Trigonometria

Vamos recordar

Ficha 1 Razões trigonométricas

1.1. **a)** [AB]
b) [AC]
c) [BC]

1.2. $\overline{AC}=\sqrt{6^2-(\sqrt{20})^2}=\sqrt{36-20}=\sqrt{16}=4$

1.3. $\sin\alpha=\frac{4}{6}=\frac{2}{3}$; $\cos\alpha=\frac{\sqrt{20}}{6}=\frac{\sqrt{5}}{3}$; $\tan\alpha=\frac{4}{\sqrt{20}}=\frac{2\sqrt{5}}{5}$

1.4. $\alpha=\sin^{-1}\left(\frac{2}{3}\right)=\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)=\tan^{-1}\left(\frac{2\sqrt{5}}{5}\right)\approx 42°$

2.1. $M\hat{R}A=90°-42°=48°$

$\cos 42°=\frac{30}{\overline{AR}} \Leftrightarrow$

$\Leftrightarrow \overline{AR}=\frac{30}{\cos 42°}\approx 40{,}4$ cm

$\tan 42°=\frac{\overline{MR}}{30} \Leftrightarrow \overline{MR}=30\tan 42°\approx 27{,}0$ cm

2.2. $R\hat{O}I=90°-27°=63°$

$\overline{OI}=10\sin 27°\approx 4{,}5$ cm

$\overline{RI}=10\cos 27°\approx 8{,}9$ cm

2.3. $\overline{RU}=\sqrt{6^2+7^2}=\sqrt{85}\approx 9{,}2$ cm

$U\hat{R}A=\tan^{-1}\left(\frac{6}{7}\right)\approx 40{,}6°$

$A\hat{U}R\approx 90°-40{,}6°=49{,}4°$

2.4. $\overline{AI}=\sqrt{18^2-15^2}=\sqrt{99}\approx 9{,}9$ cm

$T\hat{A}I=\sin^{-1}\left(\frac{15}{18}\right)\approx 56{,}4°$

$I\hat{T}A\approx 90°-56{,}4°=33{,}6°$

3.1. $\tan 30°=\frac{\overline{AC}}{6} \Leftrightarrow \overline{AC}=6\tan 30°=6\times\frac{\sqrt{3}}{3}=2\sqrt{3}$ cm

3.2. $\cos 30°=\frac{6}{\overline{BC}} \Leftrightarrow \overline{BC}=\frac{6}{\cos 30°}=\frac{6}{\frac{\sqrt{3}}{2}}=\frac{12}{\sqrt{3}}=\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{12\sqrt{3}}{3}=4\sqrt{3}$ cm

4.1. $\sin^{-1}(\sin 27°)=27°$

4.2. $\cos^{-1}(\sin 27°)=\cos^{-1}(\cos 63°)=63°$

5.1. O triângulo isósceles pode ser decomposto em dois triângulos retângulos iguais a partir da altura, $a=\overline{CD}$. Assim:

$A_{[ABC]}=\frac{b\times a}{2}$; $b=8$; $\tan 70°=\frac{\overline{CD}}{\overline{AD}} \Leftrightarrow \tan 70°=\frac{a}{4} \Leftrightarrow a=4\tan 70°$

Logo, $A_{[ABC]}=\frac{8\times 4\tan 70°}{2}=16\tan 70°\approx 43{,}96$ cm^2

5.2. $P_{[ABC]}=\overline{AB}+\overline{BC}+\overline{AC}$; $\overline{AB}=8$; $\overline{BC}=\overline{AC}$; $\cos 70°=\frac{\overline{AD}}{\overline{AC}} \Leftrightarrow \cos 70°=\frac{4}{x} \Leftrightarrow x=\frac{4}{\cos 70°}$

Logo, $P_{[ABC]}=8+2\times\frac{4}{\cos 70°}\approx 31{,}4$ cm

6.1. $\sin 20°=\frac{5}{\overline{AC}} \Leftrightarrow \overline{AC}=\frac{5}{\sin 20°}$; $P=2\times\frac{5}{\sin 20°}+10\approx 39{,}24$ cm

6.2. Seja a a altura do triângulo relativamente ao lado $[AB]$.

$$\tan 20° = \frac{5}{a} \Leftrightarrow a = \frac{5}{\tan 20°} \; ; \; A = \frac{10 \times \frac{5}{\tan 20°}}{2} \approx 68{,}69 \text{ cm}^2$$

7. $\sin 30° = \frac{10}{\overline{PM}} \Leftrightarrow \frac{1}{2} = \frac{10}{\overline{PM}} \Leftrightarrow \overline{PM} = 20$

Logo, o ornitólogo encontra-se a 20 m da ave.

Ficha 2 Resolução de problemas que envolvam triângulos

pág. 10

1.1. $A = \frac{b \times a}{2}$; $b = r$; $\sin x = \frac{a}{r} \Leftrightarrow a = r \sin x$; Logo, $A(x) = \frac{r \times r \sin x}{2} = \frac{r^2 \sin x}{2}$

1.2. $A(30°) = \frac{10^2 \sin 30°}{2} = \frac{100 \times \frac{1}{2}}{2} = 25 \text{ cm}^2$.

2.1. $\tan 55° = \frac{x + 1{,}4}{\overline{AB}} \Leftrightarrow \overline{AB} = \frac{x + 1{,}4}{\tan 55°}$; $\tan 46° = \frac{x}{\overline{AB}} \Leftrightarrow \overline{AB} = \frac{x}{\tan 46°}$.

Logo, $\frac{x + 1{,}4}{\tan 55°} = \frac{x}{\tan 46°} \Leftrightarrow x = \frac{1{,}4 \tan 46°}{\tan 55° - \tan 46°}$

2.2. Área do triângulo $[ACD] = \frac{\overline{DC} \times \overline{AB}}{2}$; $\tan 46° = \frac{\overline{BC}}{\overline{AB}}$, ou seja, $\overline{AB} = \frac{\overline{BC}}{\tan 46°}$

Como $\overline{BC} = \frac{1{,}4 \tan 46°}{\tan 55° - \tan 46°}$, $\overline{AB} = \frac{1{,}4}{\tan 55° - \tan 46°}$. Logo, $A_{[ADC]} = \frac{1{,}4 \times \overline{AB}}{2} \approx 2{,}5 \text{ cm}^2$.

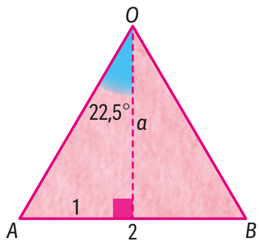
3.1. $A\hat{O}B = \frac{360°}{8} = 45°$

3.2. $V_{\text{pirâmide}} = \frac{1}{3} A_b \times h$; $h = 6$; $A_b = 8A_{[OAB]}$; Seja a a altura do triângulo $[OAB]$.

$\tan 22{,}5° = \frac{1}{a} \Leftrightarrow a = \frac{1}{\tan 22{,}5°}$;

$A_{[OAB]} = \frac{2 \times \frac{1}{\tan 22{,}5°}}{2} = \frac{1}{\tan 22{,}5°}$; $A_b = 8 \times \frac{1}{\tan 22{,}5°} = \frac{8}{\tan 22{,}5°}$;

$V_{\text{pirâmide}} = \frac{1}{3} \times \frac{8}{\tan 22{,}5°} \times 6 \approx 38{,}6 \text{ cm}^3$



pág. 11

4. $B\hat{A}C = 180° - (30° + 45° + 30°) = 75°$, pelo que o triângulo $[ABC]$ é isósceles $(\overline{AB} = \overline{BC} = 3 + \sqrt{3})$.

A projeção ortogonal do ponto D no segmento de reta $[BC]$ é o seu ponto médio. Assim,

$$\cos 30° = \frac{\frac{3+\sqrt{3}}{2}}{\overline{BD}} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{\frac{3+\sqrt{3}}{2}}{\overline{BD}} \Leftrightarrow \overline{BD} = 1 + \sqrt{3}.$$

Logo, $\overline{AD} = 3 + \sqrt{3} - (1 + \sqrt{3}) = 3 + \sqrt{3} - 1 - \sqrt{3} = 2$.

5.1. Os triângulos $[ABC]$ e $[ACD]$ são semelhantes, porque têm dois ângulos iguais:

$B\hat{A}C = D\hat{A}C$ (ângulo comum) e $C\hat{D}A = A\hat{C}B = 90°$.

5.2. Os lados dos triângulos $[ABC]$ e $[ACD]$ têm medidas de comprimento diretamente proporcionais: $\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{BC}}{\overline{CD}} = \frac{\overline{AC}}{\overline{AD}}$.

$\cos 60° = \frac{1}{\overline{AC}} \Leftrightarrow \overline{AC} = \frac{1}{\cos 60°} \Leftrightarrow \overline{AC} = 2$

Como $B\hat{A}C = D\hat{A}C = 90° - 60° = 30°$, $\cos 30° = \frac{2}{\overline{AB}} \Leftrightarrow \overline{AB} = \frac{2}{\cos 30°}$. Logo, $\overline{AB} \approx 2{,}3$ cm.

6. $\tan 72° = \frac{x}{y} \Leftrightarrow x = y \tan 72°$;

$\tan 60° = \frac{x}{30 + y} \Leftrightarrow x = 30 \tan 60° + y \tan 60°$;

Assim: $y \tan 72° = 30 \tan 60° + y \tan 60° \Leftrightarrow$

$\Leftrightarrow y(\tan 72° - \tan 60°) = 30 \tan 60° \Leftrightarrow y = \frac{30 \tan 60°}{\tan 72° - \tan 60°}$;

Logo, $x = \frac{30 \tan 60°}{\tan 72° - \tan 60°} \times \tan 72° \approx 119$ m

7.1. $A = 2 \times \frac{\overline{OC} \times \overline{CB}}{2} = \overline{OC} \times \overline{CB}$; $\overline{OC} = r \cos \alpha$; $\overline{CB} = r \sin \alpha$;

Logo, $A = r \cos \alpha \times r \sin \alpha = r^2 \sin \alpha \cos \alpha$.

7.2. $A = 1^2 \times \sin 45° \times \cos 45° = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

Ficha 3 Circunferência trigonométrica

1.1. **a)** $\dot{O}C$ **b)** $\dot{O}B$ **c)** $\dot{O}D$

1.2. **a)** $960° = 240° + 2 \times 360° = (240°, 2)$; $\dot{O}E$

b) $-2100° = -300° - 5 \times 360° = (-300°, -5)$; $\dot{O}B$

c) $4500° = 180° + 12 \times 360° = (180°, 12)$; $\dot{O}D$

d) $-1680° = -240° - 4 \times 360° = (-240°, -4)$; $\dot{O}C$

2.1. $500° = 140° + 360° = (140°, 1)$; 2.º Q

2.2. $-205°$; 2.º Q

2.3. $1660° = 220° + 4 \times 360° = (220°, 4)$; 3.º Q

2.4. $-1450° = -10° - 4 \times 360° = (-10°, -4)$; 4.º Q

3.1. $100° \in$ 2.º Q ; $\sin \alpha > 0$; $\cos \alpha < 0$

3.2. $-340° \in$ 1.º Q ; $\sin \alpha > 0$; $\cos \alpha > 0$

3.3. $-170° \in$ 3.º Q ; $\sin \alpha < 0$; $\cos \alpha < 0$

3.4. $-700° = -340° - 360° = (-340°, -1) \in$ 1.º Q ; $\sin \alpha > 0$; $\cos \alpha > 0$

4.1. Como $R(\cos 60°, \sin 60°) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, então $Q\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

4.2. $A_{[OAP]} = \frac{\overline{OA} \times \overline{AP}}{2} = \frac{1 \times \tan 60°}{2} = \frac{\sqrt{3}}{2}$

5.1. $\sin(540°) - 2\cos(-315°) + \sin(-630°) = 0 - 2 \times \frac{\sqrt{2}}{2} + 1 = 1 - \sqrt{2}$

Nota: $540° = 180° + 360°$; $-315° + 360° = 45°$; $-630° = -270° - 360° = 90° - 2 \times 360°$

Tem-se: $\sin(180°) - 2\cos(45°) + \sin(90°) = 0 - 2 \times \frac{\sqrt{2}}{2} + 1 = 1 - \sqrt{2}$

5.2. $\cos(780°) - \sin(-540°) - \sin(1170°) = \frac{1}{2} - 0 - 1 = -\frac{1}{2}$

Nota: $780° = 60° + 2 \times 360°$; $-540° = -180° - 2 \times 360°$; $1170° = 90° + 3 \times 360°$

Tem-se: $\cos 60° - \sin(-180°) - \sin(90°) = \frac{1}{2} - 0 - 1 = -\frac{1}{2}$

5.3. $\frac{\tan(-675°) + \sin(-690°)}{\sin(-270°) - \tan(420°)} = \frac{\tan(45°) + \sin(30°)}{\sin(90°) - \tan(60°)} = \frac{1 + \frac{1}{2}}{1 - \sqrt{3}} = \frac{\frac{3}{2}}{1 - \sqrt{3}} = -\frac{3}{2(1 - \sqrt{3})} =$

$= \frac{3}{2 - 2\sqrt{3}} = -\frac{3 + 3\sqrt{3}}{4}$

$-675° = 45° - 2 \times 360°$; $-690° = 30° - 2 \times 360°$; $-270° = 90° - 360°$; $420° = 60° + 360°$

6.1. $R(\cos 30°, \sin 30°) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$; $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, pois P e R são simétricos relativamente a Oy.

$Q\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, pois Q é o simétrico de P pela reflexão central de centro O.

6.2. $T(1, -\tan 30°) = \left(1, -\frac{\sqrt{3}}{3}\right)$; $\overline{QT} = \overline{OT} - 1 = \sqrt{1^2 + \left(-\frac{\sqrt{3}}{3}\right)^2} - 1 = \sqrt{\frac{4}{3}} - 1 = \frac{2\sqrt{3} - 3}{3}$

7.1. $-1 \leqslant \cos\beta \leqslant 1 \Leftrightarrow -5 \leqslant 5\cos\beta \leqslant 5$; Máximo: 5; mínimo: -5.

7.2. $-1 \leqslant \cos\beta \leqslant 1 \Leftrightarrow -2 \leqslant 2\cos\beta \leqslant 2 \Leftrightarrow -5 \leqslant 2\cos\beta - 3 \leqslant -1$; Máximo: -1; mínimo: -5.

7.3. $-1 \leqslant \sin\beta \leqslant 1 \Leftrightarrow 1 \geqslant -\sin\beta \geqslant -1 \Leftrightarrow 4 \geqslant 3 - \sin\beta \geqslant 2 \Leftrightarrow 2 \leqslant 3 - \sin\beta \leqslant 4$; Máximo: 4; mínimo: 2.

7.4. $-1 \leqslant \sin\beta \leqslant 1 \Leftrightarrow 0 \leqslant \sin^2\beta \leqslant 1$; Máximo: 1; mínimo: 0.

7.5. $\tan^2\beta \geqslant 0 \Leftrightarrow \tan^2\beta + 3 \geqslant 3$; Máximo: não existe; mínimo: 3.

8.1. $A_{[OABC]} = \frac{\overline{BC} + \overline{OA}}{2} \times \overline{AB}$; $C(2\cos\alpha, 2\sin\alpha)$; $B(2\cos\alpha, -2\sin\alpha)$; $A(0, -2\sin\alpha)$

$\overline{BC} = 2\sin\alpha - (-2\sin\alpha) = 4\sin\alpha$; $\overline{AB} = 2\cos\alpha$; $\overline{OA} = 0 - (-2\sin\alpha) = 2\sin\alpha$

Logo, $A_{[OABC]} = \frac{4\sin\alpha + 2\sin\alpha}{2} \times 2\cos\alpha = 6\sin\alpha\cos\alpha$

8.2. $\alpha = 750° = 30° + 2 \times 360° = (30°, 2)$

A área do quadrilátero é igual a $6\sin(30°)\cos(30°) = 6 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$.

Ficha 4 Radianos

1.1. $60° = \frac{60° \times \pi \text{ rad}}{180°} = \frac{\pi}{3} \text{ rad}$

1.2. $30° = \frac{30° \times \pi \text{ rad}}{180°} = \frac{\pi}{6} \text{ rad}$

1.3. $45° = \frac{90°}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4} \text{ rad}$

1.4. $240° = \frac{240° \times \pi \text{ rad}}{180°} = \frac{4\pi}{3} \text{ rad}$

1.5. $-540° = \frac{-540° \times \pi \text{ rad}}{180°} = -3\pi \text{ rad}$

1.6. $1170° = \frac{1170° \times \pi \text{ rad}}{180°} = \frac{13\pi}{2} \text{ rad}$

2.1. $\frac{7\pi}{6} \text{ rad} = \frac{7 \times 180°}{6} = 210°$

2.2. $-\frac{5\pi}{3} \text{ rad} = -\frac{5 \times 180°}{3} = -300°$

2.3. $\frac{11\pi}{2} \text{ rad} = \frac{11 \times 180°}{2} = 990°$

2.4. $-\frac{19\pi}{4} \text{ rad} = \frac{-19 \times 180°}{4} = -855°$

2.5. $\frac{3\pi}{5} \text{ rad} = \frac{3 \times 180°}{5} = 108°$

2.6. $-\frac{17\pi}{3} \text{ rad} = -\frac{17 \times 180°}{3} = -1020°$

3.1. $\frac{2\pi}{8} \text{ rad} = \frac{\pi}{4} \text{ rad}$; $\frac{3\pi}{4} \text{ rad} = 3 \times \frac{\pi}{4} \text{ rad}$; Lado extremidade: $\dot{O}G$

3.2. $-\frac{7\pi}{4} \text{ rad} = -7 \times \frac{\pi}{4} \text{ rad}$; Lado extremidade: $\dot{O}E$

3.3. $\frac{5\pi}{4} \text{ rad} = 5 \times \frac{\pi}{4} \text{ rad}$; Lado extremidade: $\dot{O}A$

3.4. $-\frac{\pi}{2} = -2 \times \frac{\pi}{4}$; Lado extremidade: $\dot{O}B$

3.5. $\frac{5\pi}{2} \text{ rad} = 10 \times \frac{\pi}{4} \text{ rad}$; Lado extremidade: $\dot{O}F$

3.6. $-\frac{15\pi}{4}\text{ rad} = -15\times\frac{\pi}{4}\text{ rad}$; Lado extremidade: $\dot{O}E$

3.7. O lado extremidade coincide com o lado origem: $\dot{O}D$

3.8. $-\frac{11\pi}{2}\text{ rad} = -22\times\frac{\pi}{4} = -(2\times8+6)\times\frac{\pi}{4}$ ou $-\frac{11\pi}{2} = -6\pi+\frac{\pi}{2} = -6\pi+2\times\frac{\pi}{4}$; Lado extremidade: $\dot{O}F$

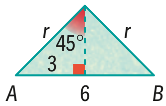
pág. 15

4.1. O arco maior da circunferência tem $\left(2\pi-\frac{\pi}{3}\right)\text{ rad} = \frac{5\pi}{3}\text{ rad}$ de amplitude.

$$\text{Área} = \frac{\frac{5\pi}{3}\times1^2}{2} = \frac{5\pi}{6}\text{ cm}^2$$

4.2. $\overline{OA} = \overline{OB} = 1$; Comprimento do arco maior $= \frac{5\pi}{3}\times1 = \frac{5\pi}{3}$ cm; $P = 2+\frac{5\pi}{3} = \frac{6+5\pi}{3}$ cm

5.1. $\sin 45° = \frac{3}{r} \Leftrightarrow r = \frac{3}{\sin 45°} \Leftrightarrow r = \frac{3}{\frac{\sqrt{2}}{2}} \Leftrightarrow r = 3\sqrt{2}$; Logo, $r = 3\sqrt{2}$ m.



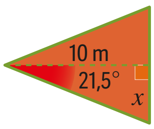
5.2. $A_{\text{secção}} = A_{\text{setor circular}} + A_{\text{triângulo}}$; $A_{\text{setor circular}} = \frac{\left(2\pi-\frac{\pi}{2}\right)\times(3\sqrt{2})^2}{2} = \frac{\frac{3\pi}{2}\times9\times2}{2} = \frac{27\pi}{2}$;

$A_t = \frac{b\times a}{2}$; $b = 6$; $a = 3$ (triângulo isósceles); $A_t = \frac{6\times3}{2} = 9$; Logo, $A_{\text{secção}} = \frac{27\pi}{2}+9 \approx 51{,}41\text{ m}^2$.

6.1. $A_{\text{jardim}} = A_{\text{setor}} - A_{\text{triângulo}}$; $43° = \frac{43°\times\pi\text{ rad}}{180°} = \frac{43\pi}{180°}\text{ rad}$;

$A_s = \frac{\frac{43\pi}{180}\times25^2}{2} = \frac{5375\pi}{72}$; $\tan 21{,}5° = \frac{x}{10} \Leftrightarrow x = 10\tan 21{,}5°$;

$A_t = \frac{20\tan 21{,}5°\times10}{2} = 100\tan 21{,}5°$; $A_{\text{jardim}} = \frac{5375\pi}{72} - 100\tan 21{,}5° \approx 195\text{ m}^2$

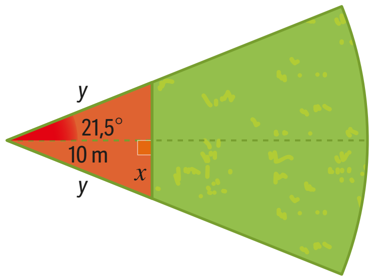


6.2. Comprimento do arco $= \frac{43\pi}{180}\times25 = \frac{215}{36}\pi$;

$\cos 21{,}5° = \frac{10}{y} \Leftrightarrow y = \frac{10}{\cos 21{,}5°}$

Logo, Perímetro $= \frac{215\pi}{36} + 20\tan 21{,}5° + 2\times\left(25-\frac{10}{\cos 21{,}5°}\right) \approx 55{,}1$

Como o valor pedido é, necessariamente, um valor aproximado por excesso, é necessário comprar, no mínimo, 56 metros de rede.



Ficha 5 Fórmulas trigonométricas

pág. 16

1. $\sin^2\alpha+\cos^2\alpha = 1$; $\left(\frac{1}{5}\right)^2+\cos^2\alpha = 1 \Leftrightarrow \frac{1}{25}+\cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = \frac{24}{25}$; Como $\alpha\in 2.°$ Q,

$\cos\alpha < 0$, pelo que $\cos\alpha = -\frac{\sqrt{24}}{5} = -\frac{2\sqrt{6}}{5}$; $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$; $\tan\alpha = \frac{\frac{1}{5}}{-\frac{2\sqrt{6}}{5}} = -\frac{1}{2\sqrt{6}} = -\frac{\sqrt{6}}{12}$

2. $\tan^2\alpha+1 = \frac{1}{\cos^2\alpha}$; $\left(-\frac{3}{2}\right)^2+1 = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{4}{13}$;

Como $\alpha\in 4.°$ Q, $\cos\alpha > 0$, pelo que $\cos\alpha = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$; $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$;

$-\frac{3}{2} = \frac{\sin\alpha}{\frac{2\sqrt{13}}{13}} \Leftrightarrow \sin\alpha = -\frac{3\sqrt{13}}{13}$; $\cos\alpha+\sin\alpha = \frac{2\sqrt{13}}{13}+\left(-\frac{3\sqrt{13}}{13}\right) = -\frac{\sqrt{13}}{13}$

3. $\tan^2\alpha+1=\frac{1}{\cos^2\alpha}$; $\left(\frac{5}{3}\right)^2+1=\frac{1}{\cos^2\alpha} \Leftrightarrow \frac{25}{9}+1=\frac{1}{\cos^2\alpha} \Leftrightarrow \frac{34}{9}=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{9}{34}$

$\sin^2\alpha+\cos^2\alpha=1$; $\sin^2\alpha+\frac{9}{34}=1 \Leftrightarrow \sin^2\alpha=1-\frac{9}{34} \Leftrightarrow \sin^2\alpha=\frac{25}{34}$

Logo, $1-2\cos^2\alpha+\sin^2\alpha=1-2\times\frac{9}{34}+\frac{25}{34}=1+\frac{7}{34}=\frac{41}{34}$

4. $P(\cos\alpha, \sin\alpha)=\left(-\frac{4}{5}, \sin\alpha\right)$. Logo, $\cos\alpha=-\frac{4}{5}$, $\alpha\in\left]\pi, \frac{3\pi}{2}\right[$.

$\sin^2\alpha+\cos^2\alpha=1$; $\sin^2\alpha+\left(-\frac{4}{5}\right)=1 \Leftrightarrow \sin^2\alpha=1-\frac{16}{25} \Leftrightarrow \sin^2\alpha=\frac{9}{25}$

Como $\alpha\in\left]\pi, \frac{3\pi}{2}\right[$, $\sin\alpha<0$, pelo que $\sin\alpha=-\frac{3}{5}$; $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$; $\tan\alpha=\frac{-\frac{3}{5}}{-\frac{4}{5}}=\frac{3}{4}$

5.1. $\frac{1-\tan^2\alpha}{1+\tan^2\alpha}+\sin^2\alpha=\frac{1-\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}+\sin^2\alpha==\frac{\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}+\sin^2\alpha=\cos^2\alpha-\sin^2+\sin^2\alpha=\cos^2\alpha$

5.2. $\sin\alpha\times\cos\alpha\times\left(\tan\alpha+\frac{1}{\tan\alpha}\right)=\sin\alpha\times\cos\alpha\times\frac{\tan^2\alpha+1}{\tan\alpha}=$

$\sin\alpha\times\cos\alpha\times\frac{\frac{1}{\cos^2\alpha}}{\frac{\sin\alpha}{\cos\alpha}}=\sin\alpha\times\cos\alpha\times\frac{1}{\sin\alpha\times\cos\alpha}=1$

5.3. $\left(2-\frac{1}{\cos^2\alpha}\right)(1-\sin^2\alpha)=\frac{2\cos^2\alpha-1}{\cos^2\alpha}\times\cos^2\alpha=2\cos^2\alpha-1$

6. $P(3\cos\theta, 3\sin\theta)=\left(-\frac{9}{5}, 3\sin\theta\right)$

Logo, $3\cos\theta=-\frac{9}{5} \Leftrightarrow \cos\theta=-\frac{9}{15} \Leftrightarrow \cos\theta=-\frac{3}{5}$.

Como $\sin^2\theta+\cos^2\theta=1$, vem: $\sin^2\theta+\left(-\frac{3}{5}\right)^2=1 \Leftrightarrow \sin^2\theta=1-\frac{9}{25} \Leftrightarrow \sin^2\theta=\frac{16}{25}$.

Como $\theta\in 3.^{\text{o}}\,Q$, $\sin\theta<0$, pelo que $\sin\theta=-\frac{4}{5}$.

Assim: $\sin\theta-\cos\theta=-\frac{4}{5}-\left(-\frac{3}{5}\right)=-\frac{4}{5}+\frac{3}{5}=-\frac{1}{5}$

7. $\cos\alpha+\frac{1}{4\cos\alpha}-1=0 \overset{\cos\alpha\neq 0}{\Leftrightarrow} 4\cos^2\alpha-4\cos\alpha+1=0 \Leftrightarrow$

$\Leftrightarrow (2\cos\alpha-1)^2=0 \Leftrightarrow 2\cos\alpha-1=0 \Leftrightarrow \cos\alpha=\frac{1}{2}$

$\sin^2\alpha+\cos^2\alpha=1$;

$\sin^2\alpha+\left(\frac{1}{2}\right)^2=1 \Leftrightarrow \sin^2\alpha+\frac{1}{4}=1 \Leftrightarrow \sin^2\alpha=1-\frac{1}{4} \Leftrightarrow \sin^2\alpha=\frac{3}{4}$

Assim: $\sin\alpha\times\tan\alpha=\sin\alpha\times\frac{\sin\alpha}{\cos\alpha}=\frac{\sin^2\alpha}{\cos\alpha}=\frac{\frac{3}{4}}{\frac{1}{2}}=\frac{\frac{3}{4}}{\frac{2}{4}}=\frac{3}{2}$

8.1. $(1-\sin\alpha)(1+\sin\alpha)+(1-\cos\alpha)(1+\cos\alpha)=1-\sin^2\alpha+1-\cos^2\alpha=1-\sin^2\alpha+\sin^2\alpha=1$

8.2. $\frac{1-\cos^2\alpha}{\tan^2\alpha}=\frac{1-\cos^2\alpha}{\tan^2\alpha}=\frac{\sin^2\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}}=\cos^2\alpha$

8.3. $(\sin\alpha+\cos\alpha)^2-(\sin\alpha-\cos\alpha)^2=\sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha-(\sin^2\alpha-2\sin\alpha\cos\alpha+\cos^2\alpha)=$
$=1+2\sin\alpha\cos\alpha-(1-2\sin\alpha\cos\alpha)=1+2\sin\alpha\cos\alpha-1+2\sin\alpha\cos\alpha=4\sin\alpha\cos\alpha$

8.4. $\frac{\sin\alpha}{1+\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha}=\frac{\sin^2\alpha+1+2\cos\alpha+\cos^2\alpha}{\sin\alpha(1+\cos\alpha)}=\frac{2+2\cos\alpha}{\sin\alpha(1+\cos\alpha)}=\frac{2(1+\cos\alpha)}{\sin\alpha(1+\cos\alpha)}=\frac{2}{\sin\alpha}$

Ficha 6 Redução ao primeiro quadrante

1.1. $\cos\left(\frac{5\pi}{4}\right)+\sin\left(-\frac{3\pi}{4}\right)=\cos\left(\pi+\frac{\pi}{4}\right)-\sin\left(\frac{3\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)-\sin\left(\pi-\frac{\pi}{4}\right)=$
$=-\frac{\sqrt{2}}{2}-\sin\left(\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}=-\sqrt{2}$

1.2. $\frac{\cos\left(-\frac{7\pi}{6}\right)+\sin\left(\frac{5\pi}{3}\right)}{\tan\left(-\frac{\pi}{3}\right)}=\frac{\cos\left(\frac{7\pi}{6}\right)+\sin\left(2\pi-\frac{\pi}{3}\right)}{-\tan\left(\frac{\pi}{3}\right)}=\frac{\cos\left(\pi+\frac{\pi}{6}\right)+\sin\left(-\frac{\pi}{3}\right)}{-\sqrt{3}}=$
$=\frac{-\cos\left(\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{3}\right)}{-\sqrt{3}}=\frac{-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}}{-\sqrt{3}}=\frac{-\sqrt{3}}{-\sqrt{3}}=1$

1.3. $\cos\left(\frac{\pi}{18}\right)+\cos\left(-\frac{17\pi}{18}\right)-\cos\left(-\frac{5\pi}{6}\right)=\cos\left(\frac{\pi}{18}\right)+\cos\left(\pi-\frac{\pi}{18}\right)-\cos\left(\frac{5\pi}{6}\right)=$
$=\cos\left(\frac{\pi}{18}\right)-\cos\left(\frac{\pi}{18}\right)-\cos\left(\pi-\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

1.4. $\frac{\tan\left(-\frac{\pi}{4}\right)+\sin^2\left(-\frac{3\pi}{4}\right)}{\cos\left(-\frac{5\pi}{6}\right)}=\frac{-\tan\left(\frac{\pi}{4}\right)+\sin^2\left(\frac{3\pi}{4}\right)}{\cos\left(\frac{5\pi}{6}\right)}=\frac{-1+\sin^2\left(\pi-\frac{\pi}{4}\right)}{\cos\left(\pi-\frac{\pi}{6}\right)}=$
$=\frac{-1+\sin^2\left(\frac{\pi}{4}\right)}{-\cos\left(\frac{\pi}{6}\right)}=\frac{-1+\left(\frac{\sqrt{2}}{2}\right)^2}{-\frac{\sqrt{3}}{2}}=\frac{-1+\frac{2}{4}}{-\frac{\sqrt{3}}{2}}=\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

2.1. $\cos(\pi-\alpha)-2\sin(\alpha-\pi)+\cos\left(\frac{\pi}{2}-\alpha\right)=$
$=-\cos\alpha+2\sin(\pi-\alpha)+\sin\alpha=-\cos\alpha+2\sin\alpha+\sin\alpha=3\sin\alpha-\cos\alpha$

2.2. $\tan(3\pi+\alpha)+\frac{\cos\left(\frac{\pi}{2}+\alpha\right)}{\sin\left(\frac{\pi}{2}+\alpha\right)}-\tan(-\pi-\alpha)=$
$=\tan(2\pi+\pi+\alpha)+\frac{-\sin\alpha}{\cos\alpha}+\tan(\pi+\alpha)=\tan\alpha-\tan\alpha+\tan\alpha=\tan\alpha$

2.3. $\sin\left(\frac{\pi}{2}-\alpha\right)+\cos(\alpha-3\pi)+2\sin\left(\alpha-\frac{\pi}{2}\right)+3\sin(\pi+\alpha)\times\tan\left(\frac{\pi}{2}-\alpha\right)=$
$=\cos\alpha+\cos(4\pi-3\pi+\alpha)-2\sin\left(\frac{\pi}{2}-\alpha\right)+3\times(-\sin\alpha)\times\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)}=$
$=\cos\alpha+\cos(\pi+\alpha)-2\cos\alpha-3\sin\alpha\times\frac{\cos\alpha}{\sin\alpha}=\cos\alpha-\cos\alpha-2\cos\alpha-3\cos\alpha=-5\cos\alpha$

3. $\tan(-\alpha)=2 \Leftrightarrow -\tan\alpha=2 \Leftrightarrow \tan\alpha=-2$
$\cos(\alpha-\pi)-\cos\left(\frac{3\pi}{2}-\alpha\right)=\cos(\pi-\alpha)+\sin\alpha=-\cos\alpha+\sin\alpha$
$\tan^2\alpha+1=\frac{1}{\cos^2\alpha}$; $(-2)^2+1=\frac{1}{\cos^2\alpha} \Leftrightarrow 5=\frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha=\frac{1}{5}$

Como $\alpha \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$, $\cos\alpha < 0$, pelo que $\cos\alpha = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$; $-2 = \frac{\sin\alpha}{-\frac{\sqrt{5}}{5}} \Leftrightarrow \sin\alpha = \frac{2\sqrt{5}}{5}$

Logo, $-\cos\alpha + \sin\alpha = -\left(\frac{-\sqrt{5}}{5}\right) + \frac{2\sqrt{5}}{5} = \frac{3\sqrt{5}}{5}$

4. $4\cos(\alpha+\pi) = -1 \Leftrightarrow -4\cos\alpha = -1 \Leftrightarrow \cos\alpha = \frac{1}{4}$

$\tan(\alpha - 3\pi) - \sin(-\alpha - \pi) = \tan(4\pi - 3\pi + \alpha) + \sin(\pi + \alpha) = \tan(\pi + \alpha) - \sin\alpha = \tan\alpha - \sin\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$; $\sin^2\alpha + \left(\frac{1}{4}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{1}{16} \Leftrightarrow \sin^2\alpha = \frac{15}{16}$

Como $\alpha \in]\pi, 2\pi[$ e $\cos\alpha > 0$, então $\alpha \in \left]\frac{3\pi}{2}, 2\pi\right[$, onde $\sin\alpha < 0$. Logo, $\sin\alpha = -\frac{\sqrt{15}}{4}$.

Assim, $\tan\alpha - \sin\alpha = \frac{\sin\alpha}{\cos\alpha} - \sin\alpha = \frac{-\frac{\sqrt{15}}{4}}{\frac{1}{4}} - \left(-\frac{\sqrt{15}}{4}\right) = -\sqrt{15} + \frac{\sqrt{15}}{4} = -\frac{3\sqrt{15}}{4}$

5. $\sin\left(\frac{3\pi}{2} - \alpha\right) = \frac{5}{8} \Leftrightarrow -\cos\alpha = \frac{5}{8} \Leftrightarrow \cos\alpha = -\frac{5}{8}$

$\cos\left(-\alpha - \frac{3\pi}{2}\right) + \tan(-\alpha) = \cos\left(\frac{3\pi}{2} + \alpha\right) - \tan\alpha = \sin\alpha - \tan\alpha$;

$\sin^2\alpha + \cos^2\alpha = 1$; $\sin^2\alpha + \left(-\frac{5}{8}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{25}{64} \Leftrightarrow \sin^2\alpha = \frac{39}{64}$;

Como $\alpha \in]-2\pi, -\pi[$ e $\sin\alpha > 0$, pelo que $\sin\alpha = \frac{\sqrt{39}}{8}$

Assim, $\sin\alpha - \tan\alpha = \frac{\sqrt{39}}{8} - \frac{\frac{\sqrt{39}}{8}}{-\frac{5}{8}} = \frac{\sqrt{39}}{8} + \frac{\sqrt{39}}{5} = \frac{13\sqrt{39}}{40}$.

6. $2\alpha - \beta = \frac{3\pi}{2} \Leftrightarrow -\beta = \frac{3\pi}{2} - 2\alpha$

Assim, $\sin(\alpha - \beta) = \sin\left(\alpha + \frac{3\pi}{2} - 2\alpha\right) = \sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos\alpha = -\left(-\frac{4}{5}\right) = \frac{4}{5}$.

Ficha 7 Funções trigonométricas

pág. 20

1.1. $P_0 = \frac{2\pi}{|-2|} = \frac{2\pi}{2} = \pi$; $f = \frac{1}{\pi}$

1.2. $P_0 = \frac{2\pi}{|4|} = \frac{2\pi}{4} = \frac{\pi}{2}$; $f = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$

1.3. $P_0 = \frac{\pi}{\left|\frac{1}{3}\right|} = \frac{\pi}{\frac{1}{3}} = 3\pi$; $f = \frac{1}{3\pi}$

1.4. $P_0 = \frac{2\pi}{|a|} = \frac{2\pi}{-a}$; $f = \frac{1}{\frac{2\pi}{-a}} = -\frac{a}{2\pi}$

2.1. Como $x \in \mathbb{R}$, então $\frac{x}{3} \in \mathbb{R}$. Assim, tem-se: $-1 \leqslant \sin\left(\frac{x}{3}\right) \leqslant 1 \Leftrightarrow$

$\Leftrightarrow 4 \geqslant -4\sin\left(\frac{x}{3}\right) \geqslant -4 \Leftrightarrow 6 \geqslant 2 - 4\sin\left(\frac{x}{3}\right) \geqslant -2 \Leftrightarrow -2 \leqslant f(x) \leqslant 6$; Logo, $D'_f = [-2, 6]$.

2.2. $f(x) = 6 \Leftrightarrow 2 - 4\sin\left(\frac{x}{3}\right) = 6 \Leftrightarrow \sin\left(\frac{x}{3}\right) = -1 \Leftrightarrow$

$\Leftrightarrow \frac{x}{3} = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{9\pi}{2} + 6k\pi, k \in \mathbb{Z}$ (máx.);

$f(x) = -2 \Leftrightarrow 2 - 4\sin\left(\frac{x}{3}\right) = -2 \Leftrightarrow \sin\left(\frac{x}{3}\right) = 1 \Leftrightarrow \frac{x}{3} = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{3\pi}{2} + 6k\pi, k \in \mathbb{Z}$ (mín.)

2.3. Sendo $x \in D_f$, então $x + 6\pi \in D_f$, porque $D_f = \mathbb{R}$.

$f(x+6\pi) = 2 - 4\sin\left(\frac{x+6\pi}{3}\right) = 2 - 4\sin\left(\frac{x}{3} + 2\pi\right) = 2 - 4\sin\left(\frac{x}{3}\right) = f(x)$;

Logo, 6π é período da função f.

2.4. $f(-a) - f(a+6\pi) = f(-a) - f(a) =$

$= 2 - 4\sin\left(\frac{-a}{3}\right) - \left[2 - 4\sin\left(\frac{a}{3}\right)\right] = 2 + 4\sin\left(\frac{a}{3}\right) - 2 + 4\sin\left(\frac{a}{3}\right) = 8\sin\left(\frac{a}{3}\right)$

3.1. Como $x \in \mathbb{R}$, então $4x \in \mathbb{R}$. Assim, tem-se:

$-1 \leqslant \cos(4x) \leqslant 1 \Leftrightarrow -2 \leqslant 2\cos(4x) \leqslant 2 \Leftrightarrow$

$\Leftrightarrow 8 \leqslant 10 + 2\cos(4x) \leqslant 12 \Leftrightarrow 8 \leqslant g(x) \leqslant 12$. Logo, $D'_g = [8, 12]$.

3.2. $g(x) = 12 \Leftrightarrow 10 + 2\cos(4x) = 12 \Leftrightarrow \cos(4x) = 1 \Leftrightarrow 4x = 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{k\pi}{2},\ k \in \mathbb{Z}$

3.3. Como $0 \notin D'_g$, então a função g não tem zeros.

3.4. Se $x \in D_g$, então $x + \frac{\pi}{2} \in D_g$, porque $D_g = \mathbb{R}$.

$g\left(x + \frac{\pi}{2}\right) = 10 + 2\cos\left[4\left(x + \frac{\pi}{2}\right)\right] = 10 + 2\cos(4x + 6\pi) = 10 + 2\cos(4x) = g(x)$;

Logo, $\frac{\pi}{2}$ é período de g.

3.5. O gráfico de g pode ser obtido do gráfico da função cosseno pela contração horizontal de coeficiente $\frac{1}{4}$, seguido da dilatação vertical de coeficiente 2 e pela translação de vetor $\vec{u}(0, 10)$.

4.1. $D_f = \left\{x \in \mathbb{R} : \frac{\pi}{6} - 2x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\} = \mathbb{R} \setminus \left\{-\frac{\pi}{6} - \frac{k\pi}{2},\ k \in \mathbb{Z}\right\}$ $\left(\text{ou } \mathbb{R} \setminus \left\{-\frac{\pi}{6} + \frac{k\pi}{2},\ k \in \mathbb{Z}\right\}\right)$

C.A.: $\frac{\pi}{6} - 2x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow -2x = \frac{\pi}{2} - \frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow -2x = \frac{\pi}{3} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$x = -\frac{\pi}{6} - \frac{k\pi}{2},\ k \in \mathbb{Z}$ (ou $x = -\frac{\pi}{6} + \frac{k\pi}{2},\ k \in \mathbb{Z}$)

4.2. $P_0 = \frac{\pi}{|-2|} = \frac{\pi}{2}$; $f = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$

4.3. $f(x) = 0 \Leftrightarrow \frac{1}{2}\tan\left(\frac{\pi}{6} - 2x\right) = 0 \Leftrightarrow \tan\left(\frac{\pi}{6} - 2x\right) = 0 \Leftrightarrow \frac{\pi}{6} - 2x = k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow -2x = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{12} - \frac{k\pi}{2},\ k \in \mathbb{Z}$ (ou $x = \frac{\pi}{12} + \frac{k\pi}{2},\ k \in \mathbb{Z}$)

5.1. $f(11) = 24{,}5 + 2{,}5\cos\left[\pi\left(\frac{11+9}{12}\right)\right] = 24{,}5 + 2{,}5\cos\left(\frac{20\pi}{12}\right) = 24{,}5 + 2{,}5\cos\left(\frac{5\pi}{3}\right) =$

$= 24{,}5 + 2{,}5\cos\left(2\pi - \frac{\pi}{3}\right) = 24{,}5 + 2{,}5\cos\left(\frac{\pi}{3}\right) =$

$= 24{,}5 + 2{,}5 \times \frac{1}{2} = 25{,}75$ °C

5.2. $\forall t \in [0, 24]: -1 \leqslant \cos\left[\frac{\pi(t+9)}{12}\right] \leqslant 1 \Leftrightarrow -2{,}5 \leqslant 2{,}5\cos\left[\frac{\pi(t+9)}{12}\right] \leqslant 2{,}5 \Leftrightarrow$

$\Leftrightarrow 22 \leqslant 24{,}5 + 2{,}5\cos\left[\frac{\pi(t+9)}{12}\right] \leqslant 27$

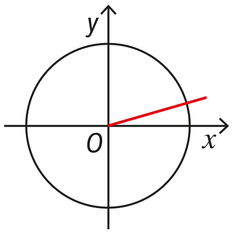
Logo, $\forall t \in [0, 24]$, $22 < f(t) \leqslant 27$, pelo que as temperaturas mínimas e máximas registadas nesse dia foram 22 °C e 27 °C, respetivamente.

6. $S =]0; 2{,}1[\cup]2\pi; 10{,}5[$

Avaliação global do tema

Ficha 8 Avaliação global do tema

1. **(A)**



$-1785° = -345° - 4 \times 360° = (-345°, -4) \in 1.° \text{ Q}$

2. **(C)** 3,53
Valor obtido através da calculadora.

3. **(C)** $\cos(2x) < 0$

$\tan x < 0$; $\sin x > 0$; $\frac{\pi}{2} < x < \frac{3\pi}{4} \Leftrightarrow \pi < 2x < \frac{3\pi}{2}$; $\sin(2x) < 0$; $\cos(2x) < 0$

4. **(B)** $a = 2$ e $c = 0{,}5$

5. **(A)** $-\frac{\sqrt{3}}{3}$

$\tan\left(\frac{17\pi}{6}\right) = \tan\left(2\pi + \pi - \frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

6. $\sin\left(-\frac{\pi}{2} - \alpha\right) = \frac{\sqrt{6}}{3} \Leftrightarrow -\sin\left(\frac{\pi}{2} + \alpha\right) = \frac{\sqrt{6}}{3} \Leftrightarrow -\cos\alpha = \frac{\sqrt{6}}{3} \Leftrightarrow \cos\alpha = -\frac{\sqrt{6}}{3}$

$\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha}$; $\tan^2\alpha + 1 = \frac{1}{\left(-\frac{\sqrt{6}}{3}\right)^2} \Leftrightarrow \tan^2\alpha = \frac{1}{2}$

Como $\alpha \in \left]\pi, \frac{3\pi}{2}\right[$, $\tan\alpha > 0$, pelo que $\tan\alpha = \frac{\sqrt{2}}{2}$. Assim, $1 - \frac{\sqrt{2}}{\tan\alpha} = 1 - \frac{\sqrt{2}}{\frac{\sqrt{2}}{2}} = 1 - 2 = -1$

7.1. $\frac{2\pi}{6} = \frac{\pi}{3}$; $A\left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $P\left(1, \tan\frac{\pi}{3}\right) = \left(1, \sqrt{3}\right)$

7.2. $A = 6 \times \frac{1 \times \frac{\sqrt{3}}{2}}{2} = \frac{3\sqrt{3}}{2}$ u. a.

8. $\frac{\cos x - 1}{\sin x} + \frac{\sin x}{1 + \cos x} = \frac{\cos^2 x - 1 + \sin^2 x}{\sin x(1 + \cos x)} = \frac{1 - 1}{\sin x(1 + \cos x)} = \frac{0}{\sin x(1 + \cos x)} = 0$

9.1. $D_f = \left\{x \in \mathbb{R} : \frac{2x + \pi}{3} \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\} = \left\{x \in \mathbb{R} : x \neq \frac{\pi}{4} + \frac{3k\pi}{2}, k \in \mathbb{Z}\right\}$; $D' = \mathbb{R}$

9.2. $1 - \tan\left(\frac{2x + \pi}{3}\right) = 1 \Leftrightarrow \tan\left(\frac{2x + \pi}{3}\right) = 0 \Leftrightarrow x = -\frac{\pi}{2} + \frac{3k\pi}{2}, k \in \mathbb{Z}$

9.3. $P_0 = \frac{\pi}{\frac{2}{3}} = \frac{3\pi}{2}$; $f = \frac{1}{\frac{3\pi}{2}} = \frac{2}{3\pi}$

10. $A(x) = \frac{2\sin x - 2\tan x + 2\sin x}{2} \times 2 = 4\sin x - \frac{2\sin x}{\cos x} = \frac{4\sin x\cos x - 2\sin x}{\cos x}$

11. $A(-0{,}52; 6)$; $B(1{,}57; 3)$; $\overline{AB} = \sqrt{(1{,}57 + 0{,}52)^2 + (3 - 6)^2} \approx 3{,}7$