-\tan\alpha\cos\alpha}{-\sin\alpha}$

$=\frac{\frac{\sin\alpha}{\cancel{\cos\alpha}}\times\cancel{\cos\alpha}}{\sin\alpha}=\frac{\sin\alpha}{\sin\alpha}=1$

5.1. $\sin\left(\pi-\alpha\right)+\cos\left(\frac{\pi}{2}-\alpha\right)+2\sin\left(-\alpha\right)$

$=\sin\alpha+\sin\alpha-2\sin\alpha=0$

5.2. $\tan\alpha\times\sin\left(\frac{\pi}{2}+\alpha\right)+\sin\left(\pi+\alpha\right)$

$=-\tan\alpha\times\cos\alpha-\sin\alpha$

$=-\frac{\sin\alpha}{\cancel{\cos\alpha}}\times\cancel{\cos\alpha}-\sin\alpha=-2\sin\alpha$

5.3. $\sin^2\left(\frac{\pi}{2}+\alpha\right)+\sin^2\left(5\pi-\alpha\right)$

$=\left[\sin\left(\frac{\pi}{2}+\alpha\right)\right]^2+\left[\sin\left(5\pi-\alpha\right)\right]^2$

$=\left(\cos\alpha\right)^2+\left[\sin\left(4\pi+\pi-\alpha\right)\right]^2$

$=\cos^2\alpha+\left[\sin\left(\pi-\alpha\right)\right]^2$

$=\cos^2\alpha+\left(\sin\alpha\right)^2$

$=\cos^2\alpha+\sin^2\alpha=1$

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6. $5\sin\left(\frac{\pi}{2}+x\right)+4=0\Leftrightarrow 5\cos x+4=0$

$\Leftrightarrow\cos x=-\frac{4}{5}$

$x\in\left]\pi,\frac{3\pi}{2}\right[\Leftrightarrow x\in 3.^\circ$ Quadrante

$3\sin\left(3\pi-x\right)+2\cos\left(\frac{3\pi}{2}+x\right)-4\tan\left(x-\pi\right)$

$=3\sin\left(2\pi+\pi-x\right)+2\sin x-4\tan\left[-\left(\pi-x\right)\right]$

$=3\sin\left(\pi-x\right)+2\sin x+4\tan\left(\pi-x\right)$

$=3\sin x+2\sin x-4\tan x$

Cálculos auxiliares

$\sin^2\alpha+\cos^2\alpha=1$

$\Leftrightarrow\sin^2\alpha=1-\left(-\frac{4}{5}\right)^2\Leftrightarrow\sin^2\alpha=1-\frac{16}{25}\Leftrightarrow$

$\Leftrightarrow\sin^2\alpha=\frac{9}{25}\Leftrightarrow\sin\alpha=\pm\frac{3}{5}$

Como $\alpha\in 3.^\circ$ Q , $\sin\alpha=-\frac{3}{5}$.

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{-\frac{3}{5}}{-\frac{4}{5}}=\frac{3}{4}$

$=5\sin x-4\tan x$

$=5\times\left(-\frac{3}{5}\right)-4\times\frac{3}{4}$

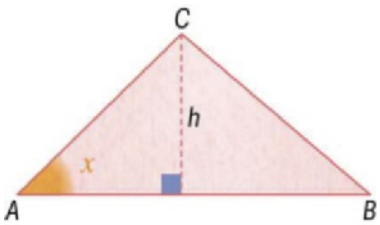
$=-3-3=-6$

7. $A_{[ABC]}=\frac{\overline{AB}\times\overline{AC}}{2}\sin x$

7.1. $x=\frac{\pi}{2}$

$A_{[ABC]}=\frac{\overline{AB}\times\overline{AC}}{2}\times\sin\frac{\pi}{2}$

7.2.

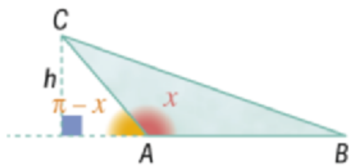


$A_{[ABC]}=\frac{\overline{AB}\times h}{2}$

$=\frac{\overline{AB}\times\overline{AC}}{2}\sin x$

$\sin x=\frac{h}{\overline{AC}}\Leftrightarrow h=\overline{AC}\ \sin x$

7.3.



$A_{[ABC]}=\frac{\overline{AB}\times h}{2}$

$=\frac{\overline{AB}\times\overline{AC}}{2}\sin x$

$\sin\left(\pi-x\right)=\frac{h}{\overline{AC}}\Leftrightarrow\sin x=\frac{h}{\overline{AC}}$

$\Leftrightarrow h=\overline{AC}\sin x$

8.1. altura $= h = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$

$$A_{[OAB]} = \frac{\overline{OA}\times\cos\alpha}{2} = \frac{\cos\alpha\times\cos\alpha}{2} = \frac{\cos^2\alpha}{2}$$

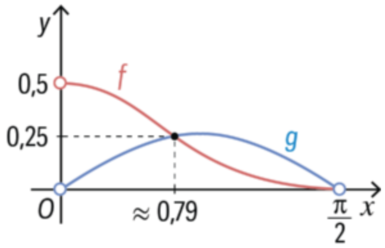
8.2. $\alpha \in 1.^\circ$ Quadrante

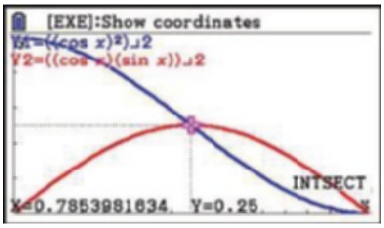
$$A_{[OAP]} = \frac{\overline{OA}\times\overline{AP}}{2} = \frac{\cos\alpha\sin\alpha}{2}$$

- $A_{[OAB]} = A_{[OAP]} \Leftrightarrow \frac{\cos^2\alpha}{2} = \frac{\cos\alpha\sin\alpha}{2}$

Designemos por $f(x)$ a área do triângulo $[OAB]$ e por $g(x)$ a área do triângulo $[OAP]$.

Assim, $f(x) = \frac{\cos^2 x}{2}$ e $g(x) = \frac{\cos x\ \sin x}{2}$.





$f(x) = g(x) \Leftrightarrow x \approx 0{,}79$

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Avaliação formativa 5

1. $A(\cos\alpha\ ,\ \sin\alpha)$

$C(-\cos\alpha\ ,\ -\sin\alpha)$

I – c)

$$B\left(\cos\left(\alpha+\frac{\pi}{2}\right)\ ,\ \sin\left(\alpha+\frac{\pi}{2}\right)\right) = (-\sin\alpha\ ,\cos\alpha)$$

II – b)

Se a ordenada de A for k, $\sin\alpha = k$

$$D\left(\cos\left(\alpha-\frac{\pi}{2}\right)\ ,\ \sin\left(\alpha-\frac{\pi}{2}\right)\right) = (\sin\alpha\ ,\ -\cos\alpha)$$

Quer-se determinar $-\cos\alpha$.

$\cos^2\alpha = 1-k^2$

$\cos\alpha = \pm\sqrt{1-k^2}$

Como $\alpha \in 2.^\circ$ Quadrante, $\cos\alpha = -\sqrt{1-k^2}$.

A abcissa de A é $-\sqrt{1-k^2}$.

A ordenada de D é $\sqrt{1-k^2}$.

III – a)

2. $A(1\ ;\ 0{,}75)$.

Como $B\left(\cos\left(\frac{\pi}{2}+\alpha\right)\ ,\ \sin\left(\frac{\pi}{2}+\alpha\right)\right)$, quer-se determinar $\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$

$\tan\alpha = 0{,}75 \Leftrightarrow \tan\alpha = \frac{3}{4}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1+\frac{9}{16} = \frac{1}{\cos^2\alpha}$

$\Leftrightarrow \frac{25}{16} = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{16}{25} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \pm\frac{4}{5}$

Como $\alpha \in 1.^\circ$ Quadrante , $\cos\alpha = \frac{4}{5}$.

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \sin^2\alpha = 1-\frac{16}{25}$

$\Leftrightarrow \sin\alpha = \pm\frac{3}{5}$

Como $\alpha \in 1.^\circ$ Q , $\sin\alpha = \frac{3}{5}$.

Logo, $\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha = -\frac{3}{5} = -0{,}6$.

A abcissa de B é $-0{,}6$.

3. $\alpha \in [\pi\ ,\ 2\pi]$

$4\cos(\pi+\alpha) = 1 \Leftrightarrow -4\cos\alpha = 1$

$\Leftrightarrow \cos\alpha = -\frac{1}{4}$

$\alpha \in 3.^\circ$ Quadrante

$\tan(\alpha-3\pi) - \sin(-\alpha-\pi) =$

$= \tan(-(3\pi-\alpha)) - \sin(-(\pi+\alpha)) =$

$= -\tan(2\pi+\pi-\alpha) + \sin(\pi+\alpha) =$

Cálculos auxiliares

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \sin^2\alpha = 1-\left(-\frac{1}{4}\right)^2 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = 1-\frac{1}{16} \Leftrightarrow \sin^2\alpha = \frac{15}{16} \Leftrightarrow$

$\Leftrightarrow \sin\alpha = \pm\frac{\sqrt{15}}{4}$

Como $\alpha \in [\pi\ ,\ 2\pi]$, $\sin\alpha = -\frac{\sqrt{15}}{4}$.

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{\sqrt{15}}{4}}{-\frac{1}{4}} = \sqrt{15}$$

$= -\tan(\pi-\alpha) - \sin\alpha = -(-\tan\alpha) - \sin\alpha =$

$= \tan\alpha - \sin\alpha = \sqrt{15} - \left(-\frac{\sqrt{15}}{4}\right) = \sqrt{15} + \frac{\sqrt{15}}{4} =$

$= \frac{5\sqrt{15}}{4}$

4. $\tan\left(\frac{\pi}{2}+\alpha\right)\cos\left(\alpha-\frac{\pi}{2}\right)+\cos(\pi-\alpha)$

$$=\frac{\sin\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{\pi}{2}+\alpha\right)}\cos\left(-\left(\frac{\pi}{2}-\alpha\right)\right)-\cos\alpha=$$

$$=\frac{\cos\alpha}{-\sin\alpha}\cos\left(\frac{\pi}{2}-\alpha\right)-\cos\alpha=$$

$$=\frac{\cos\alpha}{-\cancel{\sin\alpha}}\cancel{\sin\alpha}-\cos\alpha$$

$$=-\cos\alpha-\cos\alpha=-2\cos\alpha$$

5. $A(\cos\alpha\ ,\ \sin\alpha)$

$C\left(\cos\left(\frac{3}{2}\pi-\alpha\right)\ ,\ \sin\left(\frac{3}{2}\pi-\alpha\right)\right)$

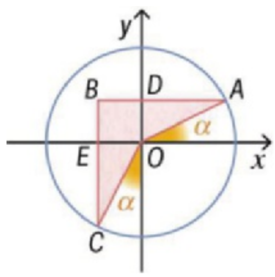
$C(-\sin\alpha\ ,\ -\cos\alpha)$

$B(-\sin\alpha\ ,\ \sin\alpha)$

$A_{[OABC]}=2\times A_{[OAD]}+A_{[ODBE]}$

$=2\times\frac{\cos\alpha\sin\alpha}{2}+|-\sin\alpha|\times\sin\alpha$

$=\cos\alpha\ \sin\alpha+\sin^2\alpha$



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Tarefa 14

1. O contradomínio de uma função f é o conjunto das imagens de f.
$D'_f=[-2\ ,\ 2]$

2. $f(-2)=0$
A imagem de -2 por f é 0.

3. Os zeros de uma função f são os objetos cuja imagem por f é zero.
Zeros em $[0\ ,\ 6]$: 0 , 2 , 4 e 6.

4. **(B)**

5. Máximo absoluto: 2
Mínimo absoluto: -2

6. Como a função descreve um movimento periódico que se repete a cada 4 unidades e 1 é o único maximizantes em $[0, 4[$, todos os maximizantes da função são obtidos a partir da soma de 1 com um múltiplo de 4.

7. $x=3+4k\ ,\ \ k\in\mathbb{Z}$.

8. $1000=2\times500$ e $500\in\mathbb{Z}$. Então, 1000 é um zero de f, ou seja, $f(1000)=0$.
$43=3+4\times10$ e $10\in\mathbb{Z}$. Então, 43 é um minimizante de f, ou seja, $f(43)=-2$.
$f(1000)+3f(43)=0+3(-2)=-6$

9. $\frac{1}{4}$. Porque $\frac{1}{4}\times4=1$.

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63.1. $f\left(\frac{7\pi}{3}\right)=\sin\left(\frac{7\pi}{3}\right)=\sin\left(\frac{6\pi}{3}+\frac{\pi}{3}\right)=\sin\left(2\pi+\frac{\pi}{3}\right)$

$=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$f\left(\frac{2\pi}{3}\right)=\sin\left(\frac{2\pi}{3}\right)=\sin\left(\frac{3\pi}{3}-\frac{\pi}{3}\right)=\sin\left(\pi-\frac{\pi}{3}\right)$

$=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

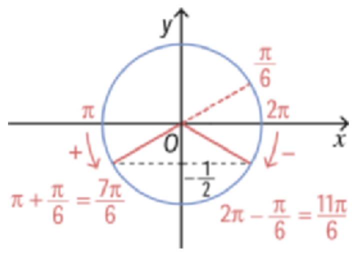
$f\left(\frac{7\pi}{3}\right)+f\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\frac{2\sqrt{3}}{2}=\sqrt{3}$

63.2. a) $f(x)=\frac{\sqrt{3}}{2}\Leftrightarrow\sin x=\frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ e $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$

ou seja, os objetos com imagem $\frac{1}{2}$, por f, em $[0\ ,\ 2\pi]$, são: $\frac{\pi}{3}$ e $\frac{2\pi}{3}$.

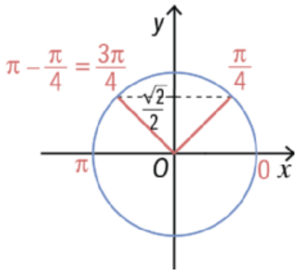
b) $f(x)=-\frac{1}{2}\Leftrightarrow\sin x=-\frac{1}{2}$



$\sin\left(\frac{7\pi}{6}\right)=-\frac{1}{2}$ e $\sin\left(\frac{11\pi}{6}\right)=-\frac{1}{2}$

ou seja, os objetos com imagem $-\frac{1}{2}$, por f, em $[0\ ,\ 2\pi]$, são $\frac{7\pi}{6}$ e $\frac{11\pi}{6}$.

c) $f(x)=\frac{\sqrt{2}}{2} \Leftrightarrow \sin x=\frac{\sqrt{2}}{2}$

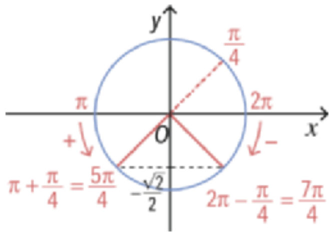


$\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ e $\sin\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{2}$,

ou seja, os objetos com imagem $\frac{\sqrt{2}}{2}$, por f,

em $[0\ ,\ 2\pi]$, são $\frac{\pi}{4}$ e $\frac{3\pi}{4}$.

d) $f(x)=-\frac{\sqrt{2}}{2} \Leftrightarrow \sin x=-\frac{\sqrt{2}}{2}$

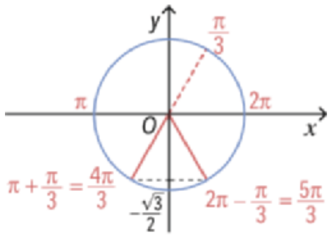


$\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$ e $\sin\left(\frac{7\pi}{4}\right)=-\frac{\sqrt{2}}{2}$,

ou seja, os objetos com imagem $-\frac{\sqrt{2}}{2}$, por f,

em $[0\ ,\ 2\pi]$, são $\frac{5\pi}{4}$ e $\frac{7\pi}{4}$.

e) $f(x)=-\frac{\sqrt{3}}{2} \Leftrightarrow \sin x=-\frac{\sqrt{3}}{2}$



$\sin\frac{4\pi}{3}=-\frac{\sqrt{3}}{2}$ e $\sin\frac{5\pi}{3}=-\frac{\sqrt{3}}{2}$,

ou seja, os objetos com imagem $-\frac{\sqrt{3}}{2}$, por f,

em $[0\ ,\ 2\pi]$, são $\frac{4\pi}{3}$ e $\frac{5\pi}{3}$.

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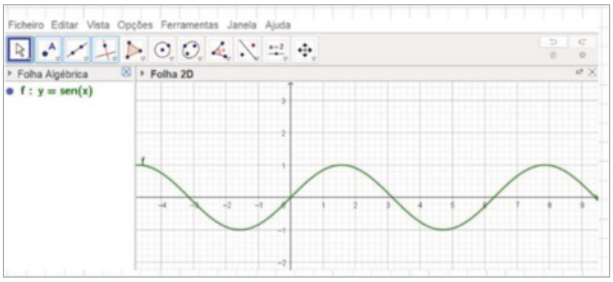
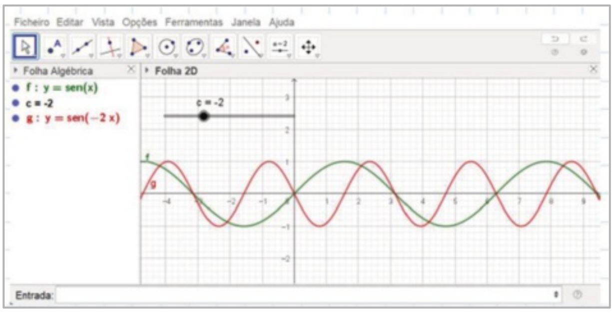
Tarefa 15

1.



2.



3.1. Não há alterações. É sempre $\mathbb{R}$.

3.2. Não há alterações. É sempre $[-1\ ,\ 1]$.

3.3. Há alterações.

3.4. Não há alterações. São sempre -1 e 1.

3.5. Não há alterações. É sempre 1.

3.6. Há alterações.

4. Se $\boxed{c=-1}$, o gráfico sofre uma reflexão de eixo vertical.

Se $\boxed{|c|>1}$, o gráfico sofre uma contração ao longo do eixo horizontal com fator $\frac{1}{c}$.

Se $\boxed{0<|c|<1}$, o gráfico sofre uma dilatação ao longo do eixo horizontal com fator $\frac{1}{c}$.

5. Se $c=2$ o gráfico sofre uma contração ao longo do eixo horizontal com fator $\frac{1}{c}$.

Então, o período fundamental é $\frac{1}{2}\times 2\pi=\pi$.

Se $c=\frac{1}{2}$, o gráfico sofre uma dilatação ao longo do eixo horizontal com fator 2.

Então, o período fundamental é $2\times 2\pi=4\pi$.

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64.1. $f\left(\frac{2\pi}{3}\right)=\sin\left(3\left(\frac{2\pi}{3}\right)\right)=\sin(2\pi)=0$

$f\left(-\frac{\pi}{6}\right)=\sin\left(3\left(-\frac{\pi}{6}\right)\right)=\sin\left(-\frac{\pi}{2}\right)=-1$

$f\left(\frac{2\pi}{3}\right)+f\left(-\frac{\pi}{6}\right)=0-1=-1$

64.2. $P_0=\frac{2\pi}{3}$

Frequência $=\frac{1}{\frac{2\pi}{3}}=\frac{3}{2\pi}$

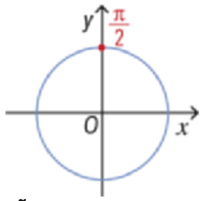
O período fundamental é $\frac{2\pi}{3}$ e a frequência é $\frac{3}{2\pi}$.

64.3. $f(x)=0 \Leftrightarrow \sin(3x)=0$

$\Leftrightarrow 3x=k\pi \ , \ k\in\mathbb{Z}$

$\Leftrightarrow x=\frac{k\pi}{3} \ , \ k\in\mathbb{Z}$

64.4. $f(x)=1 \Leftrightarrow \sin(3x)=1$



$\Leftrightarrow 3x=\frac{\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}$

$\Leftrightarrow x=\frac{\pi}{6}+\frac{2k\pi}{3} \ , \ k\in\mathbb{Z}$

Os valores de x encontrados são os maximizantes de f.

$f(x)=-1 \Leftrightarrow \sin(3x)=-1$

$\Leftrightarrow 3x=\frac{3\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}$

$\Leftrightarrow x=\frac{\pi}{2}+\frac{2k\pi}{3} \ , \ k\in\mathbb{Z}$

Os valores de x encontrados são os minimizantes de f.

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65.1. a) O gráfico de g obtém-se através de uma contração vertical do gráfico de f de coeficiente 0,5 .

b) O gráfico de g obtém-se do gráfico de f através de uma translação horizontal associada ao vetor $(-1\ ,\ 0)$.

c) O gráfico de g obtém-se do gráfico de f através de uma translação vertical associada ao vetor $(0\ ,\ -1)$.

65.2. a) $g(x)=5f(x)=5\sin x$

b) $g(x)=f(x)+5=5+\sin x$

c) $g(x)=f(5x)=\sin(5x)$

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Tarefa 16

1. De f para g_1 faz uma dilatação ou contração horizontal (se $c<0$ é feita também uma reflexão de eixo *Oy*).

De g_1 para g_2 faz uma translação horizontal associada ao vetor $(d, 0)$.

De g_2 para g_3 faz uma dilatação ou contração vertical (se $b<0$ é feita também uma reflexão de eixo Ox). Para finalizar, de g_3 para g é feita uma translaçao vertical associada ao vetor $(0, a)$.

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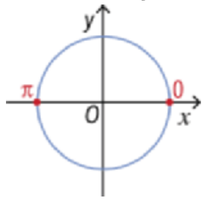
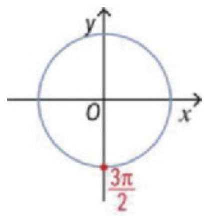
66.1. $f(x)=-4\sin\left(\frac{x+\pi}{2}\right)+3=-4\sin\left(\frac{1}{2}(x+\pi)\right)+3$

O gráfico de f obtém-se do gráfico de $y=\sin x$ através da seguinte sequência de transformações:

- Dilatação horizontal de coeficiente 2 (transforma $y=\sin x$ em $y=\sin\left(\frac{1}{2}x\right)$



- Translação horizontal associada ao vetor $(-\pi\ ,\ 0)$ (transforma $y=\sin\left(\frac{1}{2}x\right)$ em $\sin\left(\frac{1}{2}(x+\pi)\right)$



- Dilatação vertical de coeficiente 4 (transforma $y=\sin\left(\frac{1}{2}(x+\pi)\right)$ em $y=4\sin\left(\frac{1}{2}(x+\pi)\right)$)
- Reflexão de eixo *Ox* (transforma $y=4\sin\left(\frac{1}{2}(x+\pi)\right)$ em $y=-4\sin\left(\frac{1}{2}(x+\pi)\right)$)
- Translação vertical associada ao vetor $(0\ ,\ 3)$ (transforma $y=-4\sin\left(\frac{1}{2}(x+\pi)\right)$ em $f(x)$)

66.2. $f\left(\frac{\pi}{2}\right)-3=-4\sin\left(\frac{\frac{\pi}{2}+\pi}{2}\right)+3-3$

$=-4\sin\frac{3\pi}{4}=-4\sin\left(\frac{4\pi}{4}-\frac{\pi}{4}\right)$

$$= -4\sin\left(\pi - \frac{\pi}{4}\right) = -4\sin\frac{\pi}{4} = -4\times\frac{\sqrt{2}}{2} = -2\sqrt{2}$$

66.3. $D_f = \mathbb{R}$

Se $x \in D_f$, então $\frac{x+\pi}{2} \in D_f$

$$-1 \leq \sin\left(\frac{x+\pi}{2}\right) \leq 1 \Leftrightarrow 4 \geq -4\sin\left(\frac{x+\pi}{2}\right) \geq -4$$

$$\Leftrightarrow -4 \leq -4\sin\left(\frac{x+\pi}{2}\right) \leq 4$$

$$\Leftrightarrow -1 \leq -4\sin\left(\frac{x+\pi}{2}\right) + 3 \leq 7$$

$$\Leftrightarrow -1 \leq f(x) \leq 7$$

Logo, $D'_f = [-1\ ,\ 7]$.

66.4. Como $D_f = \mathbb{R}$, se $x \in D_f$ então $x + 4\pi \in D_f$.

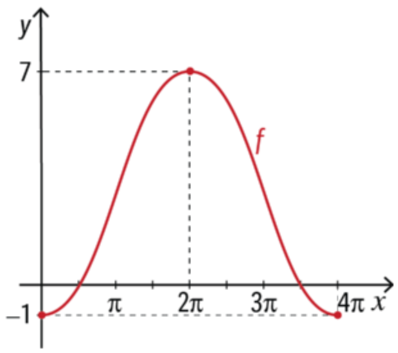
$$f(x+4\pi) = -4\sin\left(\frac{x+4\pi+\pi}{2}\right) + 3$$

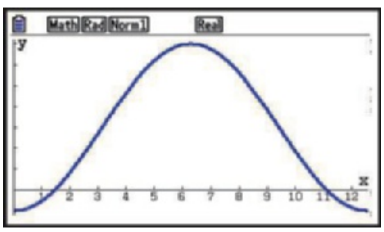
$$= -4\sin\left(\frac{x+\pi}{2} + 2\pi\right) + 3$$

$$= -4\sin\left(\frac{x+\pi}{2}\right) + 3 = f(x)$$

Logo, a função f é periódica de período 4π .

66.5.





66.6. frequência $= \frac{1}{4\pi}$

66.7. $f(\alpha - 40\pi) = -4\sin\left(\frac{\alpha - 40\pi + \pi}{2}\right) + 3$

$$= -4\sin\left(\frac{\alpha+\pi}{2} - 20\pi\right) + 3$$

$$= -4\sin\left(\frac{\alpha+\pi}{2}\right) + 3 = f(\alpha) = k$$

66.8. $f(x) = -1 \Leftrightarrow -4\sin\left(\frac{x+\pi}{2}\right) + 3 = -1$

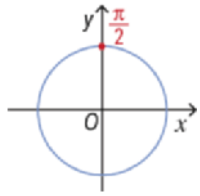
$$\Leftrightarrow -4\sin\left(\frac{x+\pi}{2}\right) = -4$$

$$\Leftrightarrow \sin\left(\frac{x+\pi}{2}\right) = 1$$

$$\Leftrightarrow \frac{x+\pi}{2} = \frac{\pi}{2} + 2k\pi \ ,\ k \in \mathbb{Z}$$

$$\Leftrightarrow x + \pi = \pi + 4k\pi \ ,\ k \in \mathbb{Z}$$

$$\Leftrightarrow x = 4k\pi \ ,\ k \in \mathbb{Z}$$



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67.1. $f(x) = 3\sin(2x+\pi) + \sin(-2x) + \cos(2\pi)$

$$= 3(-\sin(2x)) + (-\sin(2x)) + 1$$

$$= -3\sin(2x) - \sin(2x) + 1$$

$$= -4\sin(2x) + 1$$

67.2. $f\left(\frac{-7\pi}{12}\right) = -4\sin\left(2\times\left(\frac{-7\pi}{12}\right)\right) + 1$

$$= -4\sin\left(\frac{-14\pi}{12}\right) + 1 = -4\sin\left(\frac{-7\pi}{6}\right) + 1 =$$

$$= -4\sin\left(\frac{-6\pi}{6} - \frac{\pi}{6}\right) + 1 = -4\sin\left(-\pi - \frac{\pi}{6}\right) + 1 =$$

$$= -4\sin\frac{\pi}{6} + 1 = -4\times\frac{1}{2} + 1 = -2 + 1 = -1$$

-1 é a ordenada no ponto do gráfico de f com abcissa $\frac{-7\pi}{12}$.

67.3. O gráfico de f obtém-se do gráfico de $y = \sin x$ através da seguinte sequência de transformações:

- Contração horizontal de coeficiente $\frac{1}{2}$ (transforma $y = \sin(2x)$ em $y = \sin(2x)$)
- Dilatação vertical de coeficiente 4 (transforma $y = \sin(2x)$ em $y = 4\sin(2x)$)
- Reflexão de eixo Ox (transforma $y = 4\sin(2x)$ em $y = -4\sin(2x)$)
- Translação vertical associada ao vetor $(0\ ,\ 1)$ (transforma $y = -4\sin(2x)$ em $f(x)$)

67.4. Como $D_f = \mathbb{R}$, se $x \in D_f$ então $2x \in D_f$

$-1 \leq \sin(2x) \leq 1 \Leftrightarrow 4 \geq -4\sin(2x) \geq -4$

$\Leftrightarrow -4 \leq -4\sin(2x) \leq 4$

$\Leftrightarrow -3 \leq -4\sin(2x) + 1 \leq 5 \Leftrightarrow -3 \leq f(x) \leq 5$

$D'_f = [-3 \ , \ 5]$

67.5. $|-4| = 4$. Logo, a amplitude de f é 4 .

$\frac{2\pi}{|2|} = \frac{2\pi}{2} = \pi$. Logo, o período fundamental de f é π .

67.6. Máximo: 5

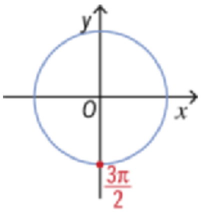
$f(x) = 5 \Leftrightarrow -4\sin(2x) + 1 = 5$

$\Leftrightarrow -4\sin(2x) = 4$

$\Leftrightarrow \sin(2x) = -1$

$\Leftrightarrow 2x = \frac{3\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{3\pi}{4} + k\pi \ , \ k \in \mathbb{Z}$



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68. $D'_f = [-2 \ , \ 0]$

Então, $a = \frac{-2+0}{2} = -1$.

$b = \frac{0-(-2)}{2} = \frac{2}{2} = 1$

O período fundamental de f é π .

Então, $\frac{2\pi}{|c|} = \pi \Leftrightarrow |c| = \frac{2\pi}{\pi} \Leftrightarrow |c| = 2 \Leftrightarrow |c| = \pm 2$

Admita-se que $c = 2$. (Para $c = -2$, obtém-se uma expressão analítica de f igualmente correta.)

$f(x) = -1 + \sin(2(x-d))$

$f\left(\frac{\pi}{2}\right) = 0 \Leftrightarrow -1 + \sin\left(2\left(\frac{\pi}{2} - d\right)\right) = 0$

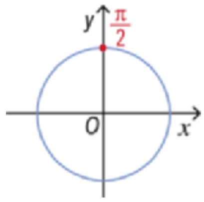
$\Leftrightarrow \sin(\pi - 2d) = 1 \Leftrightarrow \sin(2d) = 1$

Por exemplo, $\sin\frac{\pi}{2} = 1$.

$2d = \frac{\pi}{2} \Leftrightarrow d = \frac{\pi}{4}$

69.

- Amplitude de f: $|3| = 3$
- Período fundamental: $P_0 = \frac{2\pi}{|-1|} = 2\pi$
- $b = 3 > 0$. Então $D'_f = [1-3 \ , \ 1+3] = [-2 \ , \ 4]$
- Maximizantes de f:



$f(x) = 4 \Leftrightarrow 3\sin(-x+\pi) + 1 = 4$

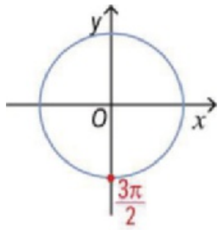
$\Leftrightarrow 3\sin(-x+\pi) = 3 \Leftrightarrow \sin(-x+\pi) = 1$

$\Leftrightarrow -x + \pi = \frac{\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$

$\Leftrightarrow -x = -\frac{\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$

- Minimizantes de f:



$f(x) = -2 \Leftrightarrow 3\sin(-x+\pi) + 1 = -2$

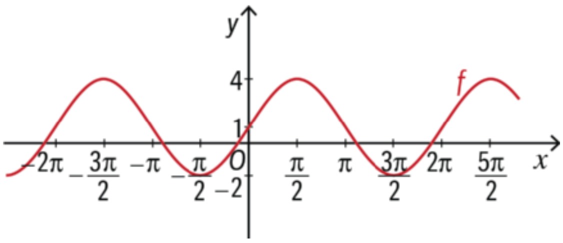
$\Leftrightarrow 3\sin(-x+\pi) = -3$

$\Leftrightarrow \sin(-x+\pi) = -1$

$\Leftrightarrow -x + \pi = \frac{3\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$

$\Leftrightarrow -x = \frac{\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$



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70.1. $f(x) = 3\sin\left(\frac{\pi}{2} + x\right) - \cos(2\pi - x) - \sin\frac{5\pi}{2}$

$= 3\cos x - \cos(-x) - \sin\left(\frac{4\pi}{2} + \frac{\pi}{2}\right)$

$= 3\cos x - \cos x - \sin\left(2\pi + \frac{\pi}{2}\right)$

$= 2\cos x - \sin\frac{\pi}{2}$

$= 2\cos x - 1$

70.2. a) $f\left(-\frac{\pi}{3}\right)=2\cos\left(-\frac{\pi}{3}\right)-1=2\cos\frac{\pi}{3}-1=2\times\frac{1}{2}-1$

$=1-1=0$

b) 1.º Processo

$D_f=\mathbb{R}$

$-1\leq \cos x\leq 1 \Leftrightarrow -2\leq 2\cos x\leq 2$

$\Leftrightarrow -3\leq 2\cos x-1\leq 1$

$\Leftrightarrow -3\leq f(x)\leq 1$

$D'_f=[-3\ ,\ 1]$

2.º Processo

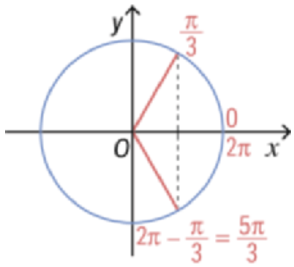
$b=2>0$. Então,

$D'_f=[-1-2\ ,\ -1+2]=[-3\ ,\ 1]$

c) Amplitude: $\frac{1-(-3)}{2}=2$

Período fundamental: $\frac{2\pi}{|1|}=2\pi$

d) $f(x)=0\wedge x\in[0\ ,\ 2\pi]$



$\Leftrightarrow 2\cos x-1=0\wedge x\in[0\ ,\ 2\pi]$

$\Leftrightarrow \cos x=\frac{1}{2}\wedge x\in[0\ ,\ 2\pi]\Leftrightarrow x=\frac{\pi}{3}\vee x=\frac{5\pi}{3}$

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71.1. $D_f=\left\{x\in\mathbb{R}:2x-\frac{\pi}{3}\neq\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\right\}$

$=\left\{x\in\mathbb{R}:2x\neq\frac{5\pi}{6}+k\pi\ ,\ k\in\mathbb{Z}\right\}$

$=\left\{x\in\mathbb{R}:x\neq\frac{5\pi}{12}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}\right\}$

$D_f=\mathbb{R}\backslash=\left\{x:x=\frac{5\pi}{12}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}\right\}$

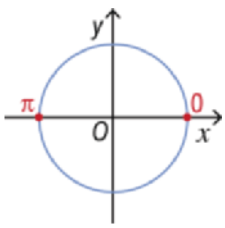
71.2. $f(x)=0\Leftrightarrow a\tan\left(2x-\frac{\pi}{3}\right)=0$

$\Leftrightarrow \tan\left(2x-\frac{\pi}{3}\right)=0$

$\Leftrightarrow 2x-\frac{\pi}{3}=k\pi\ ,\ k\in\mathbb{Z}$

$\Leftrightarrow 2x=\frac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}$

$\Leftrightarrow x=\frac{\pi}{6}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}$



71.3. $D_f=\mathbb{R}\backslash\left\{x:x=\frac{5\pi}{12}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}\right\}$

Se $x\in D_f$ então $x+\frac{\pi}{2}\in D_f$

$f\left(x+\frac{\pi}{2}\right)=a\tan\left(2\left(x+\frac{\pi}{2}\right)-\frac{\pi}{3}\right)$

$=a\tan\left(2x+\pi-\frac{\pi}{3}\right)$

$=a\tan\left(2x-\frac{\pi}{3}\right)=f(x)$

71.4. $f\left(\frac{\pi}{3}\right)=a\tan\left(2\times\frac{\pi}{3}-\frac{\pi}{3}\right)=a\tan\frac{\pi}{3}=\sqrt{3}a$

$f\left(\frac{\pi}{12}\right)=a\tan\left(2\times\frac{\pi}{12}-\frac{\pi}{3}\right)=a\tan\left(\frac{\pi}{6}-\frac{\pi}{3}\right)$

$=a\tan\left(-\frac{\pi}{6}\right)=-a\tan\frac{\pi}{6}=\frac{-\sqrt{3}}{3}a$

$f\left(\frac{\pi}{3}\right)-3f\left(\frac{\pi}{12}\right)=6\Leftrightarrow\sqrt{3}a-3\times\left(\frac{-\sqrt{3}}{3}\right)a=6$

$\Leftrightarrow\sqrt{3}a+\sqrt{3}a=6\Leftrightarrow 2\sqrt{3}a=6\Leftrightarrow$

$\Leftrightarrow a=\frac{6}{2\sqrt{3}}\Leftrightarrow a=\sqrt{3}$

72.1. $f(x)=1+3\tan(x-\pi)$

$D_f=\left\{x\in\mathbb{R}:x-\pi\neq\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\right\}$

$=\left\{x\in\mathbb{R}:x\neq\frac{3\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\right\}$

$D_f=\mathbb{R}\backslash=\left\{x:x=\frac{3\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\right\}$

72.2. $g(x)=\tan\left(-x+\frac{\pi}{2}\right)$

$D_G=\left\{x\in\mathbb{R}:-x+\frac{\pi}{2}\neq\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\right\}$

$=\{x\in\mathbb{R}:x\neq k\pi\ ,\ k\in\mathbb{Z}\}$

$D_f=\mathbb{R}\backslash\{x:x=k\pi\ ,\ k\in\mathbb{Z}\}$

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73.1. A amplitude de d é $0{,}05$ m e representa a deslocação máxima das partículas de ar com a emissão do som.

73.2. $P_0=0{,}04$

$f=\frac{1}{0{,}04}=25$

A frequência de d é 25 e representa o número de ciclos por unidade de tempo, ou seja, o número de ciclos por segundo.

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74.1. 9 horas da manhã: $t = 9$

$$h(9) = 4{,}1 + 1{,}4\sin\left(\frac{9\pi}{6} - \frac{\pi}{3}\right)$$
$$= 4{,}1 + 1{,}4\sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right) = 4{,}1 + 1{,}4\left(-\cos\frac{\pi}{3}\right)$$
$$= 4{,}1 - 1{,}4 \times \frac{1}{2} = 3{,}4$$

Às 9 h da manhã, o ponto do barco encontra-se a uma distância de $3{,}4$ m do fundo do mar.

74.2. $P_0 = \frac{2\pi}{\left|\frac{\pi}{6}\right|} = \frac{2\pi}{\frac{\pi}{6}} = 12$

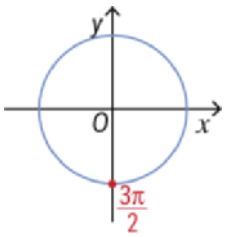
O período da função é 12. Significa que as alturas das marés se repetem de 12 em 12 horas, aproximadamente.

74.3. $-1 \le \sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) \le 1$

$$\Leftrightarrow -1{,}4 \le 1{,}4\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) \le 1{,}4$$
$$\Leftrightarrow 2{,}7 \le 4{,}1 + 1{,}4\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) \le 5{,}5$$
$$\Leftrightarrow 2{,}7 \le h(t) \le 5{,}5 \ ; \ D'_h = [2{,}7 \ ; \ 5{,}5]$$

74.4. Distância mínima: $2{,}7$

$$h(t) = 2{,}7 \Leftrightarrow 4{,}1 + 1{,}4\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = 2{,}7$$
$$\Leftrightarrow 1{,}4\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = -1{,}4 \Leftrightarrow \sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = -1$$
$$\Leftrightarrow \frac{\pi t}{6} - \frac{\pi}{3} = \frac{3\pi}{2} + 2k\pi \ , \quad k \in \mathbb{Z}$$
$$\Leftrightarrow \frac{\pi t}{6} = \frac{11\pi}{6} + 2k\pi \ , \quad k \in \mathbb{Z}$$
$$\Leftrightarrow t = 11 + 12k \ , \quad k \in \mathbb{Z}$$



Como $t \in [0 \ , \ 24]$,

Se $k = 0$, $t = 11$.

Se $k = 1$, $t = 11 + 12 \times 1 = 23$

$t = 11 \vee t = 23$

A distância do barco ao fundo do mar foi mínima às 11 h e às 23 h.

74.5. $h(t) = 4{,}1 \Leftrightarrow 4{,}1 + 1{,}4\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = 4{,}1$

$$\Leftrightarrow 1{,}4\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = 0$$
$$\Leftrightarrow \sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = 0 \Leftrightarrow \frac{\pi t}{6} - \frac{\pi}{3} = k\pi \ , \quad k \in \mathbb{Z}$$
$$\Leftrightarrow \frac{\pi t}{6} = \frac{\pi}{3} + k\pi \ , \quad k \in \mathbb{Z} \Leftrightarrow t = 2 + 6k \ , \quad k \in \mathbb{Z}$$

Como $t \in [0 \ , \ 24]$,

Se $k = 0$, $t = 2$

Se $k = 1$, $t = 2 + 6 \times 1 = 8$

Se $k = 2$, $t = 2 + 6 \times 2 = 14$

Se $k = 3$, $t = 2 + 6 \times 3 = 20$

$t = 2 \vee t = 8 \vee t = 14 \vee t = 20$

A distância do barco ao fundo do mar foi de $4{,}1$ m às 2 h, às 8 h, às 14 h e às 20 h.

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75.1. a) $22\text{ h} - 14\text{ h} = 8\text{ h}$. então, $t = 8$

$T(8) = 0{,}4\sin(0{,}3 \times 8 - 2{,}8) + 37 \approx 36{,}8$ °C

A temperatura da Rita às 22 h é $36{,}8$ °C.

b) $T(t) = 37 + 0{,}4\sin(0{,}3t - 2{,}8)$

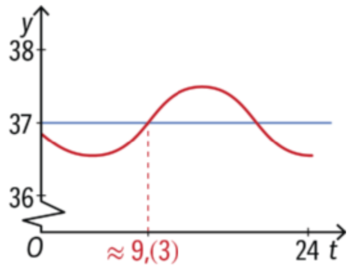
$a = 37$ e $b = 0{,}4 > 0$

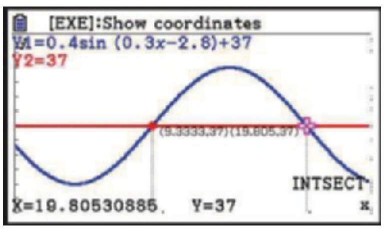
$D'_T = [37 - 0{,}4 \ ; \ 37 + 0{,}4]$

$= [36{,}6 \ ; \ 37{,}4]$

A temperatura máxima atingida pela Rita foi $37{,}4$ °C.

75.2. Pretende-se ver o número de soluções da equação $h(t) = 37$.





$0{,}(3) \times 60 = 20$ min

A Rita atingiu a temperatura de 37 °C duas vezes, tendo ocorrido a primeira 9 h e 20 min após ter iniciado as medições.

$14\text{ h} + 9\text{ h } 20\text{ m} = 23\text{ h } 20\text{ min}$

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76.1. $T(n) = 20{,}5 + 12{,}2\sin(0{,}0172n - 2)$

$a = 20{,}5$ e $b = 12{,}2 > 0$

$D'_T = [20{,}5 - 12{,}2\ ;\ 20{,}5 + 12{,}2]$

$= [8{,}3\ ;\ 32{,}7]$

Nos últimos 50 anos, a média das temperaturas máximas, em graus Celsius, registadas nessa idade portuguesa variou entre 8,3 °C e 32,7 °C .

76.2. $T(n) = 32{,}7 \Leftrightarrow 20{,}5 + 12{,}2\sin(0{,}0172n - 2) = 32{,}7$

$\Leftrightarrow 12{,}2\sin(0{,}0172n - 2) = 12{,}2$

$\Leftrightarrow \sin(0{,}0172n - 2) = 1$

$\Leftrightarrow 0{,}0172n - 2 = \frac{\pi}{2} + 2k\pi\ ,\ k \in \mathbb{Z}$

$\Leftrightarrow 0{,}0172n = 2 + \frac{\pi}{2} + 2k\pi\ ,\ k \in \mathbb{Z}$

$\Leftrightarrow n = \frac{2 + \frac{\pi}{2} + 2k\pi}{0{,}0172}\ ,\ k \in \mathbb{Z}$

Com $n \in \mathbb{N} \setminus \{0\}$ e $n \geq 1 \wedge n \leq 365$, vem que

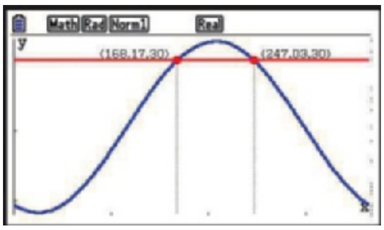
$n = \frac{2 + \frac{\pi}{2}}{0{,}0172} \approx 208$.

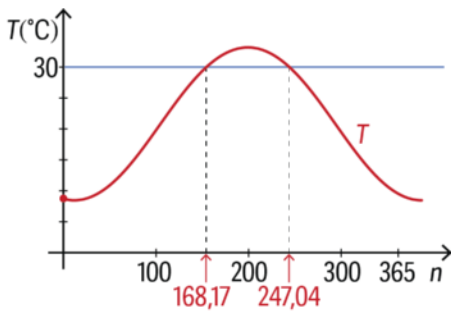
$\underset{Jan}{31} + \underset{Fev}{28} + \underset{Março}{31} + \underset{Abr}{30} + \underset{Maio}{31} + \underset{Jun}{30} = 181$

$208 - 181 = 27$

O dia mais quente do ano tem sido o dia de ordem 208 , ou seja, o dia 27 de julho.

76.3.





$247 - 168 = 79$

A média das temperaturas máximas foi superior a 30 °C entre os dias de ordem 169 e 247 , ou seja, durante 79 dias.

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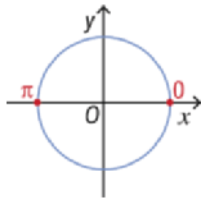
Tarefas de consolidação 6

1.1. Por exemplo, $k = -\frac{\pi}{2}$ porque

$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha$.

1.2. $f(x) = 0 \Leftrightarrow \sin x = 0$

$\Leftrightarrow x = k\pi\ ,\ k \in \mathbb{Z}$

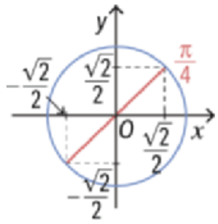


Então, $C(\pi\ ,\ 0)$.

$f(x) = g(x)$

$\Leftrightarrow \sin x = \cos x$

$\Leftrightarrow x = \frac{\pi}{4} + k\pi\ ,\ k \in \mathbb{Z}$



$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

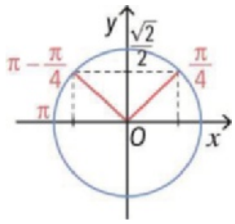
Então, $A\left(\frac{\pi}{4}\ ,\ \frac{\sqrt{2}}{2}\right)$ e

$B\left(\frac{\pi}{4}\ ,\ 0\right)$

$f(x) = \frac{\sqrt{2}}{2} \Leftrightarrow \sin x = \frac{\sqrt{2}}{2}$

$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi \vee x = \frac{3\pi}{4} + 2k\pi\ ,\ k \in \mathbb{Z}$

Então, $D\left(\frac{3\pi}{4}\ ,\ \frac{\sqrt{2}}{2}\right)$.



$\overline{BC} = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\overline{AD} = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$

$\overline{AB} = \frac{\sqrt{2}}{2}$

$A_{[ABCD]} = \frac{\overline{BC} + \overline{AD}}{2} \times \overline{AB} = \frac{\frac{3\pi}{4} + \frac{\pi}{2}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{5\pi\sqrt{2}}{16} \approx 1{,}39$ u. a.

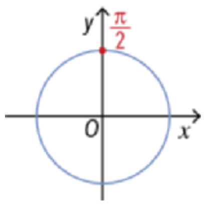
2. I – A Gabriela pode ter construído o gráfico com os valores do domínio em graus.

II – **Zeros:**

$f(x) = 0 \Leftrightarrow \cos(\pi x) = 0$

$\Leftrightarrow \pi x = \frac{\pi}{2} + k\pi\ ,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{1}{2} + k\ ,\ k \in \mathbb{Z}$



$-1 \le \cos(\pi x) \le 1$

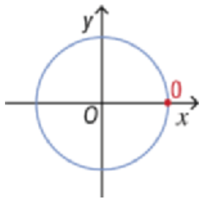
Máximo: 1

Maximizantes:

$f(x) = 1 \Leftrightarrow \cos(\pi x) = 1$

$\Leftrightarrow \pi x = 2k\pi\ ,\ k \in \mathbb{Z}$

$\Leftrightarrow x = 2k\ ,\ k \in \mathbb{Z}$



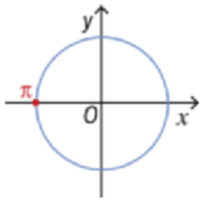
Mínimo: -1

Minimizantes:

$f(x) = -1 \Leftrightarrow \cos(\pi x) = -1$

$\Leftrightarrow \pi x = \pi + 2k\pi\ ,\ k \in \mathbb{Z}$

$\Leftrightarrow x = 1 + 2k\ ,\ k \in \mathbb{Z}$



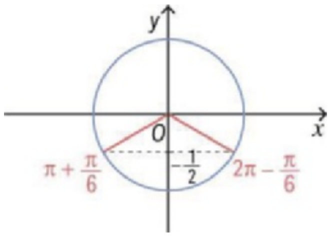
3. $f\left(-\frac{\pi}{6}\right) = \cos\left(2 \times \left(-\frac{\pi}{6}\right)\right) = \cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$f(x) = x - f\left(-\frac{\pi}{6}\right) \wedge x \in \left]0\ ,\ 2\pi\right]$

$\Leftrightarrow x + \sin x = x - \frac{1}{2} \wedge x \in \left]0\ ,\ 2\pi\right]$

$\Leftrightarrow \sin x = -\frac{1}{2} \wedge x \in \left]0,\ 2\pi\right]$

$\Leftrightarrow x = \frac{7\pi}{6} \vee x = \frac{11\pi}{6}$



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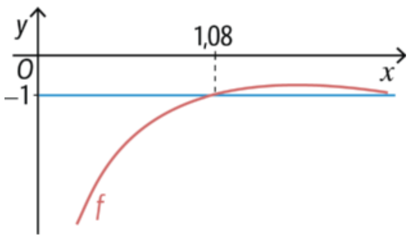
4. $P(a\ ,\ g(a)) = \left(a\ ,\ a\cos\left(a - \frac{\pi}{6}\right)\right)$ $\quad A(0\ ,\ 2)$

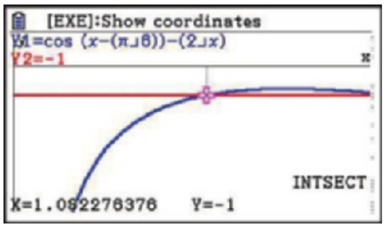
$$m_{AP} = \frac{a\cos\left(a - \frac{\pi}{6}\right) - 2}{a - 0} = \cos\left(a - \frac{\pi}{6}\right) - \frac{2}{a}$$

$m_{AP} = -1 \Leftrightarrow \cos\left(a - \frac{\pi}{6}\right) - \frac{2}{a} = -1$

Seja $f(x) = \cos\left(x - \frac{\pi}{6}\right) - \frac{2}{x}$

Pretende-se resolver graficamente a equação $f(x) = -1$.





A abcissa do ponto P é, aproximadamente $1{,}08$.

5.1. **a)** $d(t) = 50 + 20\sin\left[\frac{\pi}{6}(2t - 3)\right]$

$a = 50$ e $b = 20 > 0$

$D'_d = [50 - 20\ ,\ 50 + 20]$

$= [30\ ,\ 70]$

distância máxima de A ao solo: 70 m

distância mínima de A são solo: 30 m

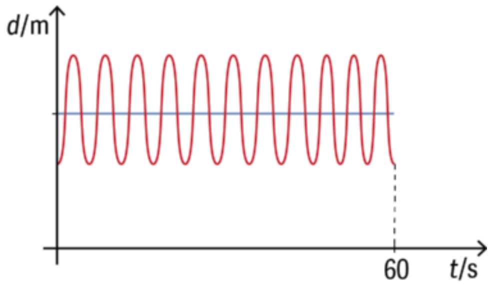
diâmetro $= 70 - 30 = 40$ m

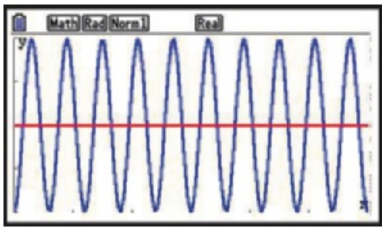
altura $= 30 + \frac{40}{2} = 50$ m

b) raio $= 20$ m

$A_{rotor} = \pi \times 20^2 \approx 1257\ \text{m}^2$

5.2. Pretende-se resolver graficamente a equação $d(t) = 50$.





Durante o primeiro minuto, a altura do ponto A ao solo é igual à altura do rotor, 20 vezes.

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1. $0 \le \cos^2(2x) \le 1 \Leftrightarrow -2 \le -2\cos^2(2x) \le 0$

$\Leftrightarrow 0 \le 2 - 2\cos^2(2x) \le 2$

$\Leftrightarrow 0 \le f(x) \le 2$

$D'_f = [0\ ,\ 2]$

Opção (A)

2. Seja p o período fundamental de g.

$g(x+p)=g(x) \Leftrightarrow 3f(2(x+p)-1)+4=3f(2x-1)+4$

$\Leftrightarrow 3f(2x+2p-1)=3f(2x-1)$

$\Leftrightarrow f(2x-1+2p)=f(2x-1)$

Como o período fundamental da função f é 2 e p é o menor valor para o qual a proposição é verdadeira, terá que ser $2p=2$, pelo que $p=1$

Opção (A)

3. Por observação do gráfico, constata-se que o período fundamental de f é 3.

$f(20)=f(2+18)=f(2+6\times 3)=f(2)$

I – b)

$f(25)=f(1+8\times 3)=f(1)$

$f(13)=f(1+4\times 3)=f(1)$

$f(25)-f(13)=f(1)-f(1)=0$

II – a)

$f(8)=f(2+2\times 3)=f(2)$

$f(6)=f(2\times 3)=f(0)$

$f(15)=f(5\times 3)=f(0)$

$f(8)+f(6)=f(2)-f(0)=f(2)-f(15)$

III – c)

R: I – b) ; II – a) ; III – c)

4.1. Seja P o período fundamental de f.

$f(x+P)=f(x)\Leftrightarrow$

$\Leftrightarrow 1-2\cos(2\pi(x+P))=1-2\cos(2\pi x)$

$\Leftrightarrow -2\cos(2\pi x+2\pi P)=-2\cos(2\pi x)$

$\Leftrightarrow \cos(2\pi x+2\pi P)=\cos(2\pi x)$

Como o período fundamental da função cosseno é 2π e P é o menor valor positivo para o qual a proposição é verdadeira, tem-se que $2\pi P=2\pi$, ou seja, $P=1$.

Alternativa:

$P_0=\frac{2\pi}{|2\pi|}=1$

4.2. $f(1)=g(1)\Leftrightarrow 1-2\cos(2\pi\times 1)=\sin(\pi+a)$

$\Leftrightarrow 1-2\times 1=-\sin a$

$\Leftrightarrow -1=-\sin a$

$\Leftrightarrow \sin a=1$

Como a é o menor valor positivo que verifica esta condição, $a=\frac{\pi}{2}$.

5. Por observação do gráfico apresentado, temos que $D'_g=[-1,\ 5]$.

Então,

$\begin{cases}a-b=-1\\a+b=5\end{cases}\Leftrightarrow\begin{cases}a=b-1\\b-1+b=5\end{cases}\Leftrightarrow\begin{cases}a=b-1\\2b=6\end{cases}$

$\Leftrightarrow\begin{cases}a=3-1\\b=3\end{cases}\Leftrightarrow\begin{cases}a=2\\b=3\end{cases}$

$g(x)=2+3\sin(cx)$

Como o período fundamental de g é 2π, tem-se que $\frac{2\pi}{|c|}=2\pi\Leftrightarrow|c|=1$

Como o gráfico de g não é obtido a partir do de f por uma reflexão de eixo Oy, $c>0$, logo, $c=1$

$a+b+c=2+3+1=6$

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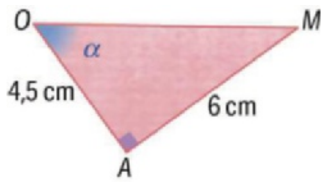
1.



1.1. Como o $[AMO]$ é retângulo vamos aplicar o Teorema de Pitágoras.

$\overline{OM}^2=4,5^2+6^2\Leftrightarrow\overline{OM}^2=20,25+36\Leftrightarrow\overline{OM}^2=56,25$

$\underset{(\overline{OM}>0)}{\Leftrightarrow}\overline{OM}=\sqrt{56,25}\Leftrightarrow\overline{OM}=7,5$

1.2. **a)** $\sin\alpha=\frac{6}{7,5}\Leftrightarrow\sin\alpha=0,8\Leftrightarrow\sin\alpha=\frac{4}{5}$

b) $\cos\alpha=\frac{4,5}{7,5}\Leftrightarrow\cos\alpha=0,6\Leftrightarrow\cos\alpha=\frac{3}{5}$

c) $\tan\alpha=\frac{6}{4,5}\Leftrightarrow\tan\alpha=\frac{60}{45}\Leftrightarrow\tan\alpha=\frac{4}{3}$

2.1. $\tan\alpha=\frac{27}{30}\Leftrightarrow\alpha=\tan^{-1}\left(\frac{27}{30}\right)$

$\alpha\approx 42,0^\circ$

A amplitude do ângulo α é, aproximadamente, 42,0º.

2.2. $\sin\alpha=\frac{6}{10}\Leftrightarrow\alpha=\sin^{-1}\left(\frac{6}{10}\right)$

$\alpha\approx 36,9^\circ$

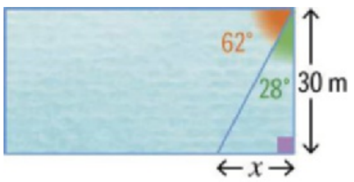
A amplitude do ângulo α é, aproximadamente, 36,9º.

2.3. $\cos\alpha=\frac{6}{8}\Leftrightarrow\alpha=\cos^{-1}\left(\frac{6}{8}\right)$

$\alpha\approx 41,4^\circ$

A amplitude do ângulo α é, aproximadamente, 41,4º.

3. $90° - 62° = 28°$

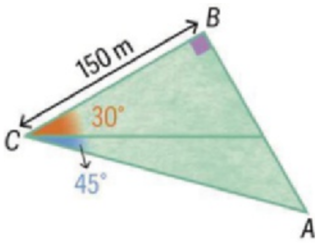


$\tan(28°) = \frac{x}{30} \Leftrightarrow x = \tan(28°) \times 30$

$x \approx 16{,}0$ m

O valor de x é, aproximadamente, 16,0 m.

4.



$\tan(30°) = \frac{\overline{BD}}{\overline{CB}} \Leftrightarrow \overline{BD} = \overline{CB}\tan(30°)$

$\Leftrightarrow \overline{BD} = 150\tan(30°)$

$\overline{BD} \approx 86{,}6$ m

Como $B\hat{A}C = 45°$, o triângulo $[ABC]$ é isósceles.

Assim, $\overline{AB} = 150$.

$\overline{DA} = \overline{AB} - \overline{BD} = 150 - 86{,}6 = 63{,}4$ m

A largura do rio, $\overline{DA}$, é 63,4 m.

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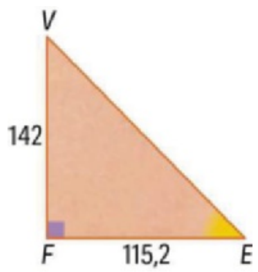
5. Cálculos auxiliares

$\overline{FE} = \frac{\overline{AB}}{2} =$

$= \frac{230{,}4}{2} = 115{,}2$ m



5.1. $\tan\left(V\hat{E}F\right) = \frac{\overline{VF}}{\overline{FE}} = \frac{142}{115{,}2}$

$V\hat{E}F = \tan^{-1}\left(\frac{142}{115{,}2}\right) \approx 50{,}95°$

5.2. $\overline{VE}^2 = \overline{VF}^2 + \overline{FE}^2 \Leftrightarrow \overline{VE}^2 = 142^2 + 115{,}2^2$

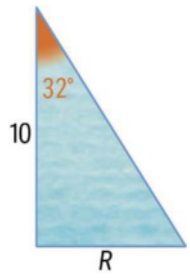
$\Leftrightarrow \overline{VE} = \sqrt{33\ 435{,}04}$

$A_{lateral} = 4 \times \frac{\overline{AB} \times \overline{VE}}{2} =$

$= 2 \times 230{,}4 \times \sqrt{33\ 435{,}04} \approx 84\ 258{,}436$

A área lateral da pirâmide é, aproximadamente, $84\ 258{,}4\ \text{m}^2$.

6. $\frac{64°}{2} = 32°$



$\tan(32°) = \frac{r}{10}$

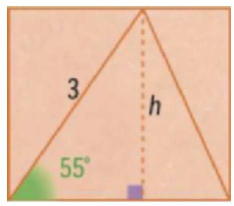
$r = 10\tan(32°)$

$V_{cone} = \frac{1}{3}A_b \times h = \frac{1}{3}\pi r^2 \times h =$

$= \frac{1}{3}\pi \times \left(10\tan(32°)\right)^2 \times 10 \approx 408{,}8905$

O volume do cone é, aproximadamente, $408{,}9\ \text{cm}^3$.

7.



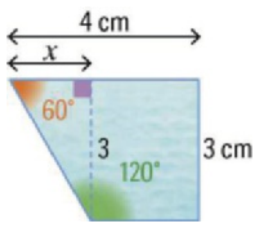
$\sin(55°) = \frac{h}{3}$

$\Leftrightarrow h = 3\sin(55°)$

$A_{[ABCD]} = \left(3\sin(55°)\right)^2 \approx 6{,}0391$

A área do quadrado $[ABCD]$ é, aproximadamente, $6{,}0\ \text{cm}^2$.

8. $360° - 120° - 90° - 90° = 60°$



$\tan(60°) = \frac{3}{x} \Leftrightarrow$

$\Leftrightarrow x = \frac{3}{\tan(60°)} \Leftrightarrow$

$\Leftrightarrow x = \frac{3}{\sqrt{3}}$

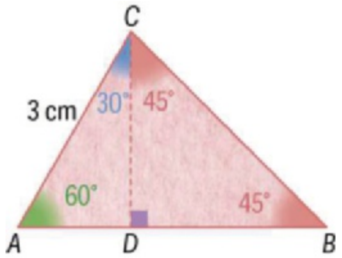
Base menor $= 4 - \frac{3}{\sqrt{3}}$

$A_{[ABCD]} = \frac{\text{Base maior} + \text{base menor}}{2} \times \text{altura}$

$= \frac{4 + 4 - \frac{3}{\sqrt{3}}}{2} \times 3 \approx 9{,}40192$

A área do trapézio é, aproximadamente, $9{,}4\ \text{cm}^2$.

9.1.



$\overline{CD} = \overline{DB}$

$D\hat{C}B = C\hat{B}D = \dfrac{180° - 90°}{2} = 45°$

$A\hat{C}D = 180° - 90° - 60° = 30°$

$C\hat{A}D = 60°$

$A\hat{C}B = 30° + 45° = 75°$

$C\hat{B}A = 45°$

O triângulo $[ABC]$ não é retângulo, porque os ângulos internos do triângulo têm amplitudes 60°, 75° e 45°.

9.2. $\sin(60°) = \dfrac{\overline{CD}}{3} \Leftrightarrow \overline{CD} = 3\sin(60°) \Leftrightarrow \overline{CD} = \dfrac{3\sqrt{3}}{2}$

$\overline{CD} = \overline{DB}$

$\overline{CB}^2 = \overline{CD}^2 + \overline{DB}^2 \Leftrightarrow$

$\Leftrightarrow \overline{CB}^2 = \overline{CD}^2 + \overline{CD}^2 \Leftrightarrow \overline{CB}^2 = 2\left(\dfrac{3\sqrt{3}}{2}\right)^2$

$\Leftrightarrow \overline{CB}^2 = 2 \times \dfrac{9 \times 3}{4} \Leftrightarrow \overline{CB}^2 = \dfrac{27}{2} \underset{(\overline{CB}>0)}{\Leftrightarrow} \overline{CB} = \sqrt{\dfrac{27}{2}}$

$\Leftrightarrow \overline{CB} = \dfrac{3\sqrt{3}}{\sqrt{2}}$ cm $\Leftrightarrow \overline{CB} = \dfrac{3\sqrt{3} \times \sqrt{2}}{2} \Leftrightarrow \overline{CB} = \dfrac{3\sqrt{6}}{2}$ cm

Cálculos auxiliares

27	3
9	3
3	3
1	

$27 = 3^2 \times 3$

$\sqrt{27} = \sqrt{3^2 \times 3} \Leftrightarrow \sqrt{27} = 3\sqrt{3}$

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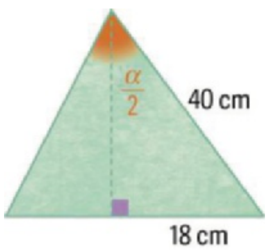
10.



$36 \text{ cm} : 2 = 18 \text{ cm}$

$\sin\left(\dfrac{\alpha}{2}\right) = \dfrac{18}{40}$

$\Leftrightarrow \dfrac{\alpha}{2} = \sin^{-1}\left(\dfrac{18}{40}\right)$

$\dfrac{\alpha}{2} \approx 26{,}744°$

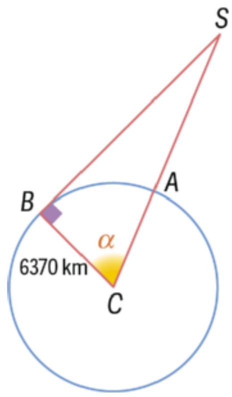
$\Leftrightarrow \alpha \approx 2 \times 26{,}744°$

$\Leftrightarrow \alpha \approx 53{,}5°$

A amplitude máxima de α é 53,5°.

11. Designemos por C o centro da circunferência.

$\alpha = 26{,}7° - 13{,}3° = 13{,}4°$



$\tan(13{,}4°) = \dfrac{\overline{BS}}{6370} \Leftrightarrow$

$\Leftrightarrow \overline{BS} = 6370 \times \tan(13{,}4°)$

$\overline{BS} \approx 1518$ km

$\cos(13{,}4°) = \dfrac{6370}{\overline{CS}} \Leftrightarrow$

$\Leftrightarrow \overline{CS} = \dfrac{6370}{\cos(13{,}4°)}$

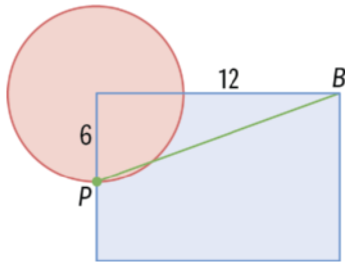
$\overline{CS} \approx 6548$ km

$\overline{AS} \approx 6548 - 6370 = 178$ km

A distância do satélite ao ponto A é 178 km.

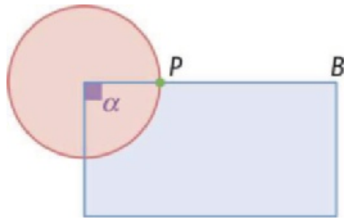
A distância do satélite ao ponto B é 1518 km.

12.1. a) $\alpha = 0°$



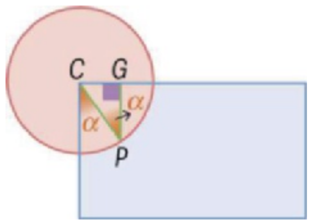
$A_{jardim} = \dfrac{6 \times 12}{2} = 36 \text{ m}^2$

b) $\alpha = 90°$



$A_{jardim} = 0 \text{ m}^2$

12.2.



$\cos\alpha = \dfrac{\overline{GP}}{6} \Leftrightarrow \overline{GP} = 6\cos\alpha$

12.3. $A_{[PBC]} = \dfrac{12 \times 6\cos\alpha}{2} = 36\cos\alpha$

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13. $\tan(27^\circ) = \dfrac{2,5}{\overline{AB}} \Leftrightarrow \overline{AB} = \dfrac{2,5}{\tan(27^\circ)}$

$\overline{AB} \approx 4,9065$

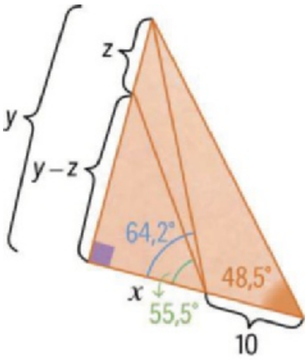
$\tan(48^\circ) = \dfrac{\overline{CB}}{\overline{AB}} \Leftrightarrow \overline{CB} = \overline{AB}\tan(48^\circ)$

$\overline{CB} \approx 4,9065\tan(48^\circ) \Leftrightarrow \overline{CB} \approx 5,4492$

$\overline{DC} = \overline{CB} - \overline{BD} \approx 5,4492 - 2,5 \approx 2,9492$

A altura do segundo andar, $\overline{DC}$, é, aproximadamente, 2,95 m.

14.



$\begin{cases} \tan(48,5^\circ) = \dfrac{y}{x+10} \\ \tan(64,2^\circ) = \dfrac{y}{x} \end{cases}$

$\begin{cases} 1,1303 = \dfrac{y}{x+10} \\ 2,0686 = \dfrac{y}{x} \end{cases}$

$\Leftrightarrow \begin{cases} 1,1303x + 11,303 = y \\ y = 2,0686x \end{cases}$

$\Leftrightarrow \begin{cases} 1,1303x + 11,303 = 2,0686x \\ \rule{5cm}{0.4pt} \end{cases}$

$\Leftrightarrow \begin{cases} 1,1303x - 2,0686x = -11,303 \\ \rule{5cm}{0.4pt} \end{cases}$

$\Leftrightarrow \begin{cases} -0,9383x = -11,303 \\ \rule{3cm}{0.4pt} \end{cases}$

$\begin{cases} x \approx 12,0463 \\ y \approx 2,0696 \times 12,0463 \end{cases}$

$\Leftrightarrow \begin{cases} x \approx 12,0463 \\ y \approx 24,9190 \end{cases}$

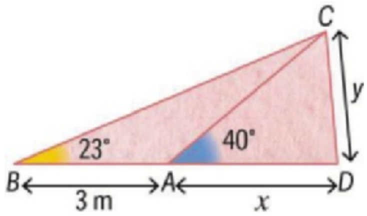
$\tan(55,5^\circ) = \left(\dfrac{y-z}{x}\right) \Leftrightarrow x\tan(55,5^\circ) = y - z$

$\Leftrightarrow z = y - x\tan(55,5^\circ)$

$z \approx 24,9190 - 12,0463 \times 1,4550 \Leftrightarrow z \approx 7,4$ m

A altura da chaminé é, aproximadamente, 7,4 m.

15.1.



$\begin{cases} \tan(40^\circ) = \dfrac{y}{x} \\ \tan(23^\circ) = \dfrac{y}{x+3} \end{cases} \Leftrightarrow \begin{cases} y = x\tan(40^\circ) \\ y = (x+3)\tan(23^\circ) \end{cases}$

$\Leftrightarrow \begin{cases} \rule{6cm}{0.4pt} \\ x\tan(40^\circ) = x\tan(23^\circ) + 3\tan(23^\circ) \end{cases}$

$\Leftrightarrow \begin{cases} \rule{6cm}{0.4pt} \\ x\left(\tan(40^\circ) - \tan(23^\circ)\right) = 3\tan(23^\circ) \end{cases}$

$\Leftrightarrow \begin{cases} \rule{4cm}{0.4pt} \\ x = \dfrac{3\tan(23^\circ)}{\tan(40^\circ) - \tan(23^\circ)} \end{cases}$

$\begin{cases} y \approx 3,0713\tan(40^\circ) \\ x \approx 3,0713 \end{cases}$

$\Leftrightarrow \begin{cases} y \approx 3,0713 \times 0,8391 \\ x \approx 3,0713 \end{cases}$

$\Leftrightarrow \begin{cases} y \approx 2,5771 \\ x \approx 3,0713 \end{cases}$

$\overline{CD} = y \approx 2,5771$

$\overline{TC} \approx 1,7 + 2,5771 \approx 4,2771$

A altura da torre, $\overline{TC}$, é, aproximadamente, 4,3 m.

15.2. $\overline{RT} = x$

A largura do rio, $\overline{RT}$, é 3,1 metros.

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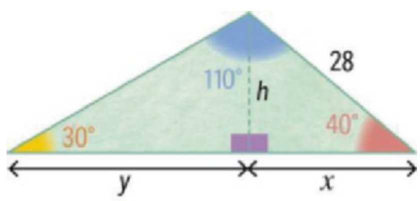
16.



$\sin(40^\circ) = \dfrac{h}{28} \Leftrightarrow h = 28\sin(40^\circ)$

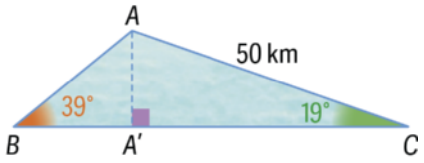
$\cos(40^\circ) = \dfrac{x}{28} \Leftrightarrow x = 28\cos(40^\circ)$

$\tan(30^\circ) = \dfrac{h}{y} \Leftrightarrow y = \dfrac{h}{\tan(30^\circ)} \Leftrightarrow y = \dfrac{28\sin(40^\circ)}{\tan(30^\circ)}$

$\overline{AB} = x + y = 28\cos(40^\circ) + \dfrac{28\sin(40^\circ)}{\tan(30^\circ)} \approx 52,6228$

A distância entre A e B é, aproximadamente, 52,6 m.

17.



$180° - 39° - 122° = 19°$

Vítor: 30 km/h

Sara: 50 km/h

Designemos por *A'* a projeção ortogonal de *A* sobre a reta *BC*.

$\cos(19°) = \frac{\overline{A'C}}{50} \Leftrightarrow \overline{A'C} = 50 \times \cos(19°)$

$\overline{A'C} \approx 47{,}276$ km

$\sin(19°) = \frac{\overline{AA'}}{50} \Leftrightarrow \overline{AA'} = 50 \times \sin(19°)$

$\overline{AA'} \approx 16{,}278$ km

$\tan(39°) = \frac{\overline{AA'}}{\overline{BA'}} \Leftrightarrow \overline{BA'} = \frac{\overline{AA'}}{\tan(39°)}$

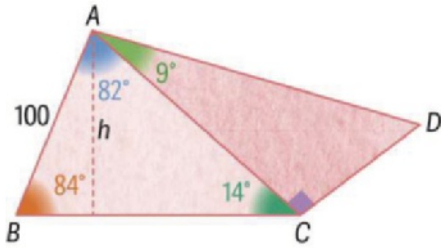
$\overline{BA'} \approx \frac{16{,}278}{\tan(39°)} \approx 20{,}102$ km

$\overline{BC} = \overline{BA'} + \overline{A'C} \approx 20{,}102 + 47{,}276 = 67{,}378$ km

Vítor:	Sara:
30 km —— 1 h	50 km —— 1 h
50 km —— x	67,378 km —— y
$x = \frac{50 \times 1}{30} \approx 1{,}67$ h	$y = \frac{67{,}378 \times 1}{50} \approx 1{,}35$ h

Quem chegou primeiro à casa *C* foi a Sara, pois demorou menos tempo a lá chegar.

18.



$180° - 84° - 82° = 14°$

Pretendemos determinar a altura da torre, isto é a distância entre *C* e *D*.

$\sin(84°) = \frac{h}{100} \Leftrightarrow h = 100 \times \sin(84°)$

$h \approx 99{,}452$

$\sin(14°) = \frac{h}{\overline{AC}} \Leftrightarrow \overline{AC} = \frac{h}{\sin(14°)}$

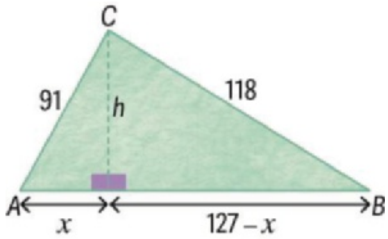
$\overline{AC} \approx \frac{99{,}452}{\sin(14°)} \Leftrightarrow \overline{AC} \approx 411{,}091$

$\tan(9°) = \frac{\overline{CD}}{\overline{AC}} \Leftrightarrow \overline{CD} = \overline{AC}\tan(9°)$

$\overline{CD} \approx 411{,}091\tan(9°) \Leftrightarrow \overline{CD} \approx 65{,}110$

A altura da torre é, aproximadamente, 65 metros.

19.



$91^2 = h^2 + x^2 \Leftrightarrow h^2 = 91^2 - x^2$ (1)

$118^2 = h^2 + (127 - x)^2 \Leftrightarrow h^2 = 118^2 - (127 - x)^2$ (2)

De (1) e (2) vem:

$91^2 - x^2 = 118^2 - (127 - x)^2$

$\Leftrightarrow 8281 \cancel{-x^2} = 13\,924 - 16\,129 + 254x \cancel{-x^2}$

$\Leftrightarrow 254x = 10\,486$

$x \approx 41{,}2835$

Vamos representar por *A*, *B* e *C* as amplitudes dos ângulos de vértices *A*, *B* e *C*, respetivamente.

$\cos A = \frac{x}{91}$

$\cos A \approx \frac{41{,}2835}{91} \Leftrightarrow A \approx \cos^{-1}\left(\frac{41{,}2835}{91}\right) \approx 63{,}0°$

$127 - x \approx 127 - 41{,}2835 \approx 85{,}7165$

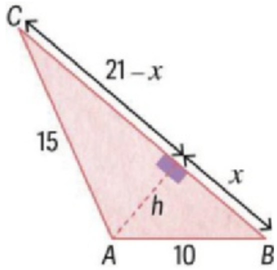
$\cos B = \frac{127 - x}{118}$

$\cos B \approx \frac{85{,}7165}{118} \Leftrightarrow B \approx \cos^{-1}\left(\frac{85{,}7165}{118}\right) \approx 43{,}4°$

$C \approx 180° - 63{,}0° - 43{,}4° \approx 73{,}6°$

$B\hat{A}C \approx 63{,}0°$; $C\hat{B}A \approx 43{,}4°$; $A\hat{C}B \approx 73{,}6°$

20.



$10^2 = h^2 + x^2 \Leftrightarrow h^2 = 10^2 - x^2$ (1)

$15^2 = h^2 + (21 - x)^2 \Leftrightarrow h^2 = 15^2 - (21 - x)^2$ (2)

De (1) e (2) vem:

$10^2 - x^2 = 15^2 - (21 - x)^2$

$\Leftrightarrow 100 \cancel{-x^2} = 225 - 441 + 42x \cancel{-x^2} \Leftrightarrow 42x = 316$

$x = \frac{316}{42} \approx 7{,}5238$

Vamos representar por *A* e *B* as amplitudes dos ângulos de vértices *A* e *B*, respetivamente.

$\cos B = \frac{x}{10}$

$\cos B \approx \dfrac{7,5238}{10} \Leftrightarrow$

$\Leftrightarrow B \approx \cos^{-1}(0,75238) \approx 41,2030^\circ \approx 41,20^\circ$

$21 - x \approx 21 - 7,5238 \approx 13,4762$

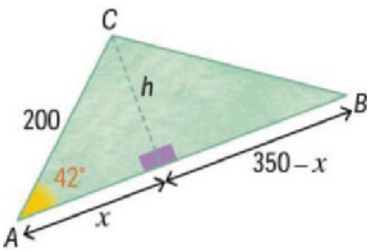
$\cos C = \dfrac{21 - x}{15}$

$\cos C \approx \dfrac{13,4762}{15} \Leftrightarrow C \approx \cos^{-1}\left(\dfrac{13,4762}{15}\right) \approx 26,0497^\circ$

$A \approx 180^\circ - 41,2030^\circ - 26,0497^\circ \approx 112,75^\circ$

$B\hat{A}C \approx 112,75^\circ$; $C\hat{B}A \approx 41,20^\circ$.

21.1.



$\cos(42^\circ) = \dfrac{x}{200} \Leftrightarrow x = 200\cos(42^\circ)$

$x \approx 148,6290$

$\sin(42^\circ) = \dfrac{h}{200} \Leftrightarrow h = 200\sin(42^\circ)$

$h \approx 133,8261$

$350 - x \approx 350 - 148,6290 = 201,3710$

$\overline{CB}^2 = h^2 + (350 - x)^2 \underset{(\overline{CB}>0)}{\Leftrightarrow} \overline{CB} = \sqrt{h^2 + (350 - x)^2}$

$\overline{CB} \approx \sqrt{133,8261^2 + 201,371^2} \Leftrightarrow \overline{CB} \approx 241,7844$

A distância entre as duas aeronaves é, aproximadamente, 241,8 km.

21.2. $\tan A\hat{B}C = \dfrac{h}{350 - x}$; $\tan A\hat{B}C \approx \dfrac{133,8261}{201,3710}$

$A\hat{B}C \approx \tan^{-1}\left(\dfrac{133,8261}{201,3710}\right) \Leftrightarrow A\hat{B}C \approx 33,6070$

$A\hat{C}B \approx 180^\circ - 42^\circ - 33,6070^\circ \approx 104,393^\circ$

A amplitude do ângulo *ACB* é, aproximadamente, 104º.

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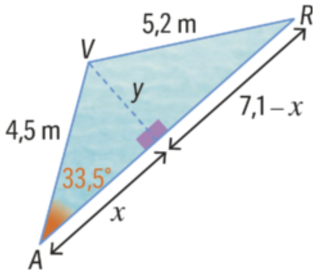
22.1.



$\sin(33,5^\circ) = \dfrac{y}{4,5} \Leftrightarrow y = 4,5\sin(33,5^\circ)$

$y \approx 2,4837$

$\cos(33,5^\circ) = \dfrac{x}{4,5} \Leftrightarrow x = 4,5\cos(33,5^\circ)$

$x \approx 3,7525$

$7,1 - x = 7,1 - 3,7525 \approx 3,3475$

$\overline{VR}^2 = y^2 + (7,1 - x)^2$

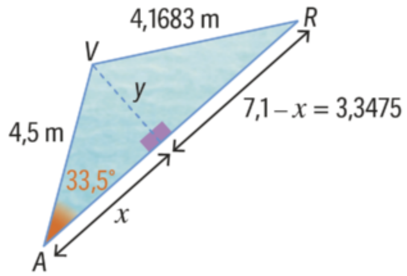
$\overline{VR}^2 \approx 2,4837^2 + 3,3475^2$

$\overline{VR}^2 \approx 17,3745$

$\Leftrightarrow \overline{VR} \approx \sqrt{17,3745} \Leftrightarrow \overline{VR} \approx 4,1683$

$\overline{VR} \neq 5,2$

22.2.



$\cos A\hat{R}V \approx \dfrac{3,3475}{4,1683}$

$A\hat{R}V \approx \cos^{-1}\left(\dfrac{3,3475}{4,1683}\right) \Leftrightarrow A\hat{R}V \approx 36,6^\circ$

$A\hat{V}R \approx 180^\circ - 33,5^\circ - 36,6^\circ \Leftrightarrow A\hat{R}V \approx 109,9^\circ$

23. $A\hat{O}C = C\hat{O}D = D\hat{O}E = E\hat{O}A = \dfrac{360^\circ}{4} = 90^\circ$

$B\hat{O}D = D\hat{O}F = F\hat{O}B = \dfrac{360^\circ}{3} = 120^\circ$

23.1. $B\hat{O}D = 120^\circ$

B é a imagem de *D* na rotação de centro *O* e amplitude $\alpha = -120^\circ$.

23.2. $F\hat{O}D = 120^\circ$

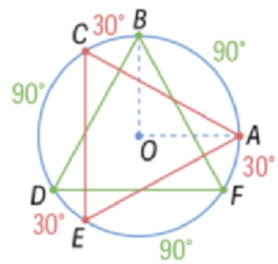
$\alpha = -120^\circ$ ou $\alpha = -120^\circ + 360^\circ = 240^\circ$

23.3. $C\hat{O}D = 90^\circ$

C é a imagem de *D* na rotação de centro *O* e amplitude -90° .

23.4. $\alpha = 90^\circ$ ou $\alpha = 90^\circ - 360^\circ = -270^\circ$

24. $B\hat{O}C = A\hat{O}C - A\hat{O}B = 120^\circ - 90^\circ = 30^\circ$



$F\hat{O}A = B\hat{O}C = D\hat{O}E = 30^\circ$

$C\hat{O}D = E\hat{O}F = A\hat{O}B = 90^\circ$

24.1. a) $A\hat{O}D = 90° + 30° + 90° = 210°$

O lado extremidade é $\dot{O}D$.

b) $A\hat{O}E = -90° - 30° = -120°$

O lado extremidade é $\dot{O}E$.

c) $A\hat{O}F = 90° + 30° + 90° + 30° + 90° = 330°$

O lado extremidade é $\dot{O}F$.

d) $A\hat{O}D = -120° - 30° = -150°$

O lado extremidade é $\dot{O}D$.

24.2. a) $960° = 240° + 2 \times 360°$

$A\hat{O}E = 120° + 120° = 240°$

O lado extremidade é $\dot{O}E$.

b) $-1470° = -30° - 4 \times 360°$

$A\hat{O}F = -30°$

O lado extremidade é $\dot{O}F$.

c) $1530° = 90° + 4 \times 360°$

$A\hat{O}B = 90°$

O lado extremidade é $\dot{O}B$.

d) $-1320° = -240° - 3 \times 360°$

$A\hat{O}C = -120° - 120° = -240°$

O lado extremidade é $\dot{O}C$.

e) $1200° = 120° + 3 \times 360°$

$A\hat{O}C = 120°$

O lado extremidade é $\dot{O}C$.

f) $-990° = -270° - 2 \times 360°$

$A\hat{O}B = -120° - 120° - 30° = -270°$

O lado extremidade é $\dot{O}B$.

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25.1. $\overline{OA} = \overline{OD}$. Então $O\hat{A}D = A\hat{D}O = 40°$

$\overset{\frown}{AB} = 2 \times A\hat{D}O = 2 \times 40° = 80°$

$A\hat{O}D = 180° - 2 \times 40° = 100°$

$\overset{\frown}{BC} = B\hat{O}C = A\hat{O}D = 100°$

$\overset{\frown}{CD} = 2 \times O\hat{A}D = 2 \times 40° = 80°$

$\overset{\frown}{DA} = A\hat{O}D = 100°$

$\overset{\frown}{AB} = 80°$; $\overset{\frown}{BC} = 100°$; $\overset{\frown}{CD} = 80°$ e $\overset{\frown}{DA} = 100°$

25.2. a) $540° = 180° + 1 \times 360°$

$A\hat{O}C = 80° + 100° = 180°$

O transformado de *A* é *C*.

b) $-640° = -280° - 1 \times 360°$

$A\hat{O}B = -100° - 80° - 100° = -280°$

O transformado de *A* é *B*.

c) $980° = 260° + 2 \times 360°$

$A\hat{O}D = 80° + 100° + 80° = 260°$

O transformado de *A* é *D*.

d) $-900° = -180° - 2 \times 360°$

$A\hat{O}C = -80° - 100° = -180°$

O transformado de *A* é *C*.

e) $1080° = 0° + 3 \times 360°$

O transformado de *A* é *A*.

f) $-1180° = -100° - 3 \times 360°$

$A\hat{O}D = -100°$

O transformado de *A* é *D*.

g) $1520° = 80° + 4 \times 360°$

$A\hat{O}B = 80°$

O transformado de *A* é *B*.

h) $-1440° = 0° - 4 \times 360°$

O transformado de *A* é *A*.

26.1. a) $\frac{3}{4} \times 360° = 270°$

P descreve um ângulo de 270º no sentido positivo.

270º

b) $-\frac{7}{12} \times 360° = -210°$

P descreve um ângulo de 210º no sentido negativo.

–210º

26.2. $r = \frac{100}{2} = 50$ m

Perímetro da circunferência $= 2\pi \times 50 = 100\pi$ m

360º —— 100π m

1200º —— x

$x = \frac{1200° \times 100\pi}{360°} \approx 1047{,}2$ m

O comprimento do arco generalizado é 1047,2 m.

27.1. $-91° \in \left]-180°, -90°\right[$

$\alpha \in 3.°$ Quadrante

27.2. α pertence ao semieixo positivo Oy

27.3. $3780° = 180° + 10 \times 360°$

α pertence ao semieixo negativo Ox

27.4. $\alpha \in 4.°$ Quadrante

27.5. $\alpha \in 2.°$ Quadrante

27.6. $\alpha \in 1.°$ Quadrante

28.1. $270° < 351° < 360°$. Logo:

- $\alpha \in 4.°$ Quadrante
- $\sin\alpha < 0$
- $\cos\alpha > 0$

28.2. $90° < 147° < 180°$. Logo:

- $\alpha \in 2.°$ Quadrante
- $\sin\alpha > 0$
- $\cos\alpha < 0$

28.3. $180° < 221° < 270°$. Logo:

- $\alpha \in 3.°$ Quadrante
- $\sin\alpha < 0$
- $\cos\alpha < 0$

28.4. $-270° < -227° < -180°$. Logo:

- $\alpha \in 2.°$ Quadrante
- $\sin\alpha > 0$
- $\cos\alpha < 0$

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29.1. Por exemplo, $\alpha = 46°$.

29.2. Por exemplo, $\alpha = -10°$.

29.3. Por exemplo, $\alpha = 190°$.

29.4. $\tan(46°) > 1$

Logo, não existe α tal que $\sin\alpha = \tan(46°)$.

30.1. $Q(\cos(130°), \sin(130°))$

$Q(-0{,}64, 0{,}77)$

30.2. $A_{[OQR]} = \frac{\overline{OR} \times y_Q}{2} = \frac{1 \times \sin(130°)}{2} \approx 0{,}4$ u. a.

31.1. $F\hat{A}D = \frac{\widehat{FD}}{2} = \frac{20°}{2} = 10°$

$B\hat{A}E = B\hat{A}D - (E\hat{A}F + F\hat{A}D) = 90° - (60° + 10°) = 20°$

$\widehat{BE} = 2 \times B\hat{A}E = 2 \times 20° = 40°$

31.2. a) $490° = 130° + 360°$

$A\hat{O}E = 90° + 40° = 130°$

O transformado de *A* é *E*.

b) $-1190° = -110° - 3 \times 360°$

$A\hat{O}F = -90° - 20° = -110°$

O transformado de *A* é *F*.

c) $2410° = 250° + 6 \times 360°$

$A\hat{O}F = 90° + 90° + 70° = 250°$

O transformado de *A* é *F*.

d) $-1670° = -230° - 4 \times 360°$

$A\hat{O}E = -90° - 90° - 50° = -230°$

O transformado de *A* é *E*.

31.3. 60º ou $60° - 360° = -300°$, por exemplo.

31.4. a) $B\hat{A}F = 20° + 60° = 80°$

O lado extremidade é $\dot{A}F$.

b) $B\hat{A}D = 90°$

O lado extremidade é $\dot{A}D$.

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32.1. a) $\widehat{BC} = 2 \times B\hat{A}C = 2 \times 20° = 40°$

$\alpha = B\hat{O}C = 40°$ ou $\alpha = 40° - 360° = -320°$, por exemplo.

b) $C\hat{B}A = 180° - (130° + 20°) = 30°$

$\widehat{CA} = 2 \times C\hat{A}B = 2 \times 30° = 60°$

$\alpha = A\hat{O}B = -60° - 40° = -100°$

ou $\alpha = -100° + 360° = 260°$, por exemplo.

32.2. $B\hat{D}A = \frac{\widehat{BA}}{2} = \frac{100°}{2} = 50°$

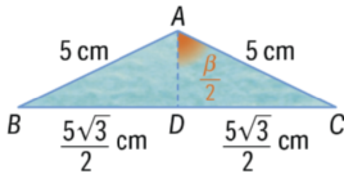
Se a imagem de *B* é *D* por uma rotação de centro em *A*, então $\overline{AB} = \overline{AD}$.

Então $A\hat{B}D = B\hat{D}A = 50°$.

$D\hat{A}B = 180° - (50° + 50°) = 80°$

$\beta = -80°$ ou $\beta = -80° + 360° = 280°$, por exemplo.

33.



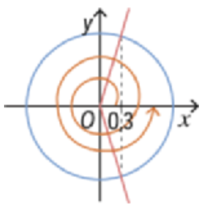
$\sin\left(\frac{\beta}{2}\right) = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$

$\frac{\beta}{2} = 60° \Leftrightarrow \beta = 120°$

$\alpha = -120°$ ou $\alpha = -120° + 360° = 240°$, por exemplo.

34. $720° = 2 \times 360°$

720º representa 2 voltas completas e em cada volta completa existem 2 valores de α para os quais $\cos\alpha = 0{,}3$.



Então, no intervalo $]0, 720°[$ existem 4 valores de α para os quais $\cos\alpha = 0{,}3$.

35.1. $\frac{\sin(30°)}{2\cos(-300°)} + \tan(45°) = \frac{\frac{1}{2}}{2 \times \cos(60° - 360°)} + 1$

$= \frac{\frac{1}{2}}{2 \times \cos(60°)} + 1 = \frac{\frac{1}{2}}{2 \times \frac{1}{2}} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$

35.2. $\cos^2(45°) + \sin^2(765°)$

$= \cos^2(45°) + \sin^2(45° + 2 \times 360°)$

$= \cos^2(45°) + \sin^2(45°)$

$= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$

35.3.

$$\frac{\tan(30^\circ)}{\sin(90^\circ)}-2\cos(150^\circ)=\frac{\frac{\sqrt{3}}{3}}{1}-2\times\left(-\frac{\sqrt{3}}{2}\right)$$

$$=\frac{\sqrt{3}}{3}+\sqrt{3}=\frac{4\sqrt{3}}{3}$$

Cálculo auxiliar:

$$\cos(150^\circ)=-\cos(-30^\circ)$$

$$=-\cos(30^\circ)=-\frac{\sqrt{3}}{2}$$

35.4. $\cos(360^\circ)\left(\tan^2(420^\circ)-\sin^4(-270^\circ)\right)$

$$=1\times\left(\tan^2(60^\circ+360^\circ)-\sin^4(90^\circ-360^\circ)\right)$$

$$=\tan^2(60^\circ)-\sin^4(90^\circ)=\left(\sqrt{3}\right)^2-1^4=3-1=2$$

35.5. $\sin(0^\circ)+\cos(0^\circ)+\tan(0^\circ)=0+1+0=1$

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36.1. *AB*: $x=\cos\alpha$, $\cos\alpha<0$

Seja *d* a distância da origem do referencial ao ponto de interseção da reta *AB* com o eixo Ox.

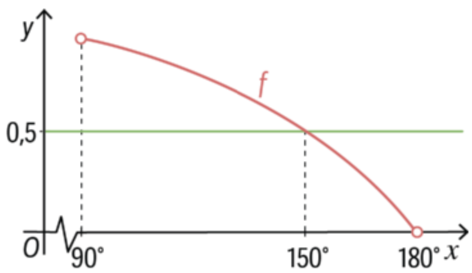
$d=-\cos\alpha$

36.2. $A_{[OAC]}=\frac{\overline{OC}\times y_A}{2}=\frac{1\times\sin\alpha}{2}=\frac{\sin\alpha}{2}$

$$A_{[AOBC]}=2A_{[OAC]}=2\times\frac{\sin\alpha}{2}=\sin\alpha$$

36.3. $A_{[AOBC]}\geq 0{,}5 \Leftrightarrow \sin\alpha\geq 0{,}5$

$A_{[AOBC]}=\sin\alpha$



$f(x)=\sin x$

A área do quadrilátero é superior ou igual a 0,5 para $\alpha\in\,]90^\circ, 150^\circ]$.

37.1. Graus radianos

$180 —— \pi$

$240 —— x$

$x=\frac{240\times\pi}{180}=\frac{4\pi}{3}$

$240^\circ=\frac{4\pi}{3}$ rad

37.2. Graus radianos

$180 —— \pi$

$210 —— x$

$x=\frac{210\times\pi}{180}=\frac{7\pi}{6}$

$210^\circ=\frac{7\pi}{6}$ rad

37.3. Graus radianos

$180 —— \pi$

$300 —— x$

$x=\frac{300\times\pi}{180}=\frac{5\pi}{3}$

$300^\circ=\frac{5\pi}{3}$ rad

37.4. Graus radianos

$180 —— \pi$

$315 —— x$

$x=\frac{315\times\pi}{180}=\frac{7\pi}{4}$

$315^\circ=\frac{7\pi}{4}$ rad

37.5. Graus radianos

$180 —— \pi$

$-288 —— x$

$x=\frac{-288\times\pi}{180}=-\frac{8\pi}{5}$

$-288^\circ=-\frac{8\pi}{5}$ rad

37.6. Graus radianos

$180 —— \pi$

$420 —— x$

$x=\frac{420\times\pi}{180}=\frac{7\pi}{3}$

$420^\circ=\frac{7\pi}{3}$ rad

38.1. Graus radianos

$180 —— \pi$

$x —— \frac{7\pi}{5}$

$x=\frac{\frac{7\pi}{5}\times 180}{\pi}=252$

$\frac{7\pi}{5}$ rad $=252^\circ$

38.2. Graus radianos

$180 —— \pi$

$x —— -\frac{5\pi}{2}$

$x=\frac{-\frac{5\pi}{2}\times 180}{\pi}=-450$

$-\frac{5\pi}{2}$ rad $=-450^\circ$

38.3. Graus radianos

$180 —— \pi$

$x —— \frac{3\pi}{8}$

$x=\frac{\frac{3\pi}{8}\times 180}{\pi}=67{,}5$

$\frac{3\pi}{8}$ rad $=67{,}5^\circ$

38.4. Graus radianos

$180 \text{ —— } \pi \qquad x = \dfrac{-\dfrac{5\pi}{16}\times 180}{\pi} = -56,25$

$x \text{ —— } -\dfrac{5\pi}{16}$

$-\dfrac{5\pi}{16}\text{ rad} = -56,25°$

38.5. Graus radianos

$180 \text{ —— } \pi \qquad x = \dfrac{\dfrac{3\pi}{32}\times 180}{\pi} = 16,875$

$x \text{ —— } \dfrac{3\pi}{32}$

$\dfrac{3\pi}{32}\text{ rad} = 16,875°$

38.6. Graus radianos

$180 \text{ —— } \pi \qquad x = \dfrac{-\dfrac{6\pi}{25}\times 180}{\pi} = -43,2$

$x \text{ —— } -\dfrac{6\pi}{25}$

$-\dfrac{6\pi}{25}\text{ rad} = -43,2°$

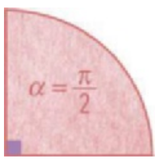
39. $A\hat{C}B = \pi - \dfrac{7\pi}{12} - \dfrac{\pi}{6} = \dfrac{12\pi}{12} - \dfrac{7\pi}{12} - \dfrac{2\pi}{12} = \dfrac{3\pi}{12} = \dfrac{\pi}{4}$

$A\hat{C}B = \dfrac{\pi}{4}\text{ rad} = 45°$

40. $\left(\dfrac{\pi}{4}, 3\right) = (45°, 3) = 45° + 3\times 360° = 1125°$

1125°

41.

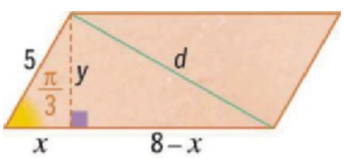


$\alpha r = \pi$

$\dfrac{\pi}{2} r = \pi \Leftrightarrow r = \dfrac{\pi}{\dfrac{\pi}{2}} = 2$

O raio do círculo é 2 cm.

42.



$\cos\left(\dfrac{\pi}{3}\right) = \dfrac{x}{5} \Leftrightarrow \dfrac{1}{2} = \dfrac{x}{5} \Leftrightarrow x = \dfrac{5}{2}$

$8 - x = 8 - \dfrac{5}{2} = \dfrac{11}{2}$

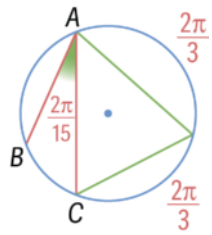
$\sin\left(\dfrac{\pi}{3}\right) = \dfrac{y}{5} \Leftrightarrow \dfrac{\sqrt{3}}{2} = \dfrac{y}{5} \Leftrightarrow y = \dfrac{5\sqrt{3}}{2}$

$d^2 = \left(\dfrac{11}{2}\right)^2 + \left(\dfrac{5\sqrt{3}}{2}\right)^2 \Leftrightarrow d^2 = \dfrac{121}{4} + \dfrac{25\times 3}{4}$

$\Leftrightarrow d^2 = \dfrac{196}{4} \Leftrightarrow d^2 = 49 \underset{(d>0)}{\Leftrightarrow} d = \sqrt{49} \Leftrightarrow d = 7$

O comprimento da diagonal menor do paralelogramo é 7 cm.

43. $\overset{\frown}{AC} = \dfrac{2\pi}{3}$



$\overset{\frown}{BC} = 2\times\dfrac{2\pi}{15} = \dfrac{4\pi}{15}$

$\overset{\frown}{AB} = \underset{(\times 5)}{\dfrac{2\pi}{3}} - \dfrac{4\pi}{15} = \dfrac{6\pi}{15} = \dfrac{2\pi}{5}$

Logo, $[AB]$ é o lado de um pentágono regular inscrito na circunferência.

44.1. $A\hat{D}E = A\hat{D}C - E\hat{D}C = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \dfrac{\pi}{6}$

44.2. $F\hat{D}E = A\hat{D}E + F\hat{D}A = \dfrac{\pi}{6} + \dfrac{\pi}{3} = \dfrac{3\pi}{6} = \dfrac{\pi}{2}$

Logo, o triângulo $[DEF]$ é retângulo.

Como o triângulo $[CDE]$ é equilátero,

$\overline{CE} = \overline{ED} = \overline{CD}$.

Como o triângulo $[ADF]$ é equilátero,

$\overline{AD} = \overline{DF} = \overline{AF}$.

Mas, $\overline{AD} = \overline{CD}$, logo $\overline{ED} = \overline{AD} = \overline{DF}$.

Assim, $\overline{ED} = \overline{DF}$ e o triângulo $[DEF]$ é isósceles.

44.3. $D\hat{E}F = \dfrac{\pi}{4}$ (o triângulo [*DEF*] é retângulo em *D* e isósceles)

$C\hat{E}D = \dfrac{\pi}{3}$ (o triângulo [*CDE*] é equilátero)

$E\hat{C}B = D\hat{C}B - D\hat{C}E = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \dfrac{\pi}{6}$

$C\hat{B}E = B\hat{E}C$ porque o triângulo [*BCE*] é isósceles, sendo $\overline{EC} = \overline{BC}$.

Logo, $B\hat{E}C = \dfrac{1}{2}\left(\pi - E\hat{C}B\right) = \dfrac{1}{2}\left(\pi - \dfrac{\pi}{6}\right) = \dfrac{5\pi}{12}$

$B\hat{E}F = D\hat{E}F + C\hat{E}D + B\hat{E}C = \dfrac{\pi}{4} + \dfrac{\pi}{3} + \dfrac{5\pi}{12} = \pi$

Como o ângulo *BEF* é raso, o ponto *E* pertence à reta *BF*.

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45. $2\pi : 20 = \frac{\pi}{10}$ ou $360^\circ : 20 = 18^\circ$

45.1. $8 \times 18^\circ + 360^\circ = 504^\circ$

$8 \times 18^\circ + 2 \times 360^\circ = 864^\circ$

Por exemplo, 504º e 864º.

45.2. 1.º jogado $810^\circ = (90^\circ, 2)$

Cálculos auxiliares

$810 \,|\, 90$ 90 2 $90 : 18 = 5$

O 1.º jogador obteve 60 pontos (5 setores).

2.º jogado $\frac{16\pi}{5} = \underbrace{2\pi}_{\text{uma volta completa}} + \frac{6}{5}\pi$

Cálculos auxiliares

$16 : 5 = 3{,}2$

$\frac{16\pi}{5} - 2\pi = \frac{6}{5}\pi$

$\frac{6}{5}\pi : \frac{\pi}{10} = 12$

O 2.º jogador obteve 25 pontos (12 setores)

3.º jogado $7\pi = \underbrace{6\pi}_{\text{três voltas completas}} + \pi$

Cálculos auxiliares: $\pi : \frac{\pi}{10} = 10$

O 3.º jogador obteve 5 pontos (10 setores)

O jogador que obteve melhor pontuação foi o 1.º jogador.

46. $2\pi : 12 = \frac{\pi}{6}$

Em cada hora, o ponteiro das horas descreve um arco de amplitude $\frac{\pi}{6}$ rad .

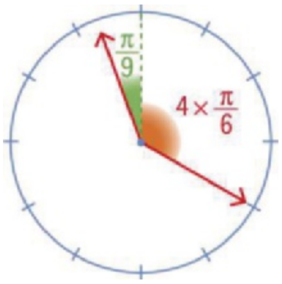
Então,

60 min —— $\frac{\pi}{6}$ rad $x = \frac{20 \times \frac{\pi}{6}}{60} = \frac{\pi}{18}$

20 min —— x

$\frac{\pi}{6} - \frac{\pi}{18} = \frac{\pi}{9}$

$\frac{\pi}{9} + 4 \times \frac{\pi}{6} = \frac{7\pi}{9}$



Às 11 h 20 m, os ponteiros formam um ângulo de amplitude $\frac{7\pi}{9}$ rad .

47.1. $\tan\alpha = \frac{\overline{BC}}{\overline{AB}} = \frac{\overline{BC}}{2\overline{BC}} = \frac{1}{2}$

47.2. $\alpha = \tan^{-1}\left(\frac{1}{2}\right) \approx 0{,}46$ rad

47.3. $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

Então, $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1 + \frac{1}{4} = \frac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow \frac{5}{4} = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{4}{5}$

Como α é um angulo agudo, $\cos\alpha > 0$, pelo que:

$\cos\alpha = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

48.1. $\sin^2\alpha + \cos^2\alpha = 1$

Então, $\left(-\frac{1}{3}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow \frac{1}{9} + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2\alpha = \frac{8}{9}$

Como $\alpha \in$ 3.º quadrante, $\cos\alpha < 0$, pelo que,

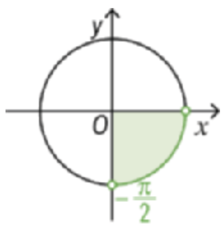
$\cos\alpha = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

48.2. $\sin^2\alpha + \cos^2\alpha = 1$

Então,

$\sin^2\alpha + \left(\frac{1}{5}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{1}{25} \Leftrightarrow \sin^2\alpha = \frac{24}{25}$



Como $\alpha \in$ 4.º quadrante, $\sin\alpha < 0$, pelo que,

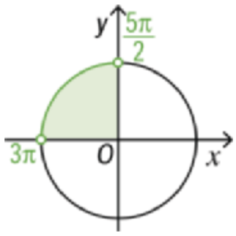
$\sin\alpha = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = -2\sqrt{6}$

48.3. $1+\tan^2\alpha=\dfrac{1}{\cos^2\alpha}$

Então, $1+\left(-\dfrac{24}{7}\right)^2=\dfrac{1}{\cos^2\alpha}\Leftrightarrow 1+\dfrac{576}{49}=\dfrac{1}{\cos^2\alpha}\Leftrightarrow$

$\Leftrightarrow\dfrac{625}{49}=\dfrac{1}{\cos^2\alpha}\Leftrightarrow\cos^2\alpha=\dfrac{49}{625}$



Como $\alpha\in 2.^{\text{o}}$ quadrante, $\cos\alpha<0$ pelo que

$\cos\alpha=-\sqrt{\dfrac{49}{625}}=-\dfrac{7}{25}$

$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}\Leftrightarrow-\dfrac{24}{7}=\dfrac{\sin\alpha}{-\dfrac{7}{25}}\Leftrightarrow$

$\Leftrightarrow\sin\alpha=-\dfrac{24}{7}\times\left(-\dfrac{7}{25}\right)\Leftrightarrow\sin\alpha=\dfrac{24}{25}$

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49.1. $T_1(1,\tan\alpha)$ e $T_1(1,2)$

Então $\tan\alpha=2$

$1+\tan^2\alpha=\dfrac{1}{\cos^2\alpha}$

$1+2^2=\dfrac{1}{\cos^2\alpha}\Leftrightarrow 1+4=\dfrac{1}{\cos^2\alpha}\Leftrightarrow 5=\dfrac{1}{\cos^2\alpha}\Leftrightarrow$

$\Leftrightarrow\cos^2\alpha=\dfrac{1}{5}$

Como $\alpha\in 1.^{\text{o}}$ quadrante, $\cos\alpha>0$, pelo que,

$\cos\alpha=\sqrt{\dfrac{1}{5}}=\dfrac{1}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$

$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}\Leftrightarrow 2=\dfrac{\sin\alpha}{\dfrac{\sqrt{5}}{5}}\Leftrightarrow\sin\alpha=\dfrac{2\sqrt{5}}{5}$

49.2. $P_2(\cos\beta,\sin\beta)$ e $P_2\left(\cos\beta,\dfrac{3}{4}\right)$

Então $\sin\beta=\dfrac{3}{4}$

$\sin^2\beta+\cos^2\beta=1$

Então,

$\left(\dfrac{3}{4}\right)^2+\cos^2\beta=1\Leftrightarrow\cos^2\beta=1-\dfrac{9}{16}\Leftrightarrow\cos^2\beta=\dfrac{7}{16}$

Como $\beta\in 2.^{\text{o}}$ quadrante, $\cos\beta<0$, pelo que,

$\cos\beta=-\sqrt{\dfrac{7}{16}}=-\dfrac{\sqrt{7}}{4}$

$\tan\beta=\dfrac{\sin\beta}{\cos\beta}=\dfrac{\dfrac{3}{4}}{-\dfrac{\sqrt{7}}{4}}=-\dfrac{3}{\sqrt{7}}=-\dfrac{3\sqrt{7}}{7}$

49.3. $P_3(\cos\theta,\sin\theta)$ e $P_3\left(-\dfrac{7}{9},\sin\theta\right)$

Então $\cos\theta=-\dfrac{7}{9}$

$\sin^2\theta+\cos^2\theta=1$

Então,

$\sin^2\theta+\left(-\dfrac{7}{9}\right)^2=1\Leftrightarrow\sin^2\theta=1-\dfrac{49}{81}\Leftrightarrow\sin^2\theta=\dfrac{32}{81}$

Como $\theta\in 3.^{\text{o}}$ quadrante, $\sin\theta<0$, pelo que,

$\sin\theta=-\sqrt{\dfrac{32}{81}}=-\dfrac{4\sqrt{2}}{9}$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{-\dfrac{4\sqrt{2}}{9}}{-\dfrac{7}{9}}=\dfrac{4\sqrt{2}}{7}$

49.4. $T_4(1,\tan\varphi)$ e $T_4\left(1,-\dfrac{3}{4}\right)$

Então $\tan\varphi=-\dfrac{3}{4}$

$1+\tan^2\varphi=\dfrac{1}{\cos^2\varphi}\Leftrightarrow\dfrac{25}{16}=\dfrac{1}{\cos^2\varphi}\Leftrightarrow\cos^2\varphi=\dfrac{16}{25}$

Como $\varphi\in 4.^{\text{o}}$ quadrante, $\cos\varphi>0$, pelo que,

$\cos\varphi=\sqrt{\dfrac{16}{25}}=\dfrac{4}{5}$

$\tan\varphi=\dfrac{\sin\varphi}{\cos\varphi}\Leftrightarrow-\dfrac{3}{4}=\dfrac{\sin\varphi}{\dfrac{4}{5}}\Leftrightarrow\sin\varphi=-\dfrac{3}{4}\times\dfrac{4}{5}\Leftrightarrow$

$\Leftrightarrow\sin\varphi=-\dfrac{3}{5}$

50.1. $\dfrac{\sin^2 x}{1-\cos x}=\dfrac{1-\cos^2 x}{1-\cos x}=\dfrac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}}=$

$=1+\cos x$

Cálculos auxiliares:

$\sin^2 x+\cos^2 x=1\Leftrightarrow\sin^2 x=1-\cos^2 x$

50.2. $\dfrac{1+\sin x}{\cos x}=\dfrac{(1+\sin x)(1-\sin x)}{\cos x(1-\sin x)}=\dfrac{1-\sin^2 x}{\cos x(1-\sin x)}=$

$=\dfrac{\cos^2 x}{\cos x(1-\sin x)}=\dfrac{\cos x}{1-\sin x}$

51. Seja α o ângulo formado pelo semieixo positivo Ox e a semirreta $\dot{O}P$.

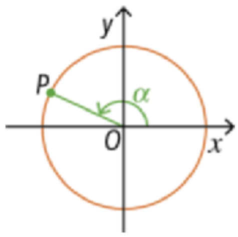
$T(1, \tan\alpha)$ e $T\left(1, -\frac{\sqrt{3}}{3}\right)$

Então, $\tan\alpha = -\frac{\sqrt{3}}{3}$.

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

Então, $1 + \left(-\frac{\sqrt{3}}{3}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow 1 + \frac{3}{9} = \frac{1}{\cos^2\alpha} \Leftrightarrow \frac{12}{9} = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{3}{4}$



Como $\alpha \in 2.^\text{o}$ quadrante, $\cos\alpha < 0$, pelo que,

$\cos\alpha = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow -\frac{\sqrt{3}}{3} = \frac{\sin\alpha}{-\frac{\sqrt{3}}{2}} \Leftrightarrow$

$\Leftrightarrow \sin\alpha = -\frac{\sqrt{3}}{3} \times \left(-\frac{\sqrt{3}}{2}\right) \Leftrightarrow \sin\alpha = \frac{3}{6} \Leftrightarrow \sin\alpha = \frac{1}{2}$

$P(\cos\alpha, \sin\alpha) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Opção (D)

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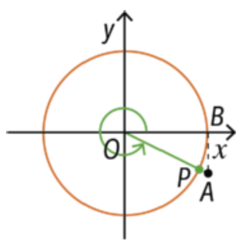
52. Seja α o ângulo formado pelo semieixo positivo Ox e a semirreta $\dot{O}P$.

$P(\cos\alpha, \sin\alpha)$ e $P\left(\cos\alpha, -\frac{1}{3}\right)$

Então, $\sin\alpha = -\frac{1}{3}$.



$\sin^2\alpha + \cos^2\alpha = 1$

Então,

$\left(-\frac{1}{3}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2\alpha = \frac{8}{9}$

Como $\alpha \in 4.^\text{o}$ quadrante, $\cos\alpha > 0$, pelo que,

$\cos\alpha = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow \tan\alpha = \frac{-\frac{1}{3}}{\frac{2\sqrt{2}}{3}} \Leftrightarrow \tan\alpha = -\frac{1}{2\sqrt{2}} \Leftrightarrow$

$\Leftrightarrow \tan\alpha = -\frac{\sqrt{2}}{4}$

$A(1, \tan\alpha) = \left(1, -\frac{\sqrt{2}}{4}\right)$

Como $[OABC]$ é um paralelogramo,

$\overline{OC} = \overline{AB} = \frac{\sqrt{2}}{4}$. Logo, $C\left(0, \frac{\sqrt{2}}{4}\right)$.

53.1. $\sin\alpha + \cos\alpha = \frac{1}{5} \Leftrightarrow \sin\alpha = \frac{1}{5} - \cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \left(\frac{1}{5} - \cos\alpha\right)^2 + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \frac{1}{25} - \frac{2}{5}\cos\alpha + \cos^2\alpha + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow 2\cos^2\alpha - \frac{2}{5}\cos\alpha - \frac{24}{25} = 0 \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \frac{\frac{2}{5} \pm \sqrt{\left(-\frac{2}{5}\right)^2 - 4 \times 2 \times \left(-\frac{24}{25}\right)}}{2 \times 2} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \frac{\frac{2}{5} \pm \frac{14}{5}}{4} \Leftrightarrow \cos\alpha = -\frac{3}{5} \vee \cos\alpha = \frac{4}{5}$

Como $\cos\alpha < 0$, $\cos\alpha = -\frac{3}{5}$

$\sin\alpha = \frac{1}{5} - \cos\alpha \Leftrightarrow \sin\alpha = \frac{1}{5} - \left(-\frac{3}{5}\right) \Leftrightarrow \sin\alpha = \frac{4}{5}$

$\sin\alpha = \frac{4}{5}$ e $\cos\alpha = -\frac{3}{5}$

53.2. $\sin\alpha > 0$. Então, $\alpha \in 1.^\text{o}$ Q ou $\alpha \in 2.^\text{o}$ Q

$\cos\alpha < 0$. Então, $\alpha \in 2.^\text{o}$ Q ou $\alpha \in 3.^\text{o}$ Q

Logo, $\alpha \in 2.^\text{o}$ Q.

54.1. $\frac{1-\tan^2\alpha}{1+\tan^2\alpha}+\sin^2\alpha=\frac{1-\left(\frac{\sin\alpha}{\cos\alpha}\right)^2}{\frac{1}{\cos^2\alpha}}+\sin^2\alpha=$

$=\frac{1-\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}+\sin^2\alpha=\frac{\frac{\cos^2\alpha-\sin^2\alpha}{\cancel{\cos^2\alpha}}}{\frac{1}{\cancel{\cos^2\alpha}}}+\sin^2\alpha=$

$=\cos^2\alpha-\cancel{\sin^2\alpha}+\cancel{\sin^2\alpha}=\cos^2\alpha$

54.2. $\sin\alpha\times\cos\alpha\times\left(\tan\alpha+\frac{1}{\tan\alpha}\right)=$

$=\sin\alpha\times\cos\alpha\times\left(\frac{\sin\alpha}{\cos\alpha}+\frac{1}{\frac{\sin\alpha}{\cos\alpha}}\right)=$

$=\sin\alpha\times\cos\alpha\times\left(\underset{\times\sin\alpha}{\frac{\sin\alpha}{\cos\alpha}}+\frac{1}{\underset{\times\cos\alpha}{\frac{\sin\alpha}{\cos\alpha}}}\right)=$

$=\cancel{\sin\alpha\times\cos\alpha}\times\left(\frac{\sin^2\alpha+\cos^2}{\cancel{\sin\alpha\times\cos\alpha}}\right)=\sin^2\alpha+\cos^2=1$

54.3. $\left(2-\frac{1}{\cos^2\alpha}\right)\left(1-\sin^2\alpha\right)=$

$=2-2\sin^2\alpha-\frac{1}{\cos^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}=$

$=2-2\left(1-\cos^2\alpha\right)-\left(1+\tan^2\alpha\right)+\left(\frac{\sin\alpha}{\cos\alpha}\right)^2=$

$=2-2+2\cos^2\alpha-1-\cancel{\tan^2\alpha}+\cancel{\tan^2\alpha}=$

$=2\cos^2\alpha-1$

55.1. $\sin\frac{7\pi}{6}=\sin\left(\frac{6\pi}{6}+\frac{\pi}{6}\right)=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

55.2. $\cos\left(-\frac{5\pi}{3}\right)=\cos\frac{5\pi}{3}=\cos\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)=$

$=\cos\left(2\pi-\frac{\pi}{3}\right)=\cos\left(-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$

55.3. $\tan\frac{4\pi}{3}=\tan\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)=\tan\left(\pi+\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

55.4. $\sin\left(-\frac{5\pi}{4}\right)=-\sin\frac{5\pi}{4}=-\sin\left(\frac{4\pi}{4}+\frac{\pi}{4}\right)=$

$=-\sin\left(\pi+\frac{\pi}{4}\right)=-\left(-\sin\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

55.5. $\cos\frac{3\pi}{4}=\cos\left(\frac{4\pi}{4}-\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)=$

$=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

55.6. $\tan\left(-\frac{11\pi}{6}\right)=-\tan\frac{11\pi}{6}=-\tan\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)=$

$=\tan\left(2\pi-\frac{\pi}{6}\right)=-\tan\left(-\frac{\pi}{6}\right)=\tan\frac{\pi}{6}=\frac{\sqrt{3}}{3}$

55.7. $\sin\left(-\frac{7\pi}{4}\right)=-\sin\frac{7\pi}{4}=-\sin\left(\frac{8\pi}{4}-\frac{\pi}{4}\right)=$

$=\sin\left(2\pi-\frac{\pi}{4}\right)=-\sin\left(-\frac{\pi}{4}\right)=-\left(-\sin\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=$

$=\frac{\sqrt{2}}{2}$

55.8. $\cos\frac{5\pi}{6}=\cos\left(\frac{6\pi}{6}-\frac{\pi}{6}\right)=\cos\left(\pi-\frac{\pi}{6}\right)=-\cos\frac{\pi}{6}=$

$=-\frac{\sqrt{3}}{2}$

55.9. $\tan\left(-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}=-\frac{\sqrt{3}}{3}$

55.10. $\sin\frac{5\pi}{3}=\sin\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)=\sin\left(2\pi-\frac{\pi}{3}\right)=\sin\left(-\frac{\pi}{3}\right)=$

$=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

55.11. $\cos\left(-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$

55.12. $\tan\left(-\frac{2\pi}{3}\right)=-\tan\frac{2\pi}{3}=-\tan\left(\frac{3\pi}{3}-\frac{\pi}{3}\right)=$

$=\tan\left(\pi-\frac{\pi}{3}\right)=-\left(-\tan\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

55.13. $\sin\left(-\frac{5\pi}{6}\right)=-\sin\left(\frac{5\pi}{6}\right)=-\sin\left(\frac{6\pi}{6}-\frac{\pi}{6}\right)=$

$=-\sin\left(\pi-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

55.14. $\cos\frac{11\pi}{6}=\cos\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)=\cos\left(2\pi-\frac{\pi}{6}\right)=$

$=\cos\left(-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

55.15. $\tan\left(-\frac{\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$

55.16. $\sin\left(-\frac{4\pi}{3}\right)=-\sin\frac{4\pi}{3}=-\sin\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)=$

$=-\sin\left(\pi+\frac{\pi}{3}\right)=-\left(-\sin\frac{\pi}{3}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

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56.1. $P(\cos 30^\circ, \sin 30^\circ)$; $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$M(1, \tan 30^\circ)$; $M\left(1, \frac{\sqrt{3}}{3}\right)$

$Q(-\cos 30^\circ, -\sin 30^\circ)$; $Q\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

56.2. $\sin 210^\circ=\sin(180^\circ+30^\circ)=-\sin 30^\circ=-\frac{1}{2}$

$\cos 210^\circ=\cos(180^\circ+30^\circ)=-\cos 30^\circ=-\frac{\sqrt{3}}{2}$

56.3. $R(\cos 150^\circ, \sin 150^\circ)$; $R\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Cálculos auxiliares:

$150^\circ=180^\circ-30^\circ$

$\cos 150^\circ=\cos(180^\circ-30^\circ)=-\cos 30^\circ=-\frac{\sqrt{3}}{2}$

$\sin 150^\circ=\sin(180^\circ-30^\circ)=\sin 30^\circ=\frac{1}{2}$

56.4. $\sin 330^\circ=\sin(360^\circ-30^\circ)=\sin(-30^\circ)=$

$=-\sin 30^\circ=-\frac{1}{2}$

$\cos 330^\circ=\cos(360^\circ-30^\circ)=\cos(-30^\circ)=$

$=\cos 30^\circ=\frac{\sqrt{3}}{2}$

56.5. $N(1, \tan 330^\circ)$; $N\left(1, -\frac{\sqrt{3}}{3}\right)$

Cálculos auxiliares:

$\tan 330^\circ=\tan(360^\circ-30^\circ)=$

$=\tan(-30^\circ)=-\tan 30^\circ=-\frac{\sqrt{3}}{3}$

57.1. $\cos(\pi-\alpha)-2\sin(\alpha-\pi)+\cos\left(\frac{\pi}{2}-\alpha\right)=$

$=-\cos\alpha+2\sin(\pi-\alpha)+\sin\alpha=$

$=-\cos\alpha+2\sin\alpha+\sin\alpha=3\sin\alpha-\cos\alpha$

57.2. $\tan(3\pi+\alpha)+\frac{\cos\left(\frac{\pi}{2}+\alpha\right)}{\sin\left(\frac{\pi}{2}+\alpha\right)}-\tan(-\pi-\alpha)=$

$=\tan(2\pi+\pi+\alpha)+\frac{-\sin\alpha}{\cos\alpha}-(-\tan(\pi+\alpha))=$

$=\tan(\pi+\alpha)-\frac{\sin\alpha}{\cos\alpha}+\tan(\pi+\alpha)=$

$=\cancel{\tan\alpha}-\cancel{\tan\alpha}+\tan\alpha=\tan\alpha$

57.3. $\sin\left(\frac{\pi}{2}-\alpha\right)+\cos(\alpha-3\pi)+2\sin\left(\alpha-\frac{\pi}{2}\right)+$

$+3\sin(\pi+\alpha)\times\tan\left(\frac{\pi}{2}-\alpha\right)=$

$=\cos\alpha+\cos(2\pi+\pi-\alpha)-2\sin\left(\frac{\pi}{2}-\alpha\right)+$

$+3(-\sin\alpha)\times\frac{\sin\alpha}{\cos\alpha}=$

$=\cos\alpha+\cos(\pi-\alpha)-2\cos\alpha-3\cancel{\sin\alpha}\times\frac{\cos\alpha}{\cancel{\sin\alpha}}=$

$=\cancel{\cos\alpha}-\cancel{\cos\alpha}-2\cos\alpha-3\cos\alpha=-5\cos\alpha$

58.1. $\left(\sin\frac{4\pi}{3}-\cos\frac{11\pi}{6}\right)\tan\left(-\frac{5\pi}{6}\right)=$

$=\left[\sin\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)-\cos\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)\right]\left(-\tan\frac{5\pi}{6}\right)=$

$=\left[\sin\left(\pi+\frac{\pi}{3}\right)-\cos\left(2\pi-\frac{\pi}{6}\right)\right]\left[-\tan\left(\frac{6\pi}{6}-\frac{\pi}{6}\right)\right]=$

$=\left[-\sin\frac{\pi}{3}-\cos\left(-\frac{\pi}{6}\right)\right]\left[-\tan\left(\pi-\frac{\pi}{6}\right)\right]=$

$=\left(-\sin\frac{\pi}{3}-\cos\frac{\pi}{6}\right)\left[-\left(-\tan\frac{\pi}{6}\right)\right]=$

$=\left(-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)\times\frac{\sqrt{3}}{3}=-\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\times\frac{\sqrt{3}}{3}=$

$=-\sqrt{3}\times\frac{\sqrt{3}}{3}=-1$

58.2. $\sqrt{2}\sin\frac{7\pi}{4}+\tan\frac{9\pi}{4}+\cos\frac{5\pi}{2}=$

$=\sqrt{2}\sin\left(\frac{8\pi}{4}-\frac{\pi}{4}\right)+\tan\left(\frac{8\pi}{4}+\frac{\pi}{4}\right)+\cos\left(\frac{4\pi}{2}+\frac{\pi}{2}\right)=$

$=\sqrt{2}\sin\left(2\pi-\frac{\pi}{4}\right)+\tan\left(2\pi+\frac{\pi}{4}\right)+\cos\left(2\pi+\frac{\pi}{2}\right)=$

$=\sqrt{2}\sin\left(-\frac{\pi}{4}\right)+\tan\frac{\pi}{4}+\cos\frac{\pi}{2}=-\sqrt{2}\sin\frac{\pi}{4}+1+0=$

$=-\sqrt{2}\times\frac{\sqrt{2}}{2}+1=-1+1=0$

58.3.

$$\frac{\cos\left(\frac{11\pi}{4}\right)\times\sin\left(-\frac{7\pi}{6}\right)}{\sqrt{2}\sin^2\frac{\pi}{5}+\sqrt{2}\cos^2\frac{\pi}{5}}=$$

$$=\frac{\cos\left(\frac{12\pi}{4}-\frac{\pi}{4}\right)\times\left[-\sin\left(\frac{6\pi}{6}+\frac{\pi}{6}\right)\right]}{\sqrt{2}\left(\sin^2\frac{\pi}{5}+\cos^2\frac{\pi}{5}\right)}=$$

$$=\frac{\cos\left(3\pi-\frac{\pi}{4}\right)\times\left[-\sin\left(\pi+\frac{\pi}{6}\right)\right]}{\sqrt{2}\times 1}=$$

$$=\frac{\cos\left(2\pi+\pi-\frac{\pi}{4}\right)\times\left[-\left(-\sin\frac{\pi}{6}\right)\right]}{\sqrt{2}}=$$

$$=\frac{\cos\left(\pi-\frac{\pi}{4}\right)\times\sin\frac{\pi}{6}}{\sqrt{2}}=\frac{-\cos\frac{\pi}{4}\times\sin\frac{\pi}{6}}{\sqrt{2}}=$$

$$=\frac{-\frac{\sqrt{2}}{2}\times\frac{1}{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{\underset{(\times\sqrt{2})}{4\sqrt{2}}}=-\frac{2}{4\times 2}=-\frac{1}{4}$$

58.4.

$$\frac{2\cos\frac{11\pi}{3}+\tan\frac{4\pi}{3}}{3\tan\left(-\frac{11\pi}{6}\right)+2\sin\frac{11\pi}{6}}=$$

$$=\frac{2\cos\left(\frac{12\pi}{3}-\frac{\pi}{3}\right)+\tan\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)}{-3\tan\frac{11\pi}{6}+2\sin\frac{11\pi}{6}}=$$

$$=\frac{2\cos\left(4\pi-\frac{\pi}{3}\right)+\tan\left(\pi+\frac{\pi}{3}\right)}{-3\tan\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)+2\sin\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)}=$$

$$=\frac{2\cos\left(-\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{3}\right)}{-3\tan\left(2\pi-\frac{\pi}{6}\right)+2\sin\left(2\pi-\frac{\pi}{6}\right)}=$$

$$=\frac{2\cos\frac{\pi}{3}+\sqrt{3}}{3\tan\frac{\pi}{6}-2\sin\frac{\pi}{6}}=\frac{1+\sqrt{3}}{\cancel{3}\times\frac{\sqrt{3}}{\cancel{3}}-\cancel{2}\times\frac{1}{\cancel{2}}}=\frac{1+\sqrt{3}}{\sqrt{3}-1}=$$

$$=\frac{\left(1+\sqrt{3}\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}=\frac{\left(\sqrt{3}\right)^2+2\times1\times\sqrt{3}+1^2}{\left(\sqrt{3}\right)^2-1^2}=$$

$$=\frac{3+2\sqrt{3}+1}{3-1}=\frac{4+2\sqrt{3}}{2}=\sqrt{3}+2$$

58.5. $\sin\left(-\frac{2\pi}{3}\right)-\cos\frac{17\pi}{6}-\tan\left(-\frac{5\pi}{4}\right)=$

$$=-\sin\frac{2\pi}{3}-\cos\left(\frac{18\pi}{6}-\frac{\pi}{6}\right)+\tan\frac{5\pi}{4}=$$

$$=-\sin\left(\frac{3\pi}{3}-\frac{\pi}{3}\right)-\cos\left(3\pi-\frac{\pi}{6}\right)+\tan\left(\frac{4\pi}{4}+\frac{\pi}{4}\right)=$$

$$=-\sin\left(\pi-\frac{\pi}{3}\right)-\cos\left(2\pi+\pi-\frac{\pi}{6}\right)+\tan\left(\pi+\frac{\pi}{4}\right)=$$

$$=-\sin\frac{\pi}{3}-\cos\left(\pi-\frac{\pi}{6}\right)+\tan\frac{\pi}{4}=$$

$$=-\frac{\sqrt{3}}{2}-\left(-\cos\frac{\pi}{6}\right)+1=-\frac{\sqrt{3}}{2}+\cos\frac{\pi}{6}+1=$$

$$=-\cancel{\frac{\sqrt{3}}{2}}+\cancel{\frac{\sqrt{3}}{2}}+1=1$$

58.6. $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}-\tan^2\frac{7\pi}{4}=$

$$=\sin^2\frac{\pi}{8}+\left[\sin\left(\frac{4\pi}{8}-\frac{\pi}{8}\right)\right]^2-\left[\tan\left(\frac{8\pi}{4}-\frac{\pi}{4}\right)\right]^2=$$

$$=\sin^2\frac{\pi}{8}+\left[\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right]^2-\left[\tan\left(2\pi-\frac{\pi}{4}\right)\right]^2=$$

$$=\sin^2\frac{\pi}{8}+\left(\cos\frac{\pi}{8}\right)^2-\left[\tan\left(-\frac{\pi}{4}\right)\right]^2=$$

$$=\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}-\left(-\tan\frac{\pi}{4}\right)^2=1-\tan^2\frac{\pi}{4}=1-1=0$$

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59.1. $\cos^2\frac{5\pi}{11}+\cos^2\frac{\pi}{22}+\left(\cos\frac{7\pi}{12}\right)^2+\cos^2\frac{\pi}{12}=$

$$=\left(\cos\frac{10\pi}{22}\right)^2+\cos^2\frac{\pi}{22}+\left(\cos\frac{7\pi}{12}\right)^2+\cos^2\frac{\pi}{12}=$$

$$=\left[\cos\left(\frac{11\pi}{22}-\frac{\pi}{22}\right)\right]^2+\cos^2\frac{\pi}{22}+\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\right]^2+$$

$$+\cos^2\frac{\pi}{12}=$$

$$=\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\right]^2+\cos^2\frac{\pi}{22}+\left(-\sin\frac{\pi}{12}\right)^2+$$

$$+\cos^2\frac{\pi}{12}=$$

$$=\left(\sin\frac{\pi}{22}\right)^2+\cos^2\frac{\pi}{22}+\sin^2\frac{\pi}{22}+\cos^2\frac{\pi}{22}=$$

$$=\sin^2\frac{\pi}{22}+\cos^2\frac{\pi}{22}+1=1+1=2$$

59.2. $\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}+\sin\frac{14\pi}{9}+\sin\frac{13\pi}{9}=$

$=\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}+\sin\left(\frac{9\pi}{9}+\frac{5\pi}{9}\right)+\sin\left(\frac{9\pi}{9}+\frac{4\pi}{9}\right)=$

$=\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}+\sin\left(\pi+\frac{5\pi}{9}\right)+\sin\left(\pi+\frac{4\pi}{9}\right)=$

$=\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}-\sin\frac{5\pi}{9}-\sin\frac{4\pi}{9}=0$

59.3. $\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}+\cos\frac{14\pi}{25}+\cos\frac{15\pi}{26}=$

$=\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}+\cos\left(\frac{25\pi}{25}-\frac{11\pi}{25}\right)+$

$+\cos\left(\frac{13\pi}{26}+\frac{2\pi}{26}\right)=$

$=\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}+\cos\left(\pi-\frac{11\pi}{25}\right)+\cos\left(\frac{\pi}{2}+\frac{\pi}{13}\right)=$

$=\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}-\cos\frac{11\pi}{25}-\sin\frac{\pi}{13}=0$

60.1. $\left[\sin\left(\frac{\pi}{2}+x\right)+\sin(x-\pi)\right]^2-1=$

$=\left[\cos x-(-\sin(\pi-x))\right]^2-1=$

$=\left[\cos x-(-\sin x)\right]^2-1=(\cos x+\sin x)^2-1=$

$=\cos^2 x+2\sin x\cos x+\sin^2 x-1=$

$=\cos^2 x+\sin^2 x-1+2\sin x\cos x=$

$=1-1+2\sin x\cos x=2\sin x\cos x$

60.2. $\cos\left(\frac{5\pi}{2}+x\right)\times\sin(\pi+x)\times\tan\left(\frac{\pi}{2}-x\right)=$

$=\cos\left(\frac{4\pi}{2}+\frac{\pi}{2}+x\right)\times(-\sin x)\times\frac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}=$

$=\cos\left(2\pi+\frac{\pi}{2}+x\right)\times(-\sin x)\times\frac{\cos x}{\sin x}=$

$=\cos\left(\frac{\pi}{2}+x\right)\times(-\cos x)=-\sin x\times(-\cos x)=$

$=\sin x\cos x$

61. $1+\cos(x-\pi)\times\sin\left(x-\frac{3\pi}{2}\right)=$

$=1+\cos(\pi-x)\times\sin\left[-\left(\frac{3\pi}{2}-x\right)\right]=$

$=1+\cos(\pi-x)\times\left[-\sin\left(\frac{3\pi}{2}-x\right)\right]=$

$=1-\cos x\times\left[-(-\cos x)\right]=$

$=1-\cos x\times\cos x=1-\cos^2 x=\sin^2 x$

Cálculos auxiliares:

$\sin^2 x+\cos^2 x=1\Leftrightarrow\sin^2 x=1-\cos^2 x$

62.1. $\cos\left(\frac{\pi}{2}+x\right)=\frac{12}{13}\Leftrightarrow-\sin x=\frac{12}{13}\Leftrightarrow\sin x=-\frac{12}{13}$

$x\in\left]\pi,\frac{3\pi}{2}\right[$

$13\sin\left(x-\frac{\pi}{2}\right)-5\tan(20+x)=-13\cos x-5\tan x=$

$=-13\times\left(-\frac{5}{13}\right)-5\times\frac{12}{5}=5-12=-7$

Cálculos auxiliares:

$\sin^2 x+\cos^2 x=1\Leftrightarrow\cos^2 x=1-\sin^2 x\Leftrightarrow$

$\Leftrightarrow\cos^2 x=1-\left(-\frac{12}{13}\right)^2\Leftrightarrow\cos^2 x=1-\frac{144}{169}\Leftrightarrow$

$\Leftrightarrow\cos^2 x=\frac{25}{169}\Leftrightarrow\cos x=\pm\frac{5}{13}$

Como $x\in 3.^{\circ}\text{Q}$, $\cos x<0$.

$\cos x=-\frac{5}{13}$

$\tan x=\frac{\sin x}{\cos x}\Leftrightarrow\tan x=\frac{-\frac{12}{13}}{-\frac{5}{13}}\Leftrightarrow\tan x=\frac{12}{5}$

62.2. $x\in\left]0,\pi\right[$

$3\sin\left(x-\frac{\pi}{2}\right)-2=0\Leftrightarrow3\left[-\sin\left(\frac{\pi}{2}-x\right)\right]-2=0\Leftrightarrow$

$\Leftrightarrow3(-\cos x)-2=0\Leftrightarrow-3\cos x=2\Leftrightarrow\cos x=-\frac{2}{3}$

$\tan(\pi-x)+\cos\left(\frac{\pi}{2}+x\right)=-\tan x-\sin x=$

$=-\left(-\frac{\sqrt5}{2}\right)-\frac{\sqrt5}{3}=\underset{(\times3)}{\frac{\sqrt5}{2}}-\underset{(\times2)}{\frac{\sqrt5}{3}}=\frac{\sqrt5}{6}$

Cálculos auxiliares:

$\sin^2 x+\cos^2 x=1\Leftrightarrow\sin^2 x=1-\cos^2 x\Leftrightarrow$

$\Leftrightarrow\sin^2 x=1-\left(-\frac{2}{3}\right)^2\Leftrightarrow\sin^2 x=1-\frac{4}{9}\Leftrightarrow$

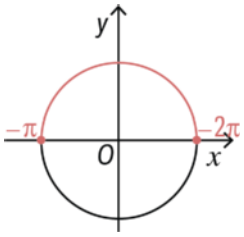
$\Leftrightarrow\sin^2 x=\frac{5}{9}\Leftrightarrow\sin x=\pm\frac{\sqrt5}{3}$

Como $x\in\left]0,\pi\right[$, $\sin x>0$

$\sin x=\frac{\sqrt5}{3}$

$$\tan x = \frac{\sin x}{\cos x} \Leftrightarrow \tan x = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} \Leftrightarrow \tan x = -\frac{\sqrt{5}}{2}$$

62.3. $\sin\left(\frac{3\pi}{2} - x\right) = \frac{5}{8}$



$x \in [-2\pi, -\pi]$

$x \in$ 1.º Q ou 2.º Quadrante

$$\sin\left(\frac{3\pi}{2} - x\right) = \frac{5}{8} \Leftrightarrow -\cos x = \frac{5}{8} \Leftrightarrow \cos x = -\frac{5}{8} \rightarrow x \in 2.º\text{Q}$$

$$\cos\left(-x - \frac{3\pi}{2}\right) + \tan(-x) = \cos\left(x + \frac{3\pi}{2}\right) - \tan x =$$

$$= \sin x - \tan x = \frac{\sqrt{39}}{8} - \left(-\frac{\sqrt{39}}{5}\right) = \underset{(\times 5)}{\frac{\sqrt{39}}{8}} + \underset{(\times 8)}{\frac{\sqrt{39}}{5}} = \frac{13\sqrt{39}}{40}$$

Cálculos auxiliares:

$$\sin^2 x + \cos^2 x = 1 \Leftrightarrow \sin^2 x = 1 - \cos^2 x \Leftrightarrow$$

$$\Leftrightarrow \sin^2 x = 1 - \left(-\frac{5}{8}\right)^2 \Leftrightarrow \sin^2 x = 1 - \frac{25}{64} \Leftrightarrow$$

$$\Leftrightarrow \sin^2 x = \frac{39}{64} \Leftrightarrow \sin x = \pm\frac{\sqrt{39}}{8}$$

Como $x \in$ 2.º quadrante, $\sin x > 0$

$$\sin x = \frac{\sqrt{39}}{8}$$

$$\tan x = \frac{\sin x}{\cos x} \Leftrightarrow \tan x = \frac{\frac{\sqrt{39}}{8}}{-\frac{5}{8}} \Leftrightarrow \tan x = -\frac{\sqrt{39}}{5}$$

63.1. a) $\sin\beta = \frac{\overline{BC}}{\overline{CE}} = \frac{4}{2\sqrt{5}} = \underset{(\times\sqrt{5})}{\frac{2}{\sqrt{5}}} = \frac{2\sqrt{5}}{5}$

b) $\cos\alpha = \cos(\pi - \beta) = -\cos\beta = -\frac{\sqrt{5}}{5}$

Cálculos auxiliares:

$$\sin^2 \beta + \cos^2 \beta = 1 \Leftrightarrow \cos^2 \beta = 1 - \sin^2 \beta \Leftrightarrow$$

$$\Leftrightarrow \cos^2 \beta = 1 - \left(\frac{2\sqrt{5}}{5}\right)^2 \Leftrightarrow \cos^2 \beta = 1 - \frac{20}{25} \Leftrightarrow$$

$$\Leftrightarrow \cos^2 \beta = \frac{5}{25} \Leftrightarrow \cos^2 \beta = \frac{1}{5} \Leftrightarrow \cos\beta = \pm\sqrt{\frac{1}{5}} \Leftrightarrow$$

$$\Leftrightarrow \cos\beta = \pm\frac{\sqrt{5}}{5}$$

Como β é agudo, $\cos\beta > 0$.

$$\cos\beta = \frac{\sqrt{5}}{5}$$

c) <u>1.º Processo</u>

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \Leftrightarrow \tan^2 \alpha + 1 = \frac{1}{\left(-\frac{\sqrt{5}}{5}\right)^2} \Leftrightarrow$$

$$\Leftrightarrow \tan^2 \alpha + 1 = \frac{1}{\frac{5}{25}} \Leftrightarrow \tan^2 \alpha + 1 = 5 \Leftrightarrow$$

$$\Leftrightarrow \tan^2 \alpha = 4 \Leftrightarrow \tan\alpha = \pm 2$$

Como α é um angulo obtuso, $\tan\alpha < 0$.

$$\tan\alpha = -2$$

<u>2.º Processo</u>

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{2\sqrt{5}}{5}}{\frac{\sqrt{5}}{5}} = 2$$

$$\tan\alpha = \tan(\pi - \beta) = -\tan\beta = -2$$

$$\alpha + \beta = \pi \Leftrightarrow \alpha = \pi - \beta$$

d) $\cos\left(\frac{3\pi}{2} - \beta\right) = -\sin\beta = -\frac{2\sqrt{5}}{5}$

e) $\sin\left(-\alpha - \frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2} + \alpha\right) = -\cos\alpha =$

$$= -\left(-\frac{\sqrt{5}}{5}\right) = \frac{\sqrt{5}}{5}$$

f) <u>1.º Processo</u>

$$\tan(-\beta) - 2\cos\left(-\alpha + \frac{\pi}{2}\right) = -\tan\beta - 2\sin\alpha =$$

$$= -\frac{\sin\beta}{\cos\beta} - 2\sin\alpha =$$

$$= -\frac{\frac{2\sqrt{5}}{5}}{\frac{\sqrt{5}}{5}} - 2 \times \frac{2\sqrt{5}}{5} = -\frac{2\sqrt{5}}{5} - \frac{4\sqrt{5}}{5} = -2 - \frac{4\sqrt{5}}{5}$$

Cálculos auxiliares:

$$\cos\alpha = -\frac{\sqrt{5}}{5}$$

$$\tan\alpha = -2$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow \tan\alpha\cos\alpha = \sin\alpha \Leftrightarrow$$

$$\Leftrightarrow -2\times\left(-\frac{\sqrt{5}}{5}\right) = \sin\alpha \Leftrightarrow \sin\alpha = \frac{2\sqrt{5}}{5}$$

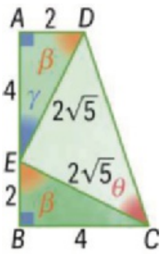
2.º Processo

$$\tan(-\beta) - 2\cos\left(-\alpha + \frac{\pi}{2}\right) = -\tan\beta - 2\sin\alpha =$$

$$= -2 - 2\sin(\pi - \beta) = -2 - 2\sin\beta = -2 - 2\times\frac{2\sqrt{5}}{5} =$$

$$= -2 - \frac{4\sqrt{5}}{5}$$

63.2.



$$\overline{EC}^2 = \overline{BC}^2 + \overline{BE}^2 \Leftrightarrow \left(2\sqrt{5}\right)^2 = 4^2 + \overline{BE}^2 \Leftrightarrow$$

$$\Leftrightarrow 20 = 16 + \overline{BE}^2 \Leftrightarrow \overline{BE}^2 = 4 \underset{\overline{BE}>0}{\Leftrightarrow} \overline{BE} = 2$$

Os triângulos $[BCE]$ e $[ADE]$ são geometricamente iguais.

$$\beta + \gamma = \frac{\pi}{2}$$

$$C\hat{E}D = \pi - \beta - \gamma = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\overline{ED} = \overline{CE}$$

$$\tan\theta = \frac{\overline{DE}}{\overline{CE}} = \frac{2\sqrt{5}}{2\sqrt{5}} = 1$$

$$\theta = \tan^{-1}(1) \Leftrightarrow \theta = \frac{\pi}{4}\text{ rad}$$

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64.1. $\sin\left(-\frac{\pi}{6}\right) + \cos\left(-\frac{\pi}{6}\right) + \cos\frac{5\pi}{3} = a - \sin\frac{4\pi}{3} \Leftrightarrow$

$$\Leftrightarrow -\sin\frac{\pi}{6} + \cos\frac{\pi}{6} + \cos\left(\frac{6\pi}{3} - \frac{\pi}{3}\right) = a - \sin\left(\frac{3\pi}{3} + \frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2} + \cos\left(2\pi - \frac{\pi}{3}\right) = a - \sin\left(\pi + \frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2} + \cos\left(-\frac{\pi}{3}\right) = a - \left(-\sin\frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} = a + \frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow a = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \Leftrightarrow a = 0$$

64.2. $\tan\frac{7\pi}{3} - 2\tan\left(-\frac{\pi}{3}\right) = 2a - 2\cos\left(-\frac{5\pi}{6}\right) \Leftrightarrow$

$$\Leftrightarrow \tan\left(\frac{6\pi}{3} + \frac{\pi}{3}\right) + 2\tan\frac{\pi}{3} = 2a - 2\cos\frac{5\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow \tan\left(2\pi + \frac{\pi}{3}\right) + 2\times\sqrt{3} = 2a - 2\times\cos\left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow \tan\frac{\pi}{3} + 2\sqrt{3} = 2a + 2\cos\frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3} + 2\sqrt{3} = 2a + 2\times\frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \sqrt{3} + 2\sqrt{3} = 2a + \sqrt{3} \Leftrightarrow 2\sqrt{3} = 2a \Leftrightarrow a = \sqrt{3}$$

64.3. $a\cos\left(-\frac{3\pi}{4}\right) = \tan\left(-\frac{7\pi}{4}\right) + \sin\left(-\frac{11\pi}{6}\right) \Leftrightarrow$

$$\Leftrightarrow a\cos\frac{3\pi}{4} = -\tan\frac{7\pi}{4} - \sin\frac{11\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow a\cos\left(\frac{4\pi}{4} - \frac{\pi}{4}\right) = -\tan\left(\frac{8\pi}{4} - \frac{\pi}{4}\right) - \sin\left(\frac{12\pi}{6} - \frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow a\cos\left(\pi - \frac{\pi}{4}\right) = -\tan\left(2\pi - \frac{\pi}{4}\right) - \sin\left(2\pi - \frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow a\left(-\cos\frac{\pi}{4}\right) = -\tan\left(-\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow -a\times\frac{\sqrt{2}}{2} = \tan\frac{\pi}{4} + \sin\frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow -\frac{\sqrt{2}}{2}a = 1 + \frac{1}{2} \Leftrightarrow -\frac{\sqrt{2}}{2}a = \frac{3}{2} \Leftrightarrow -\sqrt{2}a = 3 \Leftrightarrow$$

$$\Leftrightarrow a = -\frac{3}{\underset{(\times\sqrt{2})}{\sqrt{2}}} \Leftrightarrow a = -\frac{3\sqrt{2}}{2}$$

64.4. $a\sin\left(-\frac{5\pi}{6}\right) + a\cos(-\pi) = \tan\left(-\frac{3\pi}{4}\right) + \cos\frac{2\pi}{3} \Leftrightarrow$

$$\Leftrightarrow -a\sin\frac{5\pi}{6} + a\cos\pi = -\tan\frac{3\pi}{4} + \cos\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -a\sin\left(\frac{6\pi}{6} - \frac{\pi}{6}\right) - a = -\tan\left(\frac{4\pi}{4} - \frac{\pi}{4}\right) + \cos\left(\pi - \frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -a\sin\left(\pi - \frac{\pi}{6}\right) - a = -\tan\left(\pi - \frac{\pi}{4}\right) + \cos\left(\pi - \frac{\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -a\sin\frac{\pi}{6} - a = -\left(-\tan\frac{\pi}{4}\right) - \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow -a\times\frac{1}{2} - a = \tan\frac{\pi}{4} - \frac{1}{2} \Leftrightarrow -\frac{3}{2}a = \frac{1}{2} \Leftrightarrow a = -\frac{1}{3}$$

65.1. $\sin\frac{11\pi}{12}=\sin\left(\frac{12\pi}{12}-\frac{\pi}{12}\right)=\sin\left(\pi-\frac{\pi}{12}\right)=\sin\frac{\pi}{12}=a$

65.2. $\sin\frac{13\pi}{12}=\sin\left(\frac{12\pi}{12}+\frac{\pi}{12}\right)=\sin\left(\pi+\frac{\pi}{12}\right)=-\sin\frac{\pi}{12}=$

$=-a$

65.3. $\cos\frac{5\pi}{12}=\cos\left(\frac{6\pi}{12}-\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)=\sin\frac{\pi}{12}=$

$=a$

65.4. $\cos\frac{7\pi}{12}=\cos\left(\frac{6\pi}{12}+\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)=-\sin\frac{\pi}{12}=$

$=-a$

66. $\cos\alpha=-\frac{4}{5}$

$2\alpha-\beta=\frac{3\pi}{2}\Leftrightarrow-\beta=\frac{3\pi}{2}-2\alpha\Leftrightarrow\beta=-\frac{3\pi}{2}+2\alpha$

$\sin(\alpha-\beta)=\sin\left(\alpha-\left(-\frac{3\pi}{2}+2\alpha\right)\right)=$

$=\sin\left(\alpha+\frac{3\pi}{2}-2\alpha\right)=\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha=\frac{4}{5}$

67. Designemos por α o ângulo AOB.

$\tan\left(\frac{\pi}{2}+\alpha\right)=-2\Leftrightarrow\frac{\sin\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{\pi}{2}+\alpha\right)}=-2\Leftrightarrow$

$\Leftrightarrow\frac{\cos\alpha}{-\sin\alpha}=-2\Leftrightarrow\cos\alpha=2\sin\alpha$

$\sin^2\alpha+\cos^2\alpha=1\Leftrightarrow\sin^2\alpha+(2\sin\alpha)^2=1\Leftrightarrow$

$\Leftrightarrow\sin^2\alpha+4\sin^2\alpha=1\Leftrightarrow5\sin^2\alpha=1\Leftrightarrow\sin^2\alpha=\frac{1}{5}\Leftrightarrow$

$\Leftrightarrow\sin\alpha=\pm\frac{1}{\sqrt{5}}\Leftrightarrow\sin\alpha=\pm\frac{\sqrt{5}}{5}$

Como α é ângulo agudo, $\sin\alpha=\frac{\sqrt{5}}{5}$.

$A_{[OAB]}=\frac{1\times\frac{\sqrt{5}}{5}}{2}=\frac{\sqrt{5}}{10}$ u. a.

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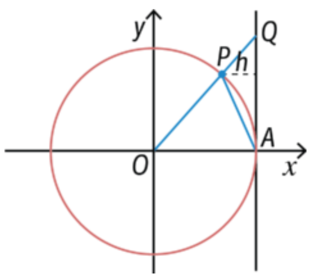
68.1. $A(1, 0)$; $P(\cos x, \sin x)$; $Q(1, \tan x)$



Seja h a altura do triângulo $[AQP]$ relativamente à base $[AQ]$.

$h=1-\cos x$

$A_{[AQP]}=\frac{\overline{AQ}\times h}{2}=\frac{\tan x\times(1-\cos x)}{2}=$

$=\frac{\tan x-\tan x\cos x}{2}==\frac{\tan x-\frac{\sin x}{\cancel{\cos x}}\cancel{\cos x}}{2}=$

$=\frac{\tan x-\sin x}{2}$

68.2. $\overline{AP}=\overline{OP}=1$

O triângulo $[AOP]$ é equilátero e $x=\frac{\pi}{3}$.

a)

$A\left(\frac{\pi}{3}\right)=\frac{\tan\frac{\pi}{3}-\sin\frac{\pi}{3}}{2}=\frac{\sqrt{3}-\frac{\sqrt{3}}{2}}{2}=\frac{\frac{\sqrt{3}}{2}}{2}=\frac{\sqrt{3}}{4}$ u. a.

b) $\overline{AP}=1$; $\overline{AQ}=\tan\frac{\pi}{3}=\sqrt{3}$

$\cos\frac{\pi}{3}=\frac{\overline{OA}}{\overline{OQ}}\Leftrightarrow\frac{1}{2}=\frac{1}{\overline{OQ}}\Leftrightarrow\overline{OQ}=2$

$\overline{PQ}=\overline{OQ}-\overline{OP}=2-1=1$

$P=\overline{AP}+\overline{AQ}+\overline{PQ}=1+\sqrt{3}+1=\left(\sqrt{3}+2\right)$ u.c.

68.3. $\sin\left(\frac{\pi}{2}+x\right)-\cos(\pi-x)=\frac{8}{5}\Leftrightarrow$

$\Leftrightarrow\cos x-(-\cos x)=\frac{8}{5}\Leftrightarrow$

$\Leftrightarrow2\cos x=\frac{8}{5}\Leftrightarrow\cos x=\frac{8}{10}\Leftrightarrow\cos x=\frac{4}{5}$

$\sin^2x+\cos^2x=1\Leftrightarrow\sin^2x=1-\cos^2x\Leftrightarrow$

$\Leftrightarrow\sin^2x=1-\left(\frac{4}{5}\right)^2\Leftrightarrow\sin^2x=1-\frac{16}{25}\Leftrightarrow$

$\Leftrightarrow\sin^2x=\frac{9}{25}\Leftrightarrow\sin x=\pm\frac{3}{5}$

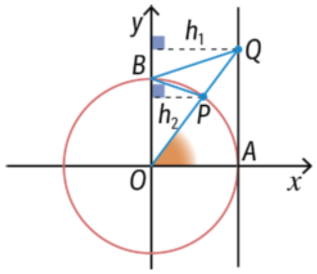
Como $x\in1.^{\text{o}}$ quadrante, $\sin x=\frac{3}{5}$

$\tan x=\frac{\sin x}{\cos x}=\frac{\frac{3}{5}}{\frac{4}{5}}=\frac{3}{4}$

$A_{[AQP]}=\frac{\frac{3}{4}-\frac{3}{5}}{2}=\frac{\frac{3}{20}}{2}=\frac{3}{40}$ u. a.

69.1. $A(1, 0)$; $B(0, 1)$

$P(\cos x, \sin x)$; $Q(1, \tan x)$



$A_{[PQB]} = A_{[OQB]} - A_{[OPB]} =$

$= \frac{\overline{OB} \times h_1}{2} - \frac{\overline{OB} \times h_2}{2} =$

$= \frac{1 \times 1}{2} - \frac{1 \times \cos x}{2} = \frac{1}{2} - \frac{\cos x}{2} = \frac{1 - \cos x}{2}$

$A(x) = \frac{1 - \cos x}{2}$

69.2. $9\sin(\pi + x) + 8\cos\left(\frac{\pi}{2} + x\right) + 15 = 0 \Leftrightarrow$

$\Leftrightarrow -9\sin x - 8\sin x + 15 = 0 \Leftrightarrow$

$\Leftrightarrow -17\sin x = -15 \Leftrightarrow$

$\Leftrightarrow \sin x = \frac{15}{17}$

$\sin^2 x + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \sin^2 x \Leftrightarrow$

$\Leftrightarrow \cos^2 x = 1 - \left(\frac{15}{17}\right)^2 \Leftrightarrow \cos^2 x = 1 - \frac{225}{289} \Leftrightarrow$

$\Leftrightarrow \cos^2 x = \frac{64}{289} \Leftrightarrow \cos x = \pm\frac{8}{17}$

Como $x \in \left]0, \frac{\pi}{2}\right[$, $\cos x = \frac{8}{17}$.

$A = \frac{1 - \frac{8}{17}}{2} = \frac{\frac{17}{17} - \frac{8}{17}}{2} = \frac{\frac{9}{17}}{2} = \frac{9}{34}$ u. a.

69.3. $\cos\frac{\pi}{3} = \frac{\overline{OA}}{\overline{OQ}} \Leftrightarrow \frac{1}{2} = \frac{1}{\overline{OQ}} \Leftrightarrow \overline{OQ} = 2$

$\overline{PQ} = \overline{OQ} - \overline{OP} = 2 - 1 = 1$ u. c.

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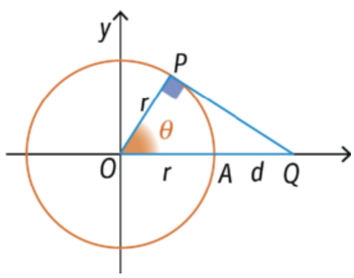
70.1.



$\cos\theta = \frac{\overline{OP}}{\overline{OQ}} \Leftrightarrow \cos\theta = \frac{r}{r + d} \underset{(r+d>0)}{\Leftrightarrow} \cos\theta(r + d) = r \Leftrightarrow$

$\Leftrightarrow r\cos\theta + d\cos\theta = r \Leftrightarrow d\cos\theta = r - r\cos\theta \Leftrightarrow$

$\Leftrightarrow d = \frac{r - r\cos\theta}{\cos\theta} \Leftrightarrow d = \frac{r(1 - \cos\theta)}{\cos\theta}$

70.2. $d = r$

$\cos\theta = \frac{r}{2r} \Leftrightarrow \cos\theta = \frac{1}{2}$

Como $\theta \in \left]0, \frac{\pi}{2}\right[$, $\theta = \frac{\pi}{3}$

70.3. $r = 1$; $d = 2$

$P(\cos\theta, \sin\theta)$

$2 = \frac{1(1 - \cos\theta)}{\cos\theta} \underset{(\cos\theta>0)}{\Leftrightarrow} 2\cos\theta = 1 - \cos\theta \Leftrightarrow$

$\Leftrightarrow 3\cos\theta = 1 \Leftrightarrow \cos\theta = \frac{1}{3}$

$\sin^2\theta + \cos^2\theta = 1 \Leftrightarrow \sin^2\theta = 1 - \cos^2\theta \Leftrightarrow$

$\Leftrightarrow \sin^2\theta = 1 - \left(\frac{1}{3}\right)^2 \Leftrightarrow \sin^2\theta = 1 - \frac{1}{9} \Leftrightarrow$

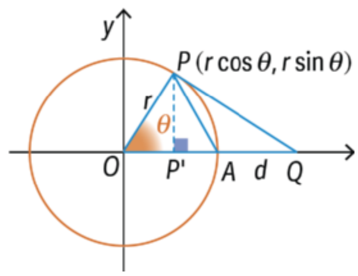
$\Leftrightarrow \sin^2\theta = \frac{8}{9} \Leftrightarrow \sin\theta = \pm\frac{\sqrt{8}}{3} \Leftrightarrow \sin\theta = \pm\frac{2\sqrt{2}}{3}$

Como $\theta \in 1.^{o}$ quadrante, $\sin\theta = \frac{2\sqrt{2}}{3}$.

$P\left(\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$

70.4.

a)



Seja P' a projeção ortogonal de P sobre Ox .

Cálculos auxiliares:

$\sin\theta = \frac{\overline{PP'}}{\overline{OP}}$; $\sin\theta = \frac{\overline{PP'}}{r}$

$\overline{PP'} = r\sin\theta$

$d = \frac{r(1 - \cos\theta)}{\cos\theta}$

$$A_{[AQP]}=\frac{\overline{AQ}\times\overline{PP'}}{2}=\frac{d\times r\sin\theta}{2}=$$

$$=\frac{\frac{r(1-\cos\theta)}{\cos\theta}\times r\sin\theta}{2}=\frac{r(1-\cos\theta)\times r\frac{\sin\theta}{\cos\theta}}{2}=$$

$$=\frac{r(1-\cos\theta)\times r\tan\theta}{2}=\frac{r^2}{2}(1-\cos\theta)\times\tan\theta=$$

$$=\frac{1}{2}r^2(1-\cos\theta)\tan\theta$$

b) $r=2$

$\tan^2\alpha=\tan\alpha\Leftrightarrow\tan^2\alpha-\tan\alpha=0\Leftrightarrow$

$\Leftrightarrow\tan\alpha(\tan\alpha-1)=0\Leftrightarrow$

$\Leftrightarrow\tan\alpha=0\vee\tan\alpha-1=0\Leftrightarrow$

$\Leftrightarrow\tan\alpha=0\vee\tan\alpha=1\Leftrightarrow$

Como $\alpha\in\left]0,\frac{\pi}{2}\right[$, $\tan\alpha>0$ logo, $\tan\alpha=1$ e

$\alpha=\frac{\pi}{4}$.

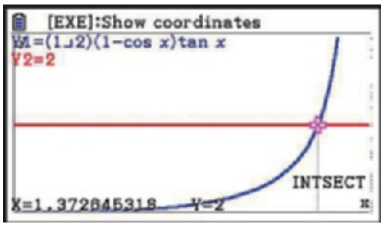
$\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$$A(\alpha)=\frac{1}{2}r^2(1-\cos\alpha)\tan\alpha=$$

$$=\frac{1}{2}\times2^2\left(1-\frac{\sqrt{2}}{2}\right)\times1=2\left(1-\frac{\sqrt{2}}{2}\right)=\left(2-\sqrt{2}\right)\text{ u. a.}$$

c) $r=1$

$$A(\theta)=2\Leftrightarrow\frac{1}{2}(1-\cos\theta)\tan\theta=2\Leftrightarrow\theta\approx1{,}37\text{ rad}$$



71.1. A função g não é periódica.

71.2. Período fundamental de f: 8

Período fundamental de h: π

72.1. $D_f=\mathbb{R}$

Assim, se $x\in D_f$ então $x+\pi\in D_f$

$$f(x+\pi)=\sin(2(x+\pi))=\sin(2x+2\pi)=$$

$$=\sin(2x)=f(x)$$

Logo, f é periódica de período π.

72.2. $D_g=\mathbb{R}$

Assim, se $x\in D_g$ então $x+\frac{\pi}{2}\in D_g$.

$$g\left(x+\frac{\pi}{2}\right)=\sin\left(4\left(x+\frac{\pi}{2}\right)-\frac{\pi}{5}\right)=\cos\left(4x+2\pi-\frac{\pi}{5}\right)=$$

$$=\cos\left(4x-\frac{\pi}{5}\right)=g(x)$$

Logo, g é periódica de período $\frac{\pi}{2}$.

72.3. Se $x\in D_h$ então $x+3\pi\in D_h$

$$h(x+3\pi)=\tan\left(\frac{x+3\pi}{3}\right)=\tan\left(\frac{x}{3}+\pi\right)=\tan\left(\frac{x}{3}\right)=h(x)$$

Logo, h é periódica de período 3π.

72.4. $D_i=\mathbb{R}$. Assim, se $x\in D_i$ então $x+\frac{2\pi}{|a|}\in D_i$.

$$i\left(x+\frac{2\pi}{|a|}\right)=\cos\left(a\left(x+\frac{2\pi}{|a|}\right)+b\right)=$$

$$=\cos\left(ax+2\pi\frac{a}{|a|}+b\right)=$$

$$=\cos(ax\pm2\pi+b)=\cos(ax+b)=i(x)$$

Logo, i é periódica de período $\frac{2\pi}{|a|}$.

73.1. <u>1.º processo</u>:

f é uma função da forma $f(x)=a+b\sin(c(x-d))$

$b=-2<0$. Então, $D'_f=[3+(-2),3-(-2)]=[1,5]$

<u>2.º processo</u>:

$D_f=\mathbb{R}$. Se $x\in D_f$ então $\frac{x}{2}\in D_f$

$$-1\le\sin\left(\frac{x}{2}\right)\le1\Leftrightarrow2\ge-2\sin\left(\frac{x}{2}\right)\ge-2\Leftrightarrow$$

$$\Leftrightarrow-2\le-2\sin\left(\frac{x}{2}\right)\le2\Leftrightarrow1\le3-2\sin\left(\frac{x}{2}\right)\le5\Leftrightarrow$$

$\Leftrightarrow1\le f(x)\le5$; $\quad D'_f=[1,5]$

73.2. $D'_f=[1,5]$

Então, $0\notin D'_f$, ou seja, 0 não pertence ao conjunto das imagens de f. Logo, f não tem zeros.

73.3. <u>1.º processo:</u>

$y=\sin(cx)$

$$P_0=\frac{2\pi}{|c|}=\frac{2\pi}{\frac{1}{2}}=4\pi$$

2.º processo:

$D_f = \mathbb{R}$

Assim, se $x \in D_f$ então $x + \pi \in D_f$

$f(x + 4\pi) = 3 - 2\sin\left(\frac{x + 4\pi}{2}\right) = 3 - 2\sin\left(\frac{x}{2} + 2\pi\right) =$

$= 3 - 2\sin\left(\frac{x}{2}\right) = f(x)$

Logo, 4π é o período de *f*.

73.4. $f(a) = 3 - 2\sin\left(\frac{a}{2}\right)$

$f(-a) = 3 - 2\sin\left(-\frac{a}{2}\right) = 3 + 2\sin\left(\frac{a}{2}\right)$

$f(a) + f(-a) = 3 - 2\sin\left(\frac{a}{2}\right) + 3 + 2\sin\left(\frac{a}{2}\right) =$

$= 3 + 3 = 6$

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74. O gráfico de *g* obtém-se do gráfico de $y = \sin x$ através da seguinte sequência de transformações:

Translação horizontal associada ao vetor $\left(\frac{\pi}{2}, 0\right)$

(transforma $y = \sin x$ em $y = \sin\left(x - \frac{\pi}{2}\right)$)

Dilatação vertical de coeficiente 2 (transforma $y = \sin\left(x - \frac{\pi}{2}\right)$ em $y = 2\sin\left(x - \frac{\pi}{2}\right)$)

Translação vertical associada ao vetor $(0, 1)$

(transforma $y = 2\sin\left(x - \frac{\pi}{2}\right)$ em $g(x)$)

75.1. $b = 2 > 0$. Então $D'_h = [1 - 2, 1 + 2] = [-1, 3]$

75.2. Seja P_0 o período fundamental de *h*.

$D_h = \mathbb{R}$. Se $x \in D_h$, então $x + P_0 \in D_h$.

$\forall x \in \mathbb{R}$, $h(x + P_0) = h(x) \Leftrightarrow$

$\Leftrightarrow 1 + 2\cos\left(\frac{x + P_0}{3}\right) = 1 + 2\cos\left(\frac{x}{2}\right) \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{x}{3} + \frac{P_0}{3}\right) = \cos\left(\frac{x}{3}\right)$

2π é o período fundamental da função cosseno.

Então, $\frac{P_0}{3} = 2\pi \Leftrightarrow P_0 = 6\pi$

O período positivo mínimo de *h* é 6π.

75.3. O gráfico de *h* obtém-se do gráfico de $y = \cos x$ através da segunda sequência de transformações:

Dilatação horizontal de coeficiente 3;

Dilatação vertical de coeficiente 2;

Translação vertical associada ao vetor $(0, 1)$.

76. $k = \frac{2 - (-2)}{2} = 2$

$P_0 = 4\pi$. Então $\frac{2\pi}{|t|} = 4\pi \Leftrightarrow |t| = \frac{2\pi}{4\pi} = \frac{1}{2}$

Opção (B)

77.1. Seja P_0 o período fundamental de *f*.

Se $x \in D_f$, então $x + P_0 \in D_f$

$\forall_{x \in D_f}$, $f(x + P_0) = f(x) \Leftrightarrow$

$\Leftrightarrow \frac{1}{2}\tan\left(\frac{\pi}{6} - 2(x + P_0)\right) = \frac{1}{2}\tan\left(\frac{\pi}{6} - 2x\right) \Leftrightarrow$

$\Leftrightarrow \tan\left(\frac{\pi}{6} - 2x - 2P_0\right) = \tan\left(\frac{\pi}{6} - 2x\right)$

π é o período fundamental da função tangente.

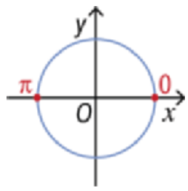
Então, $2P_0 = \pi \Leftrightarrow P_0 = \frac{\pi}{2}$.

Logo, *f* é uma função periódica de período positivo mínimo igual a $\frac{\pi}{2}$.

77.2. $f(x) = 0 \Leftrightarrow \frac{1}{2}\tan\left(\frac{\pi}{6} - 2x\right) = 0 \Leftrightarrow$

$\Leftrightarrow \tan\left(\frac{\pi}{6} - 2x\right) = 0 \Leftrightarrow$

$\Leftrightarrow \frac{\pi}{6} - 2x = 0 + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$



$\Leftrightarrow -2x = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$x = \frac{\pi}{12} + \frac{k\pi}{2},\ k \in \mathbb{Z}$

77.3. $f\left(\frac{5\pi}{12}\right) = \frac{1}{2}\tan\left(\frac{\pi}{6} - 2 \times \frac{5\pi}{12}\right) = \frac{1}{2}\tan\left(\frac{\pi}{6} - \frac{5\pi}{6}\right) =$

$= \frac{1}{2}\tan\left(-\frac{2\pi}{3}\right) = \frac{1}{2}\tan\left(-\pi + \frac{\pi}{3}\right) = \frac{1}{2}\tan\left(\frac{\pi}{3}\right) =$

$= \frac{1}{2} \times \sqrt{3} = \frac{\sqrt{3}}{2}$

$f\left(-\frac{\pi}{3}\right) = \frac{1}{2}\tan\left(\frac{\pi}{6} - 2 \times \left(-\frac{\pi}{3}\right)\right) = \frac{1}{2}\tan\left(\frac{\pi}{6} + \frac{2\pi}{3}\right) =$

$= \frac{1}{2}\tan\left(\frac{5\pi}{6}\right) = \frac{1}{2}\tan\left(\pi - \frac{\pi}{6}\right) =$

$= -\frac{1}{2}\tan\left(\frac{\pi}{6}\right) = -\frac{1}{2}\times\frac{\sqrt{3}}{3} = -\frac{\sqrt{3}}{6}$

$f\left(\frac{5\pi}{12}\right) + f\left(-\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} =$

$= \frac{3\sqrt{3}-\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

78. $f(x) = 4\sin\left(3x - \frac{\pi}{7}\right)$; $D_f = \mathbb{R}$

$b = 4 > 0$, então, $D'_f = [0-4, 0+4] = [-4, 4]$

$g(x) = 2\tan^2\left(\frac{2\pi x}{5}\right)$

Cálculos auxiliares:

$\frac{2\pi x}{5} = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 4\pi x = 5\pi + 10k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{5}{4} + \frac{5}{2}k,\ k \in \mathbb{Z}$

$D_g = \left\{x \in \mathbb{R} : \frac{2\pi x}{5} \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\} \Leftrightarrow$

$\Leftrightarrow D_g = \mathbb{R} \setminus \left\{x : x = \frac{5}{4} + \frac{5}{2}k,\ k \in \mathbb{Z}\right\}$

$\tan^2\left(\frac{2\pi x}{5}\right) \geq 0 \Leftrightarrow 2\tan^2\left(\frac{2\pi x}{5}\right) \geq 0 \Leftrightarrow g(x) \geq 0$

$D'_g = \mathbb{R}_0^+$

79. $h(t) = 12 - 4\cos(2t)$

79.1. $h(t+\pi) = 12 - 4\cos(2(t+\pi)) =$

$= 12 - 4\cos(2t + 2\pi) = 12 - 4\cos(2t) = h(t)$

Logo, h é uma função periódica de período π.

79.2. $a = 12$ e $b = -4 < 0$

$D'_h = [12-4, 12+4] = [8, 16]$

8 é o mínimo da função *h*.

Então, a distância do ponto do casco do navio ao fundo do mar no momento da maré baixa é 8 m.

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80. $f(t) = 24{,}5 + 2{,}5\cos\left[\frac{\pi(t+9)}{12}\right]$

80.1. $f(13) = 24{,}5 + 2{,}5\cos\left[\frac{\pi(13+9)}{12}\right] =$

$= 24{,}5 + 2{,}5\cos\left(\frac{22\pi}{12}\right) \approx 26{,}7\ °C$

Às 13 horas desse dia, a temperatura no interior da habitação é de $26{,}7\ °C$.

80.2. $a = 24{,}5$ e $b = 2{,}5 > 0$

$D'_f = [24{,}5 - 2{,}5;\ 24{,}5 + 2{,}5] = [22, 27]$

A temperatura mínima e máxima no interior da habitação são de $22\ °C$ e $27\ °C$, respetivamente.

81. $f(t) = 4\cos\left(\frac{\pi}{6}t\right)$

81.1. $a = 0$ e $b = 4 > 0$

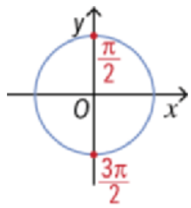
$D'_f = [0-4;\ 0+4] = [-4, 4]$

4 é o máximo da função *f*.

A altura máxima que o nível da água atingiu foi 4 metros.

81.2. $f(t) = 0 \Leftrightarrow 4\cos\left(\frac{\pi}{6}t\right) = 0 \Leftrightarrow \cos\left(\frac{\pi}{6}t\right) = 0 \Leftrightarrow$

$\Leftrightarrow \frac{\pi}{6}t = \frac{\pi}{2} + kt,\ k \in \mathbb{Z} \Leftrightarrow t = 3 + 6k,\ k \in \mathbb{Z}$



Como $t \in [0, 24]$,

Tem-se que:

Se $k = 0$, $t = 3$.

Se $k = 1$, $t = 3 + 6 = 9$.

Se $k = 2$, $t = 3 + 6 \times 2 = 15$.

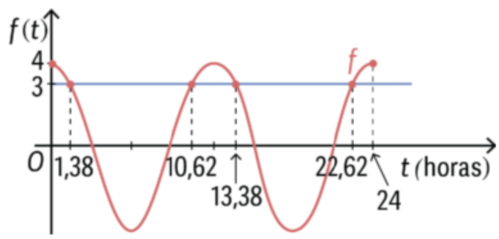
Se $k = 3$, $t = 3 + 6 \times 3 = 21$.

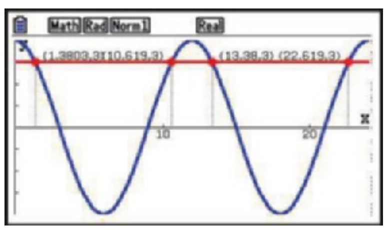
A altura da água no porto esteve ao nível médio da água do mar às 3 h, às 9 h, e às 21 h, ou seja, 4 vezes.

81.3. $D'_f = [-4, 4]$; -4 é mínimo da função *f*.

Então, a altura mínima que a água atingiu foi -4 m.

81.4. $f(t) \geq 3$





$0{,}38 \times 60 \approx 23$

$1{,}38 \text{ h} = 1 \text{ h } 23 \text{ min}$

$13{,}38 \text{ h} = 13 \text{ h } 23 \text{ min}$

$0{,}62 \times 60 \approx 37 \text{ min}$

$10{,}62 \text{ h} = 10 \text{ h } 37 \text{ min}$

$22{,}62 \text{ h} = 22 \text{ h } 37 \text{ min}$

Foi possível o Júlio entrar no posto das 0 h às 1 h 23 min, das 10 h 37 min às 13 h 23 min e das 22 h 37 min às 24 h do dia 21 de janeiro.

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82.1. $h(0) = \overline{OQ} = 4\text{cm}$

$h\left(\frac{\pi}{2}\right) = \overline{OQ} + \text{raio} = 4 + 2 = 6\text{m}$

$h(\pi) = \overline{OQ} = 4\text{m}$

$h\left(\frac{3\pi}{2}\right) = \overline{OQ} - \text{raio} = 4 - 2 = 2\text{m}$

$h(2\pi) = h(0) = 4\text{m}$

α	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$h(\alpha)$	4	6	4	2	4

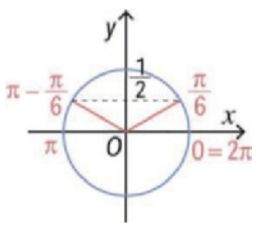
82.2. $\sin\alpha = \frac{\overline{BP}}{\overline{OP}} \Leftrightarrow \sin\alpha = \frac{\overline{BP}}{2} \Leftrightarrow \overline{BP} = 2\sin\alpha$

$\overline{PR} = \overline{BR} + \overline{BP}$

$h(\alpha) = \overline{OQ} + 2\sin\alpha = 4 + 2\sin\alpha$

82.3. $h(\alpha) = 5 \Leftrightarrow 4 + 2\sin\alpha = 5 \Leftrightarrow$

$\Leftrightarrow 2\sin\alpha = 1 \Leftrightarrow \sin\alpha = \frac{1}{2}$



Como $\alpha \in [0, 2\pi]$

$\alpha = \frac{\pi}{6} \wedge \alpha = \pi - \frac{\pi}{6} \Leftrightarrow \alpha = \frac{\pi}{6} \wedge \alpha = \frac{5\pi}{6}$

Quando $\alpha = \frac{\pi}{6}$ e $\alpha = \frac{5\pi}{6}$, o ponto P está a 5 metros do solo.

83. $f(x) = \frac{1 - \sin(4x)}{2} \Leftrightarrow f(x) = \frac{1}{2} - \frac{1}{2}\sin(4x)$

83.1. $a = \frac{1}{2}$ e $b = -\frac{1}{2} < 0$

$D'_f = \left[\frac{1}{2} - \frac{1}{2}, \frac{1}{2} + \frac{1}{2}\right] = [0, 1]$

83.2. $D_f = \mathbb{R}$

Se $x \in D_f$, $x + \frac{\pi}{2} \in D_f$

$f\left(x + \frac{\pi}{2}\right) = \frac{1}{2} - \frac{1}{2}\sin\left(4\left(x + \frac{\pi}{2}\right)\right) =$

$= \frac{1}{2} - \frac{1}{2}\sin(4x + 2\pi) = \frac{1}{2} - \frac{1}{2}\sin(4x) = f(x)$

Logo, f é periódica de período $\frac{\pi}{2}$

83.3. $f(\pi + \alpha) = \frac{1 - \sin(4(\pi + \alpha))}{2} = \frac{1 - \sin(4\pi + 4\alpha)}{2} =$

$= \frac{1 - \sin(4\alpha)}{2}$

$f(\pi - \alpha) = \frac{1 - \sin(4(\pi - \alpha))}{2} = \frac{1 - \sin(4\pi - 4\alpha)}{2} =$

$= \frac{1 - \sin(-4\alpha)}{2}$

$f(\pi + \alpha) \times f(\pi - \alpha) = \frac{4}{25} \Leftrightarrow$

$\Leftrightarrow \frac{1 - \sin(4\alpha)}{2} \times \frac{1 + \sin(4\alpha)}{2} = \frac{4}{25} \Leftrightarrow$

$\Leftrightarrow \frac{1 - \sin^2(4\alpha)}{4} = \frac{4}{25} \Leftrightarrow \cos^2(4\alpha) = \frac{16}{25}$

$\sin^2(4\alpha) + \cos^2(4\alpha) = 1 \Leftrightarrow \sin^2(4\alpha) = 1 - \frac{16}{25} \Leftrightarrow$

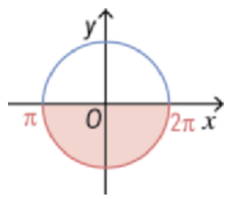
$\Leftrightarrow \sin^2(4\alpha) = \frac{9}{25} \Leftrightarrow \sin(4\alpha) = \pm\frac{3}{5}$

Cálculos auxiliares:

$\frac{\pi}{4} < \alpha < \frac{\pi}{2} \Leftrightarrow$

$\Leftrightarrow 4 \times \frac{\pi}{4} < 4 \times \alpha < 4 \times \frac{\pi}{2} \Leftrightarrow$

$\Leftrightarrow \pi < \alpha < 2\pi$



Como $4\alpha \in]\pi, 2\pi[$, $\sin(4\alpha) = -\frac{3}{5}$.

$f(\alpha) = \frac{1 - \sin(4\alpha)}{2} = \frac{1 - \left(-\frac{3}{5}\right)}{2} = \frac{\frac{8}{5}}{2} = \frac{8}{10} = \frac{4}{5}$

84. $f(x)=2\left(1-2\cos\frac{\pi x}{3}\right) \Leftrightarrow f(x)=2-4\cos\frac{\pi x}{3}$

84.1. $a=2$ e $b=-4<0$

$D'_f=[2-4,\ 2+4]=[-2,\ 6]$

84.2. $D_f=\mathbb{R}$

Se $x\in D_f$, $x+6\in D_f$

$f(x+6)=2\left[1-2\cos\frac{\pi(x+6)}{3}\right]=$

$=2\left[1-2\cos\left(\frac{\pi x}{3}+2\pi\right)\right]=2\left[1-2\cos\frac{\pi x}{3}\right]=f(x)$

Logo, *f* é periódica de período 6.

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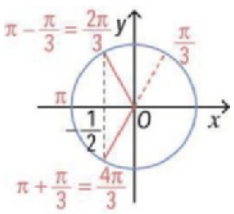
84.3. $f(x)=4 \Leftrightarrow 2\left(1-2\cos\frac{\pi x}{3}\right)=4 \Leftrightarrow$

$\Leftrightarrow 1-2\cos\frac{\pi x}{3}=2 \Leftrightarrow$

$\Leftrightarrow -2\cos\frac{\pi x}{3}=1 \Leftrightarrow \cos\frac{\pi x}{3}=-\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \frac{\pi x}{3}=\frac{2\pi}{3}+2k\pi \vee \frac{\pi x}{3}=\frac{4\pi}{3}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=2+6k \vee x=4+6k,\ k\in\mathbb{Z}$



Como $x\in[0,\ 6]$, tem-se que: $x=2 \vee x=4$

$\overline{BC}=4-2=2$

Altura $=4$

$A_{[ABC]}=\frac{\overline{BC}\times \text{altura}}{2}=\frac{2\times 4}{2}=4$ u. a.

85. $f(x)=3\cos\left(\frac{x}{2}\right)$; $g(x)=\frac{1}{3}\sin(3x)$

85.1. $D_f=\mathbb{R}$

Se $x\in D_f$, $x+4\pi\in D_f$.

$f(x+4\pi)=3\cos\left(\frac{x+4\pi}{2}\right)=3\cos\left(\frac{x}{2}+2\pi\right)=$

$=3\cos\left(\frac{x}{2}\right)=f(x)$

Logo, *f* é uma função periódica de período 4π.

85.2. Seja P_0 o período positivo mínimo de *g*.

$D_g=\mathbb{R}$

Se $x\in D_g$, então $x+P_0\in D_g$

$g(x+P_0)=g(x) \Leftrightarrow$

$\Leftrightarrow \frac{1}{3}\sin(3(x+P_0))=\frac{1}{3}\sin(3x) \Leftrightarrow$

$\Leftrightarrow \sin(3x+3P_0)=\sin(3x)$

2π é o período da função seno.

Então, $3P_0=2\pi \Leftrightarrow P_0=\frac{2\pi}{3}$.

Logo, *g* é uma função periódica de período positivo mínimo $P_0=\frac{2\pi}{3}$.

Alternativa:

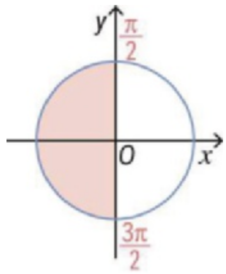
$P_0=\frac{2\pi}{|3|}=\frac{2\pi}{3}$

85.3. $f(\pi-2\alpha)=1 \Leftrightarrow 3\cos\left(\frac{\pi-2\alpha}{2}\right)=1 \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi}{2}-\alpha\right)=\frac{1}{3} \Leftrightarrow \sin\alpha=\frac{1}{3}$

$\sin^2\alpha+\cos^2\alpha=1 \Leftrightarrow \left(\frac{1}{3}\right)^2+\cos^2\alpha=1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha=1-\frac{1}{9} \Leftrightarrow \cos^2\alpha=\frac{8}{9} \Leftrightarrow \cos\alpha=\pm\frac{2\sqrt{2}}{3}$



Como $\alpha\in\left]\frac{\pi}{2},\frac{3\pi}{2}\right[$, tem-se que $\cos\alpha=-\frac{2\sqrt{2}}{3}$

$g\left(\frac{\pi}{2}+\frac{\alpha}{3}\right)=\frac{1}{3}\sin\left[3\left(\frac{\pi}{2}+\frac{\alpha}{3}\right)\right]=\frac{1}{3}\sin\left(\frac{3\pi}{2}+\alpha\right)=$

$=-\frac{1}{3}\cos\alpha=-\frac{1}{3}\times\left(-\frac{2\sqrt{2}}{3}\right)=\frac{2\sqrt{2}}{9}$

86.1. $P(\cos\alpha,\ \sin\alpha)$; $B(\cos\alpha,\ 1)$

$\overline{OC}=1$

$\overline{CB}=\cos\alpha$

$\overline{BP}=1-\sin\alpha$

$$A_{[OPBC]} = \frac{\overline{OC} + \overline{BP}}{2} \times \overline{CB} = \frac{1 + 1 - \sin\alpha}{2} \times \cos\alpha =$$

$$= \left(1 - \frac{1}{2}\sin\alpha\right) \times \cos\alpha = \cos\alpha - \frac{1}{2}\sin\alpha\cos\alpha$$

86.2. $\overline{OC} = \overline{OP} = 1$

Então, $O\hat{C}P = C\hat{P}O = \frac{\pi}{3}$.

$$P\hat{O}C = \pi - 2 \times \frac{\pi}{3} = \frac{3\pi - 2\pi}{3} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi - 2\pi}{6} = \frac{\pi}{6}$$

$$A\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} - \frac{1}{2}\sin\frac{\pi}{6}\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} =$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8} = \frac{4\sqrt{3} - \sqrt{3}}{8} = \frac{3\sqrt{3}}{8} \text{ u. a.}$$

87. $f(x) = 3\cos(2x + c) - 1$

87.1. $a = -1$ e $b = 3 > 0$

$D'_f = [-1 - 3, -1 + 3] = [-4, 2]$

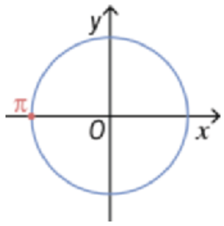
2 é máximo de *f*.

$$f\left(\frac{\pi}{2}\right) = 2 \Leftrightarrow 3\cos\left(2\frac{\pi}{2} + c\right) - 1 = 2 \Leftrightarrow$$

$$\Leftrightarrow 3\cos(\pi + c) = 3 \Leftrightarrow -3\cos c = 3 \Leftrightarrow \cos c = -1 \Leftrightarrow$$

$$\Leftrightarrow c = \pi + 2k\pi, k \in \mathbb{Z}$$

Logo, o menor valor positivo de *c* é π.



87.2. $f(x) = 3\cos(2x + \pi) - 1$

Seja *P* um ponto do gráfico de *f* cuja distância à origem é igual a 2.

$P(x, f(x))$

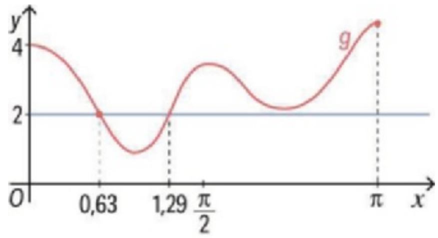
$$\overline{OP} = 2 \Leftrightarrow \sqrt{x^2 + (f(x))^2} = 2 \Leftrightarrow$$

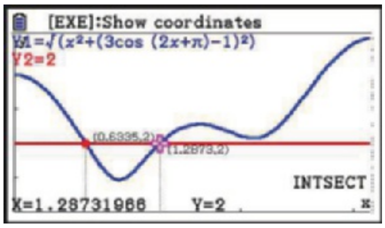
$$\Leftrightarrow \sqrt{x^2 + (3\cos(2x + \pi) - 1)^2} = 2$$

Seja *g* a função definida por

$$g(x) = \sqrt{x^2 + (3\cos(2x + \pi) - 1)^2}$$

Pretende se resolver graficamente a equação $g(x) = 2$.





$$f(0{,}63) = 3\cos(2 \times 0{,}63 + \pi) - 1 \approx -1{,}9$$

$$f(1{,}29) = 3\cos(2 \times 1{,}29 + \pi) - 1 \approx 1{,}5$$

$A(0{,}6; -1{,}9)$; $B(1{,}3; 1{,}5)$

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88. $h(x) = 2\tan\left(\frac{x}{2} + \frac{\pi}{2}\right)$

88.1. $D_h = \left\{x \in \mathbb{R} : \frac{x}{2} + \frac{\pi}{2} \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\} =$

$$= \mathbb{R} \setminus \{x : x = 2k\pi, k \in \mathbb{Z}\}$$

Cálculos auxiliares:

$$\frac{x}{2} + \frac{\pi}{2} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \Leftrightarrow \frac{x}{2} = k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = 2k\pi, k \in \mathbb{Z}$$

88.2. $h(\pi - x) = 2\tan\left(\frac{\pi - x}{2} + \frac{\pi}{2}\right) = 2\tan\left(\frac{\pi - x + \pi}{2}\right) =$

$$= 2\tan\left(\pi - \frac{x}{2}\right) = 2\tan\left(-\frac{x}{2}\right) = -2\tan\frac{x}{2}$$

$$h(\pi - x) \times h(x) = -2\tan\frac{x}{2} \times 2\tan\left(\frac{x}{2} + \frac{\pi}{2}\right) =$$

$$= -4 \times \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} \times \frac{\sin\left(\frac{x}{2} + \frac{\pi}{2}\right)}{\cos\left(\frac{x}{2} + \frac{\pi}{2}\right)} = -4 \times \frac{\sin\frac{x}{2} \times \cos\frac{x}{2}}{-\cos\frac{x}{2} \times \sin\frac{x}{2}} =$$

$$= -4 \times (-1) = 4$$

88.3. $D_h = \mathbb{R} \setminus \{x : x = 2k\pi, k \in \mathbb{Z}\}$

Se $x \in D_h$, $x + 2\pi \in D_h$

$$h(x + 2\pi) = 2\tan\left(\frac{x + 2\pi}{2} + \frac{\pi}{2}\right) = 2\tan\left(\frac{x}{2} + \frac{\pi}{2} + \pi\right) =$$

$$= 2\tan\left(\frac{x}{2} + \frac{\pi}{2}\right) = h(x)$$

Logo, *h* é uma função periódica de período 2π.

89.1. $\sin x = \dfrac{\overline{SO}}{\overline{AO}} = \dfrac{\overline{SO}}{1} = \overline{SO}$

$\cos x = \dfrac{\overline{SA}}{\overline{AO}} = \dfrac{\overline{SA}}{1} = \overline{SA}$

$g(x) = \dfrac{\overline{SP} \times \overline{SO}}{2} = \dfrac{\overline{SA} \times \overline{SO}}{2} =$

$= \dfrac{\cos x \times \sin x}{2} = \dfrac{1}{2}\sin x \cos x$

89.2. $g(x) = \dfrac{1}{4} \times 2\sin x \cos x = \dfrac{1}{4} \times \sin(2x)$

$0 < x < \dfrac{\pi}{2} \Leftrightarrow 0 < 2x < \pi$

Então, $0 \leq \sin(2x) \leq 1 \Leftrightarrow 0 \leq \dfrac{1}{4}\sin(2x) \leq \dfrac{1}{4}$

$D'_f = \left]0, \dfrac{1}{4}\right]$

A área máxima do triângulo [*OPS*] é $\dfrac{1}{4}$ u. a.

89.3. $\overline{AS} = \cos(x) = 0,8$

$\sin^2(x) + \cos^2(x) = 1 \Leftrightarrow \sin^2(x) + 0,8^2 = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2(x) = 1 - 0,64 \Leftrightarrow \sin^2(x) = 0,36 \Leftrightarrow$

$\Leftrightarrow \sin(x) = \pm 0,6$

Como $x \in \left]0, \dfrac{\pi}{2}\right[$, tem-se que: $\sin(x) = 0,6$

$A_{[OPS]} = \dfrac{1}{2}\sin(x)\cos(x) = \dfrac{1}{2} \times 0,6 \times 0,8 = 0,24$ u. a.

90.1. [*ABP*] é um triângulo inscrito numa semicircunferência de diâmetro [*AB*]. Então, é um triangulo retângulo em *P*.

90.2. $B\hat{A}P = \dfrac{1}{2} B\hat{O}P = \dfrac{1}{2}x = \dfrac{x}{2}$

$\sin\left(\dfrac{x}{2}\right) = \dfrac{\overline{PB}}{\overline{AB}} \Leftrightarrow \sin\left(\dfrac{x}{2}\right) = \dfrac{\overline{PB}}{2} \Leftrightarrow \overline{PB} = 2\sin\left(\dfrac{x}{2}\right)$

$f(x) = \overline{PB} = 2\sin\left(\dfrac{x}{2}\right)$

$\cos\left(\dfrac{x}{2}\right) = \dfrac{\overline{AP}}{\overline{AB}} \Leftrightarrow \cos\left(\dfrac{x}{2}\right) = \dfrac{\overline{AP}}{2} \Leftrightarrow \overline{AP} = 2\cos\left(\dfrac{x}{2}\right)$

$g(x) = 2\cos\left(\dfrac{x}{2}\right)$

90.3. $f(\pi - 2\alpha) = 1 \Leftrightarrow 2\sin\left(\dfrac{\pi - 2\alpha}{2}\right) = 1 \Leftrightarrow$

$\Leftrightarrow 2\sin\left(\dfrac{\pi}{2} - \alpha\right) = 1 \Leftrightarrow 2\cos\alpha = 1 \Leftrightarrow \cos\alpha = \dfrac{1}{2}$

Como $\alpha \in \left]0, \pi\right[$, tem-se que $\alpha = \dfrac{\pi}{3}$.

$\overline{AP} = g\left(\dfrac{\pi}{3}\right) = 2\cos\left(\dfrac{\frac{\pi}{3}}{2}\right) = 2\cos\left(\dfrac{\pi}{6}\right) = 2 \times \dfrac{\sqrt{3}}{2} = \sqrt{3}$

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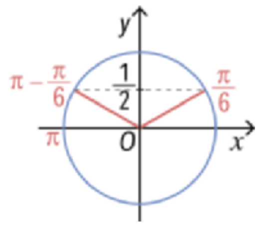
91.1. $f(x) = 1 - 2\cos\left(\dfrac{2x - 3\pi}{2}\right) = 1 - 2\cos\left(x - \dfrac{3\pi}{2}\right) =$

$= 1 - 2\cos\left(x + \dfrac{\pi}{2}\right) = 1 - 2(-\sin x) = 1 + 2\sin x$

91.2. $f(x) = 2 \Leftrightarrow 1 + 2\sin x = 2 \Leftrightarrow 2\sin x = 1 \Leftrightarrow \sin x = \dfrac{1}{2}$



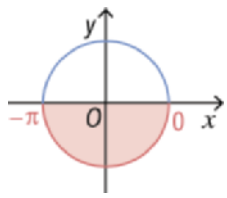
Como $x \in \left]0, 2\pi\right[$, tem-se que: $x = \dfrac{\pi}{6} \vee x = \dfrac{5\pi}{6}$

$A = \left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}$

91.3. $\cos(-\pi - \alpha) = \dfrac{\sqrt{5}}{3} \Leftrightarrow -\cos\alpha = \dfrac{\sqrt{5}}{3} \Leftrightarrow \cos\alpha = -\dfrac{\sqrt{5}}{3}$

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \sin^2\alpha + \left(-\dfrac{\sqrt{5}}{3}\right)^2 = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = 1 - \dfrac{5}{9} \Leftrightarrow \sin^2\alpha = \dfrac{4}{9} \Leftrightarrow \sin\alpha = \pm\dfrac{2}{3}$



Como $\alpha \in \left]-\pi, 0\right[$, tem-se que $\sin\alpha = -\dfrac{2}{3}$.

$f(\alpha) = 1 + 2\sin\alpha = 1 + 2\left(-\dfrac{2}{3}\right) = 1 - \dfrac{4}{3} = -\dfrac{1}{3}$

91.4. $f\left(\frac{\pi}{6}\right)=1+2\sin\frac{\pi}{6}=1+2\times\frac{1}{2}=2\,;\quad A\left(\frac{\pi}{6},2\right)$

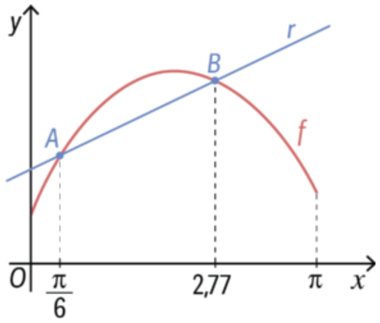
$r: y=\frac{1}{2}x+b$ e $A\in r$

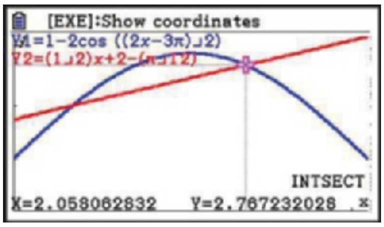
$2=\frac{1}{2}\times\frac{\pi}{6}+b\Leftrightarrow 2-\frac{\pi}{12}=b$

$r: y=\frac{1}{2}x+2-\frac{\pi}{12}$

Pretende-se resolver graficamente a equação

$f(x)=\frac{1}{2}x+2-\frac{\pi}{12}$





$x_B\approx 2{,}06$

92. $f(x)=\frac{\sin x}{1+\cos x}$

92.1. $f\left(\frac{5\pi}{6}\right)=\frac{\sin\left(\frac{5\pi}{6}\right)}{1+\cos\left(\frac{5\pi}{6}\right)}=\frac{\sin\left(\pi-\frac{\pi}{6}\right)}{1+\cos\left(\pi-\frac{\pi}{6}\right)}=$

$=\frac{\sin\frac{\pi}{6}}{1-\cos\frac{\pi}{6}}=\frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}}=\frac{\frac{1}{2}}{\frac{2-\sqrt{3}}{2}}=\frac{1}{2-\sqrt{3}}=$

$=\frac{2+\sqrt{3}}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}=\frac{2+\sqrt{3}}{2^2-\sqrt{3}^2}=$

$=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$

92.2.

$f\left(\frac{\pi}{2}-\alpha\right)=\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{1+\cos\left(\frac{\pi}{2}-\alpha\right)}=\frac{\cos\alpha}{1+\sin\alpha}$

$f\left(\frac{\pi}{2}+\alpha\right)=\frac{\sin\left(\frac{\pi}{2}+\alpha\right)}{1+\cos\left(\frac{\pi}{2}+\alpha\right)}=\frac{\cos\alpha}{1-\sin\alpha}$

$f\left(\frac{\pi}{2}-\alpha\right)\times f\left(\frac{\pi}{2}+\alpha\right)=$

$=\frac{\cos\alpha}{1+\sin\alpha}\times\frac{\cos\alpha}{1-\sin\alpha}=$

$=\frac{\cos^2\alpha}{(1+\sin\alpha)(1-\sin\alpha)}=$

$=\frac{\cos^2\alpha}{1-\sin^2\alpha}=\frac{\cos^2\alpha}{\cos^2\alpha}=1$

92.3. $f(x+1)=f(x)+1\Leftrightarrow\frac{\sin(x+1)}{1+\cos(x+1)}=\frac{\sin x}{1+\cos x}+1$

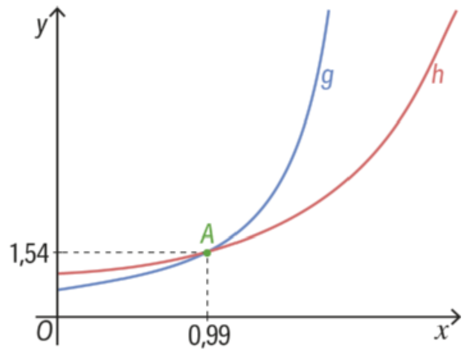
Sejam g e h as funções definidas por

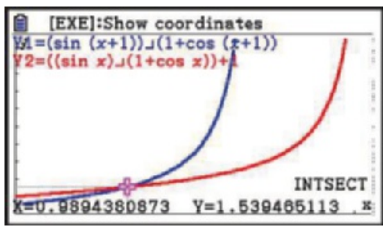
$g(x)=\frac{\sin(x+1)}{1+\cos(x+1)}$ e $h(x)=\frac{\sin x}{1+\cos x}+1$,

respetivamente.

Pretende-se resolver graficamente a equação

$g(x)=h(x)$.





$x_A\approx 0{,}99$

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93.1. $\cos\alpha = \frac{\overline{OP}}{\overline{OA}} \Leftrightarrow \cos\alpha = \frac{\overline{OP}}{r} \Leftrightarrow \overline{OP} = r\cos\alpha$

$\sin\alpha = \frac{\overline{AP}}{\overline{OA}} \Leftrightarrow \sin\alpha = \frac{\overline{AP}}{r} \Leftrightarrow \overline{AP} = r\sin\alpha$

$A_{[OAP]} = \frac{\overline{OP}\times\overline{AP}}{2} = \frac{r\cos\alpha\times r\sin\alpha}{2} = \frac{r^2}{2}\sin\alpha\cos\alpha$

93.2. $3\overline{AP} = \sqrt{3}\,\overline{OP} \Leftrightarrow \frac{\overline{AP}}{\overline{OP}} = \frac{\sqrt{3}}{3} \Leftrightarrow \tan\alpha = \frac{\sqrt{3}}{3}$

Como $\alpha \in \left]0, \frac{\pi}{2}\right[$, tem-se que $\alpha = \frac{\pi}{6}$.

$\overline{OP} = r\cos\left(\frac{\pi}{6}\right) = r\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}r$

Outro processo:

$3\overline{AP} = \sqrt{3}\,\overline{OP} \Leftrightarrow \overline{AP} = \frac{\sqrt{3}}{3}\overline{OP}$

$\overline{OA}^2 = \overline{OP}^2 + \overline{AP}^2 \Leftrightarrow r^2 = \overline{OP}^2 + \left(\frac{\sqrt{3}}{3}\overline{OP}\right)^2 \Leftrightarrow$

$\Leftrightarrow r^2 = \overline{OP}^2 + \frac{3}{9}\overline{OP}^2 \Leftrightarrow r^2 = \overline{OP}^2 + \frac{1}{3}\overline{OP}^2 \Leftrightarrow$

$\Leftrightarrow r^2 = \frac{4}{3}\overline{OP}^2 \Leftrightarrow \overline{OP}^2 = \frac{3}{4}r^2 \underset{(\overline{OP}>0)}{\Leftrightarrow} \overline{OP} = \sqrt{\frac{3}{4}r^2} \Leftrightarrow$

$\underset{(r>0)}{\Leftrightarrow} \overline{OP} = \frac{\sqrt{3}}{2}r$

93.3. $\overline{OP} = \frac{r}{2} \Leftrightarrow r\cos\alpha = \frac{r}{2} \Leftrightarrow \cos\alpha = \frac{1}{2}$

$\alpha \in \left]0, \frac{\pi}{2}\right[$. Então $\alpha \in \frac{\pi}{3}$

$A_{[OAP]} = \frac{r^2}{2}\sin\frac{\pi}{3}\cos\frac{\pi}{3} = \frac{r^2}{2}\times\frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{8}r^2$

93.4. Seja B a projeção ortogonal do ponto P em Ox.

Os triângulos $[OBP]$ e $[OAQ]$ são semelhantes.

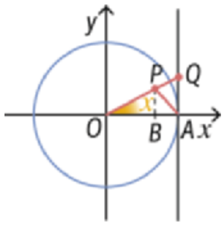
Então, $\frac{\overline{BP}}{\overline{AQ}} = \frac{\overline{OB}}{\overline{OA}} \Leftrightarrow \frac{\overline{BP}}{\tan\alpha} = \frac{\frac{3}{4}}{1} \Leftrightarrow \overline{BP} = \frac{3}{4}\tan\alpha$

$A_{[OAP]} = \frac{1^2}{2}\sin\alpha\cos\alpha \Leftrightarrow \frac{\overline{OA}\times\overline{BP}}{2} = \frac{1}{2}\sin\alpha\cos\alpha \Leftrightarrow$

$\Leftrightarrow 1\times\frac{3}{4}\tan\alpha = \sin\alpha\cos\alpha \Leftrightarrow$

$\Leftrightarrow \frac{3\sin\alpha}{4\cos\alpha} = \sin\alpha\cos\alpha \Leftrightarrow \frac{3}{4} = \frac{\cancel{\sin\alpha}\cos^2\alpha}{\cancel{\sin\alpha}} \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = \frac{3}{4} \Leftrightarrow \cos\alpha = \pm\sqrt{\frac{3}{4}} \Leftrightarrow \cos\alpha = \pm\frac{\sqrt{3}}{2}$



Como $\alpha = \left]0, \frac{\pi}{2}\right[$, tem-se que $\cos\alpha = \frac{\sqrt{3}}{2}$

Então, $\alpha = \frac{\pi}{6}$.

$\overline{BP} = \frac{3}{4}\tan\frac{\pi}{6} = \frac{3}{4}\times\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{4}$

$\alpha = \frac{\pi}{6}$ e a ordenada do ponto P é $\frac{\sqrt{3}}{4}$.

Outro processo:

$\cos\alpha = \frac{\overline{OB}}{\overline{OP}} \wedge \cos\alpha = \frac{\overline{OP}}{\overline{OA}} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \frac{\frac{3}{4}}{\overline{OP}} \wedge \cos\alpha = \frac{\overline{OP}}{1} \Leftrightarrow$

$\Leftrightarrow \overline{OP} = \frac{3}{4\cos\alpha} \wedge \overline{OP} = \cos\alpha \Leftrightarrow \cos\alpha = \frac{3}{4\cos\alpha} \Leftrightarrow$

$\underset{(\cos\alpha\neq 0)}{\Leftrightarrow} 4\cos^2\alpha = 3 \Leftrightarrow \cos^2\alpha = \frac{3}{4} \Leftrightarrow \cos\alpha = \pm\sqrt{\frac{3}{4}} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \pm\frac{\sqrt{3}}{2}$

Como $\alpha \in \left]0, \frac{\pi}{2}\right[$, tem-se que: $\cos\alpha = \frac{\sqrt{3}}{2}$

Então, $\alpha = \frac{\pi}{6}$. $\overline{OP} = \cos\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \frac{\overline{PB}}{\overline{OP}} \Leftrightarrow \sin\frac{\pi}{6} = \frac{\overline{PB}}{\frac{\sqrt{3}}{2}} \Leftrightarrow$

$\Leftrightarrow \overline{PB} = \frac{1}{2}\times\frac{\sqrt{3}}{2} \Leftrightarrow \overline{PB} = \frac{\sqrt{3}}{4}$

$\alpha = \frac{\pi}{6}$ e a ordenada do ponto P é $\frac{\sqrt{3}}{4}$.

93.5. $\overline{OA} = r$; $\overline{AQ} = r\tan\alpha$

$A_{[OAQ]} = \frac{\overline{OA}\times\overline{AQ}}{2} = \frac{r\times r\tan\alpha}{2} = \frac{r^2\tan\alpha}{2}$

$$A_{[AQP]} = A_{[OAQ]} - A_{[OAP]} = \frac{r^2 \tan\alpha}{2} - \frac{r^2}{2}\sin\alpha\cos\alpha =$$

$$= \frac{r^2}{2}\left(\frac{\sin\alpha}{\cos\alpha} - \sin\alpha\cos\alpha\right) =$$

$$= \frac{r^2}{2}\left(\frac{\sin\alpha - \sin\alpha\cos^2\alpha}{\cos\alpha}\right) =$$

$$= \frac{r^2}{2} \times \frac{\sin\alpha\left(1-\cos^2\alpha\right)}{\cos\alpha} = \frac{r^2\sin\alpha\sin^2\alpha}{2\cos\alpha} =$$

$$= \frac{r^2\sin^3\alpha}{2\cos\alpha} = A(\alpha)$$

93.6. $\tan\alpha = \sqrt{8}$

$$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow$$

$$\Leftrightarrow 1+\sqrt{8}^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow 9 = \frac{1}{\cos^2\alpha} \Leftrightarrow$$

$$\Leftrightarrow \cos^2\alpha = \frac{1}{9} \Leftrightarrow \cos\alpha = \pm\frac{1}{3}$$

Como $\alpha = \left]0, \frac{\pi}{2}\right[$, tem-se que $\cos\alpha = \frac{1}{3}$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow \sqrt{8} = \frac{\sin\alpha}{\frac{1}{3}} \Leftrightarrow$$

$$\Leftrightarrow \sin\alpha = \frac{\sqrt{8}}{3} \Leftrightarrow \sin\alpha = \frac{2\sqrt{2}}{3}$$

$$A(\alpha) = \frac{1^2\sin^3\alpha}{2\cos\alpha} = \frac{\left(\frac{2\sqrt{2}}{3}\right)^3}{2\times\frac{1}{3}} = \frac{\frac{16\sqrt{2}}{27}}{\frac{2}{3}} = \frac{8\sqrt{2}}{9} \text{ u. a.}$$

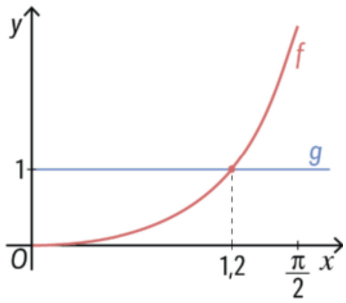
$$A_{[OAP]} = \frac{8\sqrt{2}}{9} \text{ u. a.}$$

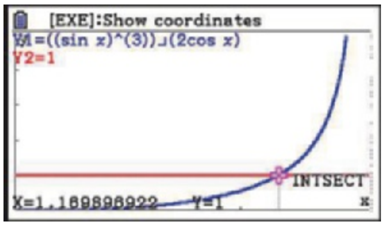
93.7. Pretende-se resolver graficamente a equação

$$A(\alpha) = 1 \Leftrightarrow \frac{\sin^3\alpha}{2\cos\alpha} = 1$$

Sejam f e g as funções definidas por

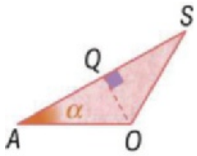
$f(x) = \frac{\sin^3\alpha}{2\cos\alpha}$ e $g(x) = 1$, respetivamente.





$\alpha \approx 1{,}2$ rad

94.1. Seja Q a projeção ortogonal de O sobre $[AS]$.



Como $[AOS]$ é um triângulo isósceles ($\overline{AO} = \overline{OS}$), Q é o ponto médio de $[AS]$.

$$\cos\alpha = \frac{\overline{AQ}}{\overline{AO}} \Leftrightarrow \overline{AQ} = r\cos\alpha$$

$$\overline{AS} = 2\overline{AQ} = 2r\cos\alpha$$

$$P = \overline{AO} + \overline{OS} + \overline{AS} = r + r + 2r\cos\alpha =$$

$$= 2r + 2r\cos\alpha = 2r(1+\cos\alpha)$$

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94.2. $\widehat{BS} + \widehat{SA} = \pi \Leftrightarrow \widehat{BS} + \frac{1}{2}\widehat{BS} = \pi \Leftrightarrow \widehat{BS} = \frac{2\pi}{3}$

$$\alpha = \frac{\widehat{BS}}{2} = \frac{\frac{2\pi}{3}}{2} = \frac{\pi}{3}$$

$$P = 2\times1\left(1+\cos\frac{\pi}{3}\right) = 2\left(1+\frac{1}{2}\right) = 2\times\frac{3}{2} = 3 \text{ u. c.}$$

94.3. $P = 3r \Leftrightarrow 2r(1+\cos\alpha) = 3r \Leftrightarrow 2(1+\cos\alpha) = 3 \Leftrightarrow$

$$\Leftrightarrow 2 + 2\cos\alpha = 3 \Leftrightarrow 2\cos\alpha = 1 \Leftrightarrow \cos\alpha = \frac{1}{2}$$

Como $\alpha = \left]0, \frac{\pi}{2}\right[$, tem-se que $\alpha = \frac{\pi}{3}$.

$$B\hat{O}S = 2\alpha = \frac{2\pi}{3}$$

$$S\left(r\cos\frac{2\pi}{3}, r\sin\frac{2\pi}{3}\right)$$

$$y_s = r\sin\frac{2\pi}{3} = r\sin\left(\pi - \frac{\pi}{3}\right) = r\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}r$$

$$A_{[AOS]} = \frac{\overline{AO}\times y_s}{2} = \frac{r\times\frac{\sqrt{3}}{2}r}{2} = \frac{\sqrt{3}}{4}r^2 \text{ u. a.}$$

94.4. $P = 2+\sqrt{3} \Leftrightarrow 2\times 1(1+\cos\alpha) = 2+\sqrt{3} \Leftrightarrow$

$\Leftrightarrow 2\times\cos\alpha = \sqrt{3} \Leftrightarrow \cos\alpha = \frac{\sqrt{3}}{2}$

Como $\alpha = \left]0, \frac{\pi}{2}\right[$, tem-se que $\alpha = \frac{\pi}{6}$

$\widehat{BS} = 2B\hat{A}S = 2\alpha = 2\times\frac{\pi}{6} = \frac{\pi}{3}$

Comprimento do arco $\widehat{BS} = 2\alpha\times r = \frac{\pi}{3}\times 1 = \frac{\pi}{3}$ u. c.

94.5. a) $\tan\alpha = \frac{\overline{OQ}}{\overline{AO}} \Leftrightarrow \overline{OQ} = r\tan\alpha$

$S\left(r\cos(2\alpha), r\sin(2\alpha)\right)$

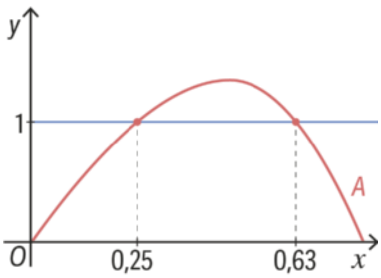
$A_{[OSQ]} = \frac{\overline{OQ}\times x_S}{2} = \frac{r\tan\alpha\times r\cos(2\alpha)}{2} =$

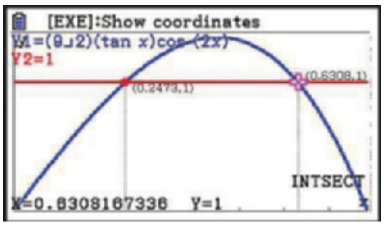
$\underset{(r>0)}{=} \frac{1}{2}r\tan\alpha\times r\cos(2\alpha) = \frac{1}{2}r^2\tan\alpha\cos(2\alpha)$

b) $A(x) = \frac{1}{2}\times 3^2\tan x\cos(2x) = \frac{9}{2}\tan x\cos(2x)$

Pretende-se resolver graficamente a equação

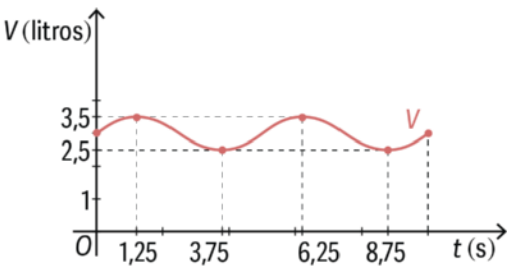
$A(x) = 1$

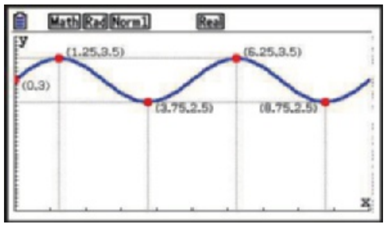




$\alpha \approx 0,25$ rad $\vee\ \alpha \approx 0,63$ rad

95. $v(t) = 3+0,5\sin\left(\frac{2\pi t}{5}\right)$





$8,75-3,75 = 5$ seg.

$6,25-3,75 = 2,5$ seg.

$6,25-2,5 = 1$ litro

Durante os 10 segundos que durou o exame, houve 2 processos de respiração (inspiração e expiração) com a duração de 5 segundos cada. Cada inspiração e cada expiração de ar dos pulmões teve uma duração de 2,5 segundos. O volume máximo de ar nos pulmões do paciente foi 3,5 litros e o volume mínimo foi de 2,5 litros. Em cada processo de respiração, o paciente inspirou um litro de ar.

Máximo On

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Tarefa 1

1.1. $O(0,0)$, $A\left(\frac{\pi}{4},\frac{\sqrt{2}}{2}\right)$, $B\left(\frac{\pi}{2},1\right)$, $C\left(\frac{3\pi}{4},\frac{\sqrt{2}}{2}\right)$ e $D(\pi,0)$

1.2. $\overline{OA} = \sqrt{\left(\frac{\pi}{4}-0\right)^2+\left(\frac{\sqrt{2}}{2}-0\right)^2} \approx 1,057$

$\overline{AB} = \sqrt{\left(\frac{\pi}{2}-\frac{\pi}{4}\right)^2+\left(1-\frac{\sqrt{2}}{2}\right)^2} \approx 0,838$

$\overline{BC} = \sqrt{\left(\frac{3\pi}{4}-\frac{\pi}{2}\right)^2+\left(\frac{\sqrt{2}}{2}-1\right)^2} \approx 0,838$

$\overline{CD} = \sqrt{\left(\pi-\frac{3\pi}{4}\right)^2+\left(0-\frac{\sqrt{2}}{2}\right)^2} \approx 1,057$

$\overline{OA}+\overline{OA}+\overline{OA}+\overline{OA} \approx$

$\approx 1,057+0,838+0,838+1,057 \approx 3,79$

2. **a)** $\frac{\pi}{n}$

b) *delta*

c) $\sin(x2)$

d) $\sqrt{(x2-x1)^2+(y2-y1)^2}$

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3.1.

i	0	1	2	3
$x1$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$x2$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y1$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$
$y2$	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

3.2. a)

```
Número de divisões do intervalo?10
O comprimento arredondado da sinusoide é: 3.815283
```

O comprimento da linha poligonal é $3{,}815283$.

b)

```
Número de divisões do intervalo?20
O comprimento arredondado da sinusoide é: 3.818967
```

O comprimento da linha poligonal é $3{,}818967$

3.3. a)

```
Número de divisões do intervalo?4
O comprimento arredondado da sinusoide é: 3.790091
```

```
Número de divisões do intervalo?5
O comprimento arredondado da sinusoide é: 3.800655
```

Diferença entre os comprimentos das linhas poligonais quando $n=4$ e $n=5$

$3{,}800655-3{,}790091=0{,}010564>10^{-2}=0{,}01$

```
Número de divisões do intervalo?6
O comprimento arredondado da sinusoide é: 3.806608
```

Diferença entre os comprimentos das linhas poligonais quando $n=5$ e $n=6$

$3{,}806608-3{,}800655=0{,}005953<10^{-2}=0{,}01$

Logo, o valor mínimo de n para o qual a diferença entre os comprimentos das linhas poligonais é inferior a 10^{-2}, é $n=6$.

b) Para $n=1305$, tem-se:

```
Número de divisões do intervalo?1305
O comprimento arredondado da sinusoide é: 3.820197
```

Para $n=1306$, tem-se o valor do comprimento arredondado $3{,}820198$ que se mantém inalterado para valores de n ainda maiores.

```
Número de divisões do intervalo?1306
O comprimento arredondado da sinusoide é: 3.820198
```

3.4. Pelo resultado da questão 3.3.b), conclui-se que para valores grandes de n , que é a forma de termos o valor de *delta* cada vez mais pequeno, tem-se que o comprimento da linha poligonal se aproxima de $3{,}82$.

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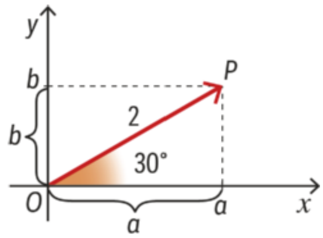
Tarefa 2

1.1. Por exemplo:

a) $A,3;G,45;A,2;G,135;A,4$

b) $G,90;A,2;G,-90;A,3;G,-120;A,3$



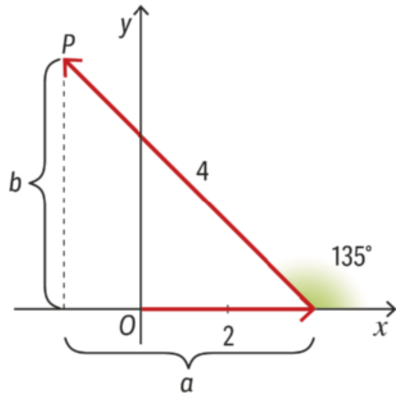
1.2. **a)** $\cos(30°)=\frac{a}{2}\Leftrightarrow a=2\cos(30°)$

Abcissa de P: $0+2\cos(30°)=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$

$\sin(30°)=\frac{b}{2}\Leftrightarrow b=2\sin(30°)$

Ordenada de P: $0+2\sin(30°)=2\times\frac{1}{2}=1$

Logo, as coordenadas de P são: $P(\sqrt{3},1)$



b)

- $\cos(135°)=-\cos(45°)$

$\cos(45°)=\frac{a}{4}\Leftrightarrow a=4\cos(45°)$

Abcissa de P:

$2-4\cos(45°)=2+4\cos(135°)=$

$=2-4\times\frac{\sqrt{2}}{2}=2-2\sqrt{2}$

- $\sin(135°)=\sin(45°)$

$\sin(45°)=\frac{b}{4}\Leftrightarrow b=4\sin(45°)$

Ordenada de P:

$0+4\cos(45°)=4\cos(135°)=$

$=4\times\frac{\sqrt{2}}{2}=2\sqrt{2}$

Logo, as coordenadas de P são: $P\left(2-2\sqrt{2},2\sqrt{2}\right)$

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2.1. **a)** Etapa 2
b) Etapa 1
c) Etapa 3
d) Etapa 4

2.2. **a)** Com a instrução $G,180$, o robô muda o seu sentido de movimento - passa a andar no sentido do semieixo negativo n Ox.

Com a instrução $A,5$, anda para o ponto de coordenadas $(-5;0)$.

Com a instrução $G,120$, muda apenas a sua direção, não alterando a sua posição.

Coordenadas da posição final do robô: $(-5\ ,\ 0)$.

b) direcão $=180+120=300$

2.3.

```
Quantas instruções deseja dar? 5
Instrução 1:
  Tipo (A para andar, G para girar): A
  Valor: 10
Instrução 2:
  Tipo (A para andar, G para girar): G
  Valor: 70
Instrução 3:
  Tipo (A para andar, G para girar): A
  Valor: -1
Instrução 4:
  Tipo (A para andar, G para girar): G
  Valor: -120
Instrução 5:
  Tipo (A para andar, G para girar): A
  Valor: 15
Posição final: (19.3, -12.43)
```

Coordenadas da posição do robô: $(19{,}3;\ -12{,}43)$.

Avaliação global 1

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1. $-1785°=-345°-4\times360°$

$-360°<-345°<-270°$

$-345°\in 1.°$ Quadrante

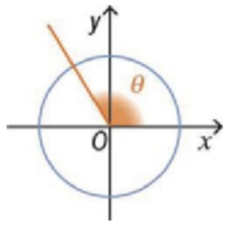
$-1785°\in 1.°$ Quadrante

(A)

2. $\frac{5\pi}{2}-\theta=\frac{4\pi}{2}+\frac{\pi}{2}-\theta=2\pi+\frac{\pi}{2}-\theta$

$\frac{\pi}{2}-\theta\in 4.°$ quadrante

$\frac{5\pi}{2}-\theta\in 4.°$ quadrante



(D)

3. $\alpha+\beta=\pi\Leftrightarrow\alpha=\pi-\beta$

$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha=-\sin(\pi-\beta)=-\sin\beta$

$\cos\beta=-\frac{5}{7}$

$\sin^2\beta+\cos^2\beta=1\Leftrightarrow\sin^2\beta=1-\cos^2\beta\Leftrightarrow$

$\Leftrightarrow\sin^2\beta=1-\left(-\frac{5}{7}\right)^2\Leftrightarrow\sin^2\beta=1-\frac{25}{49}\Leftrightarrow$

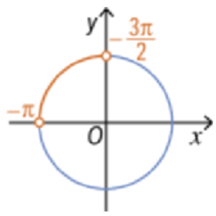
$\Leftrightarrow\sin^2\beta=\frac{24}{49}\Leftrightarrow\sin\beta=\pm\frac{2\sqrt{6}}{7}$

Como β é um ângulo obtuso, $\sin\beta=\frac{2\sqrt{6}}{7}$

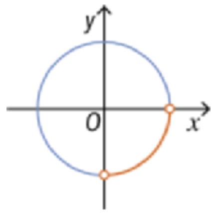
$\cos\left(\frac{\pi}{2}+\beta\right)=-\sin\beta=-\frac{2\sqrt{6}}{7}$

(D)

4. $\alpha\in\left]-\frac{3\pi}{2},-\pi\right[$; $\alpha\in 2.°$ quadrante



$\beta\in\left]-\frac{\pi}{2},0\right[$; $\beta\in 4.°$ quadrante



$\underbrace{\sin\alpha}_{>0}\times\underbrace{\sin\beta}_{<0}<0$

$\underset{<0}{\sin\beta} + \underset{<0}{\tan\alpha} < 0$

$\underset{>0}{\cos\beta} - \underset{<0}{\tan\alpha} > 0$

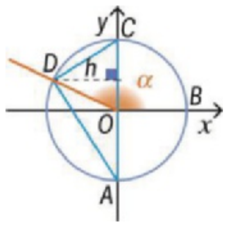
$\underset{>0}{\sin\alpha} \times \underset{<0}{\cos\alpha} < 0$

(C)

5. $A(0,-1);\ B(1,0);\ C(0,1);\ D(\cos\alpha, \sin\alpha)$

$$A_{[ACD]} = \frac{\overline{AC} \times h}{2} = \frac{\overline{AC} \times (-\cos\alpha)}{2} = \frac{-2\cos\alpha}{2} =$$

$= -\cos\alpha$



(B)

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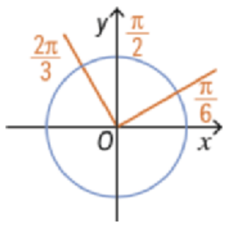
6. $f(x) = \sin x$

$D_f = \left[\frac{\pi}{6}, \frac{2\pi}{3}\right]$



$\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\frac{\pi}{2} = 1$

$\sin\frac{2\pi}{3} = \sin\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\frac{1}{2} \leq \sin x \leq 1$

$D'_f = \left[\frac{1}{2}, 1\right]$

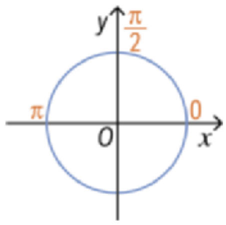
(C)

7. $f(x) = \cos(3x)$

$D'_f = [-1, 1]$

$x \in \left[0, \frac{\pi}{3}\right]$

$3x \in [0, \pi]$



$\cos 0 = 1$; $\cos\frac{\pi}{2} = 0$; $\cos\pi = -1$

A função $f(x) = \cos(3x)$ é decrescente em

$\left[0, \frac{\pi}{3}\right]$.

$f(x + P_0) = f(x)$, $x \in D_f$ e $x + P_0 \in D_f$

$f(x + P_0) = \cos[3(x + P_0)] = \cos(3x + P_0)$

$3P_0 = 2\pi \Leftrightarrow P_0 = \frac{2\pi}{3}$ (2π é o período fundamental da função cosseno)

$x \in \left]-\pi, -\frac{\pi}{2}\right[$

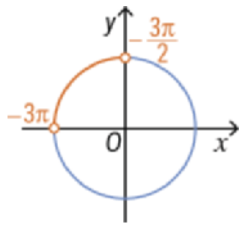
$3x \in \left]-3\pi, -\frac{3\pi}{2}\right[$

$3x \in$ 2.º quadrante

$\cos(3x) < 0$

f é negativa.

(B)



8. $f(x) = 2k + a\cos(ax)$, $a, k \in \mathbb{N} \setminus \{0\}$

$-1 \leq \cos(ax) \leq 1 \Leftrightarrow -a \leq a\cos(ax) \leq a \Leftrightarrow$

$\Leftrightarrow -a + 2k \leq 2k + a\cos(ax) \leq a + 2k$

Como $D'_f = [-2, 8]$,

$$\begin{cases} a + 2k = 8 \\ -a + 2k = -2 \end{cases} \Leftrightarrow \begin{cases} a = 8 - 2k \\ -(8 - 2k) + 2k = -2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a = 8 - 2k \\ -8 + 2k + 2k = -2 \end{cases} \Leftrightarrow \begin{cases} a = 8 - 2k \\ 4k = 6 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a = 8 - 2 \times \frac{3}{2} \\ k = \frac{3}{2} \end{cases} \Leftrightarrow \begin{cases} a = 5 \\ k = \frac{3}{2} \end{cases}$$

$f(x) = 2 \times \frac{3}{2} + 5\cos(5x) \Leftrightarrow f(x) = 3 + 5\cos(5x)$

$f(x + P_0) = f(x)$, $x \in D_f$, $x + P_0 \in D_f$

$f(x + P_0) = 3 + 5\cos(5(x + P_0)) = 3 + 5\cos(5x + 5P_0)$

$5P_0 = 2\pi \Leftrightarrow P_0 = \frac{2\pi}{5}$ (2π é o período fundamental da função cosseno)

Alternativa:

$P_0 = \frac{2\pi}{|5|} = \frac{2\pi}{5}$

(D)

9. $f(x) = \cos(\pi x)$

Se a abcissa de A for $\frac{3}{4}$, a de B tem de ser $\frac{3}{4} + \frac{1}{2}$.

$f\left(\frac{3}{4}\right) = \cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) =$

$= -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$f\left(\frac{3}{4} + \frac{1}{2}\right) = f\left(\frac{5}{4}\right) = \cos\frac{5\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right) =$

$= -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

Como $f\left(\frac{3}{4}\right) = f\left(\frac{3}{4} + \frac{1}{2}\right)$, a abcissa de A é $\frac{3}{4}$.

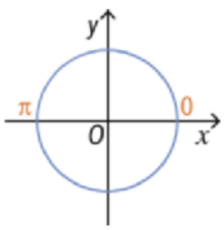
(C)

10. $f(x) = \cos^2 x$

$g(x) = 1$

$f(x) = g(x) \Leftrightarrow \cos^2 x = 1 \Leftrightarrow \cos x = \pm 1$

$[0, 1000\pi]$



$\cos 0 = 1$; $\cos\pi = -1$

No intervalo $[0, 2\pi[$ a equação $\cos^2 x = 1$ tem duas soluções $x = 0 \vee x = \pi$

$[0, 2\pi[$ a equação tem duas soluções

$[2\pi, 4\pi[$ a equação tem duas soluções

$[4\pi, 6\pi[$ a equação tem duas soluções

…

$[998\pi, 1000\pi[$ a equação tem duas soluções

Em cada um dos 500 intervalos, $[0, 2\pi[$, $[2\pi, 4\pi[$, …, $[998\pi, 1000\pi[$ a equação tem duas soluções.

$\cos^2(1000\pi) = \cos^2 0 = 1$

1000π também é solução da equação.

Logo, o número de soluções da equação $\cos^2 x = 1$ no intervalo $[0, 1000\pi[$ é

$500 \times 2 + 1 = 1001$

(D)

Avaliação global 2

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1.1. $P = 2\pi r = 2\pi \times 7 \times 10^3 = 14 \times 10^3 \pi \approx 43\,982$ km

O comprimento da órbita do satélite é aproximadamente 43 982 km.

1.2.

120 minutos — $14 \times 10^3 \pi$

10 minutos — x km

$x = \frac{10 \times 14 \times 10^3 \pi}{120} = \frac{3500\pi}{3}$

$\alpha r = \frac{3500\pi}{3} \Leftrightarrow \alpha \times 7 \times 10^3 = \frac{3500\pi}{3} \Leftrightarrow$

$\Leftrightarrow \alpha = \frac{3500\pi}{21 \times 10^3} \Leftrightarrow$

$\Leftrightarrow \alpha = \frac{\pi}{6}$ rad

O satélite descreve um anglo de $\frac{\pi}{6}$ rad em 10 minutos.

2.1. $A(\cos 45°, \sin 45°) \Leftrightarrow A\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$M(1, \tan 45°) \Leftrightarrow M(1, 1)$

$C(\cos(180° + 45°), \sin(180° + 45°)) \Leftrightarrow$

$\Leftrightarrow C(-\cos 45°, -\sin 45°) \Leftrightarrow C\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

2.2. $B(\cos(180° - 45°), \sin(180° - 45°)) \Leftrightarrow$

$\Leftrightarrow B(-\cos 45°, \sin 45°) \Leftrightarrow B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

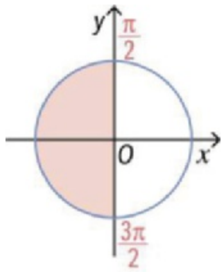
$N(1, \tan(-45°)) \Leftrightarrow N(1, -\tan 45°) \Leftrightarrow N(1, -1)$

3. $\tan(-\alpha)=2 \Leftrightarrow -\tan\alpha = 2 \Leftrightarrow \tan\alpha=-2$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1+(-2)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow 1+4=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{1}{5} \Leftrightarrow$

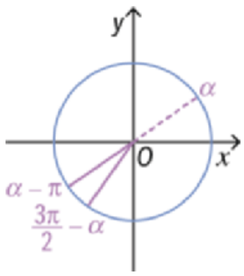
$\Leftrightarrow \cos\alpha = \pm\sqrt{\frac{1}{5}} \Leftrightarrow \cos\alpha = \pm\frac{\sqrt{5}}{5}$



Como $\alpha \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$ tem-se que $\cos\alpha < 0$.

Logo, $\cos\alpha = -\frac{\sqrt{5}}{5}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow -2 = \frac{\sin\alpha}{-\frac{\sqrt{5}}{5}} \Leftrightarrow \sin\alpha = \frac{2\sqrt{5}}{5}$



$\cos(\alpha-\pi)-\cos\left(\frac{3\pi}{2}-\alpha\right) = -\cos\alpha-(-\sin\alpha)=$

$=-\cos\alpha+\sin\alpha = -\left(-\frac{\sqrt{5}}{5}\right)+\frac{2\sqrt{5}}{5} = \frac{\sqrt{5}}{5}+\frac{2\sqrt{5}}{5}=$

$=\frac{3\sqrt{5}}{5}$

4.1. $\tan^2\alpha - \sin^2\alpha = \tan^2\alpha\left(1-\frac{\sin^2\alpha}{\tan^2\alpha}\right)=$

$=\tan^2\alpha\left(1-\frac{\sin^2\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}}\right) = \tan^2\alpha\left(1-\frac{\cancel{\sin^2\alpha}\cos^2\alpha}{\cancel{\sin^2\alpha}}\right)=$

$=\tan^2\alpha\left(1-\cos^2\alpha\right) = \tan^2\alpha\times\sin^2\alpha$

4.2. $\frac{\sin\alpha}{1-\cos\alpha}-\frac{1}{\tan\alpha} = \frac{\sin\alpha}{1-\cos\alpha}-\frac{1}{\frac{\sin\alpha}{\cos\alpha}}=$

$=\frac{\sin\alpha}{\underset{(\times\sin\alpha)}{1-\cos\alpha}}-\frac{\cos\alpha}{\underset{(\times(1-\cos\alpha))}{\sin\alpha}} = \frac{\sin^2\alpha-\cos\alpha(1-\cos\alpha)}{\sin\alpha(1-\cos\alpha)}=$

$=\frac{\sin^2\alpha-\cos\alpha+\cos^2\alpha}{\sin\alpha(1-\cos\alpha)} = \frac{\cancel{1-\cos\alpha}}{\sin\alpha\,\cancel{(1-\cos\alpha)}} = \frac{1}{\sin\alpha}$

5. $f(x)=2-4\sin\left(\frac{x}{3}\right)$

5.1. $a=2$ e $b=-4<0$

$D'_f=[2-4, 2+4]=[-2, 6]$

5.2. $D_f=\mathbb{R}$

Se $x\in D_f$, $x+6\pi\in D_f$.

$f(x+6\pi)=2-4\sin\left(\frac{x+6\pi}{3}\right)=$

$=2-4\sin\left(\frac{x}{3}+2\pi\right)=2-4\sin\left(\frac{x}{3}\right)=f(x)$

Logo, *f* é periódica de período 6π.

5.3. $f(-a)=2-4\sin\left(\frac{-a}{3}\right)=2-4\left(-\sin\left(\frac{a}{3}\right)\right)=$

$=2+4\sin\left(\frac{a}{3}\right)$

$f(a+6\pi)=f(a)=2-4\sin\left(\frac{a}{3}\right)$

$f(-a)-f(a+6\pi)=2+4\sin\left(\frac{a}{3}\right)-\left(2-4\sin\left(\frac{a}{3}\right)\right)=$

$=\cancel{2}+4\sin\left(\frac{a}{3}\right)-\cancel{2}+4\sin\left(\frac{a}{3}\right)=8\sin\left(\frac{a}{3}\right)$

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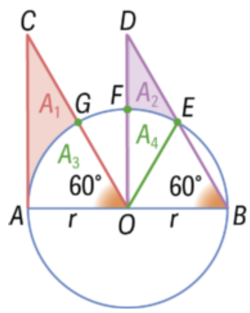
6. Sejam, respetivamente, A_1 e A_2 as áreas dos triângulos $[AOC]$ e $[OBD]$ que não estão contidas no círculo e sejam A_3 e A_4 as áreas dos setores circulares correspondentes aos arcos *GA* e *EF*, respetivamente.



$$\tan 60^\circ = \frac{\overline{AC}}{\overline{AO}} \Leftrightarrow \sqrt{3} = \frac{\overline{AC}}{r} \Leftrightarrow \overline{AC} = \sqrt{3}r$$

$$A_{[AOC]} = A_{[BOD]} = \frac{\overline{AO} \times \overline{AC}}{2} = \frac{r \times \sqrt{3}r}{2} = \frac{\sqrt{3}r^2}{2}$$

$$60^\circ = \frac{\pi}{3} \text{ rad.}$$

$$A_3 = \frac{\frac{\pi}{3}r^2}{2} = \frac{\pi r^2}{6}$$

$$A_1 = A_{[AOC]} - A_3 = \frac{\sqrt{3}r^2}{2} - \frac{\pi r^2}{6} = \frac{\left(3\sqrt{3} - \pi\right)r^2}{6}$$

$$\widehat{EA} = 2E\hat{B}A = 2 \times 60^\circ = 120^\circ$$

$$E\hat{O}F = 120^\circ - 90^\circ = 30^\circ = \frac{\pi}{6} \text{ rad.}$$

$$B\hat{O}E = 90^\circ - 30^\circ = 60^\circ$$

$[OBE]$ é um triângulo equilátero, uma vez que todos os seus ângulos internos são iguais a 60º.

Seja h a altura do triângulo $[OBE]$.

$$\sin 60^\circ = \frac{h}{\overline{OE}} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{h}{r} \Leftrightarrow h = \frac{\sqrt{3}}{2}r$$

$$A_{[OBE]} = \frac{\overline{OB} \times h}{2} = \frac{r \times \frac{\sqrt{3}}{2}r}{2} = \frac{\sqrt{3}r^2}{4}$$

$$A_4 = \frac{\frac{\pi}{6} \times r^2}{2} = \frac{\pi \times r^2}{12}$$

$$A_2 = A_{[OBD]} - A_{[OBE]} - A_4 = \frac{\sqrt{3}r^2}{2} - \frac{\sqrt{3}r^2}{4} - \frac{\pi r^2}{12} =$$

$$= \frac{6\sqrt{3}r^2 - 3\sqrt{3}r^2 - \pi r^2}{12} = \frac{\left(3\sqrt{3} - \pi\right)r^2}{12}$$

$$\frac{A_1}{A_2} = \frac{\frac{\left(3\sqrt{3} - \pi\right)r^2}{6}}{\frac{\left(3\sqrt{3} - \pi\right)r^2}{12}} = \frac{12\cancel{\left(3\sqrt{3} - \pi\right)}\cancel{r^2}}{6\cancel{\left(3\sqrt{3} - \pi\right)}\cancel{r^2}} = 2$$

7. $\cos^2(0^\circ) + \cos^2(1^\circ) + \ldots + \cos^2(89^\circ) + \cos^2(90^\circ) =$

$$= \cos^2(0^\circ) + \cos^2(1^\circ) + \cos^2(2^\circ) + \ldots + \cos^2(45^\circ) +$$

$$+ \cos^2(90^\circ - 44^\circ) + \cos^2(90^\circ - 43^\circ) + \ldots$$

$$\ldots + \cos^2(90^\circ - 1^\circ) + \cos^2(90^\circ - 0^\circ) =$$

$$= \cos^2(0^\circ) + \cos^2(1^\circ) + \cos^2(2^\circ) + \ldots + \left(\frac{\sqrt{2}}{2}\right)^2 +$$

$$+ \sin^2(44^\circ) + \sin^2(43^\circ) + \ldots + \sin^2(1^\circ) + \sin^2(0^\circ) =$$

$$= \cos^2(0^\circ) + \sin^2(0^\circ) + \cos^2(1^\circ) + \sin^2(1^\circ) + \ldots +$$

$$+ \cos^2(43^\circ) + \sin^2(43^\circ) + \cos^2(44^\circ) +$$

$$+ \sin^2(44^\circ) + \frac{2}{4} = \underbrace{1 + 1 + 1 + \ldots + 1 + 1}_{45 \text{ parcelas}} + \frac{1}{2} =$$

$$= 45 \times 1 + 0{,}5 = 45{,}5$$

8.1. $C\left(2\cos\left(\frac{\pi}{2} + \alpha\right), 2\sin\left(\frac{\pi}{2} + \alpha\right)\right) \Leftrightarrow$

$$\Leftrightarrow C\left(-2\sin\alpha, 2\cos\alpha\right)$$

$$B\left(-2\sin\alpha, 2\right)$$

$$\overline{BC} = 2 - 2\cos\alpha$$

$$\overline{OA} = 2$$

$$\overline{AB} = 0 - \left(-2\sin\alpha\right) = 2\sin\alpha$$

$$A_{[OABC]} = \frac{\overline{BC} + \overline{OA}}{2} \times \overline{AB} = \frac{2 - 2\cos\alpha + 2}{2} \times 2\sin\alpha =$$

$$= \left(4 - 2\cos\alpha\right) \times \sin\alpha = 2\left(2 - \cos\alpha\right) \times \sin\alpha =$$

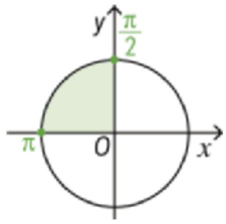
$$= 2\sin\alpha\left(2 - \cos\alpha\right)$$

8.2. $3\tan\left(\pi - \alpha\right) = 4 \Leftrightarrow 3\left(-\tan\alpha\right) = 4 \Leftrightarrow$

$$\Leftrightarrow -3\tan\alpha = 4 \Leftrightarrow \tan\alpha = -\frac{4}{3}$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1 + \left(-\frac{4}{3}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow$$

$$\Leftrightarrow 1 + \frac{16}{9} = \frac{1}{\cos^2\alpha} \Leftrightarrow \frac{25}{9} = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{9}{25}$$



Como $\alpha \in \left]\frac{\pi}{2}, \pi\right[$, tem-se que $\cos\alpha < 0$

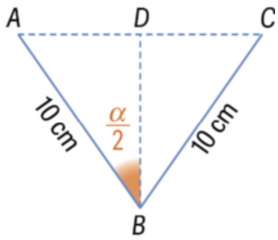
Logo, $\cos\alpha = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$.

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \Leftrightarrow -\frac{4}{3} = \frac{\sin\alpha}{-\frac{3}{5}} \Leftrightarrow \sin\alpha = \frac{4}{5}$$

$$A_{[OABC]} = A(\alpha) = 2 \times \frac{4}{5}\left(2 - \left(-\frac{3}{5}\right)\right) =$$

$$= \frac{8}{5}\left(2 + \frac{3}{5}\right) = \frac{8}{5} \times \frac{13}{5} = \frac{104}{25} \text{ u. a.}$$

9.1. Considere-se a figura que representa a base do prisma triangular (caleira).



$$\sin\frac{\alpha}{2}=\frac{\overline{AD}}{\overline{AB}} \Leftrightarrow \sin\frac{\alpha}{2}=\frac{\overline{AD}}{10} \Leftrightarrow \overline{AD}=10\sin\frac{\alpha}{2}$$

$$\overline{AC}=2\times\overline{AD}=2\times 10\sin\frac{\alpha}{2}=20\sin\frac{\alpha}{2}$$

$$\cos\frac{\alpha}{2}=\frac{\overline{BD}}{\overline{AB}} \Leftrightarrow \cos\frac{\alpha}{2}=\frac{\overline{BD}}{10} \Leftrightarrow \overline{BD}=10\cos\frac{\alpha}{2}$$

$$A_{\text{base}}=\frac{20\sin\frac{\alpha}{2}\times 10\cos\frac{\alpha}{2}}{2}=100\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

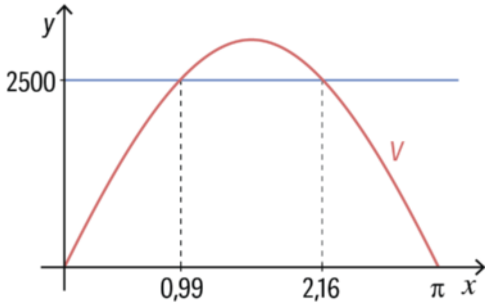
$$V(\alpha)=A_{\text{base}}\times\text{altura}=100\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\times 60=$$

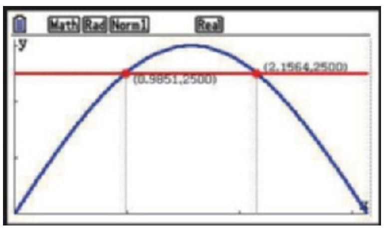
$$=6000\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

9.2. $2{,}5\text{ l}=2{,}5\text{ dm}^3=2500\text{ cm}^3$

Pretende-se resolver graficamente a inequação

$v(\alpha)\geq 2500$





$0{,}99\text{ rad}<\alpha<2{,}16\text{ rad}$.